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Q. Show that for an otto cycle in which the pts are 1, 2, 3 and 4 and the upper & lower temperature limit are T_3 & T_1 resp, for maximum work per kg of air to be done the intermediate temp is given by

$$\boxed{T_2 = T_4 = \sqrt{T_1 T_3}}$$

Sol^n Work done per kg in an otto cycle is given by,

$$W = C_v (T_3 - T_2) - C_v (T_4 - T_1)$$

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} = (R_c)^{\gamma-1}$$

$$\Rightarrow T_2 = T_1 (R_c)^{\gamma-1}$$

Similarly,

$$T_3 = T_4 (R_c)^{\gamma-1}$$

$$W = C_v \left[\left(T_3 - T_1 (R_c)^{\gamma-1} \right) - \frac{T_3}{R_c^{\gamma-1}} + T_1 \right]$$

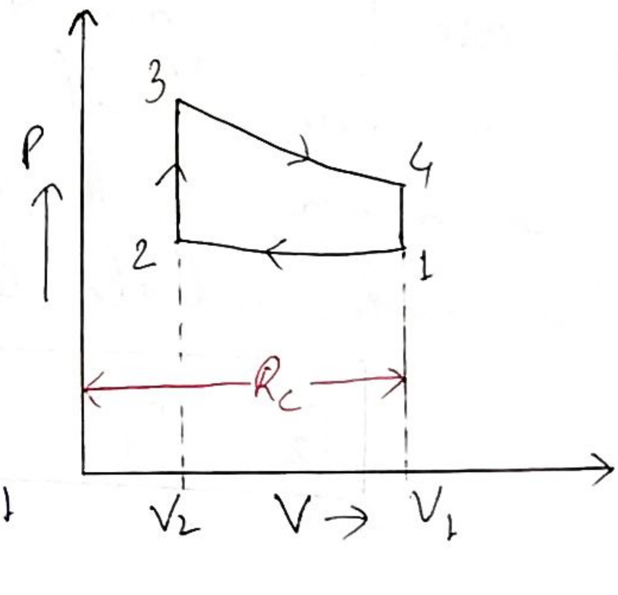
In the above eqt^n, an upper and lower tempt^r are fixed. Therefore, W is a function of R_c. For maximum value of W,

$$\frac{dW}{dR_c} = -T_1(\gamma-1)(R_c)^{\gamma-2} - T_3(1-\gamma)(R_c)^{-\gamma} = 0$$

$$T_3 (R_c)^{-\gamma} = T_1 (R_c)^{\gamma-2}$$

$$\Rightarrow \frac{T_3}{T_1} = (R_c)^{2(\gamma-1)}$$

$$\Rightarrow R_c = \left(\frac{T_3}{T_1} \right)^{\frac{1}{2(\gamma-1)}}$$



$$\Rightarrow \left(\frac{T_2}{T_1} \right)^{\frac{1}{\gamma-1}} = \left(\frac{T_3}{T_1} \right)^{\frac{1}{2(\gamma-1)}}$$

$$\Rightarrow \frac{T_2}{T_1} = \left(\frac{T_3}{T_1} \right)^{\frac{1}{2(\gamma-1)} \times \gamma-1}$$

$$\Rightarrow \frac{T_2}{T_1} = \left(\frac{T_3}{T_1} \right)^{1/2}$$

$$\Rightarrow \frac{T_2^2}{T_1^2} = \frac{T_3}{T_1}, \Rightarrow T_2^2 = T_3 T_1, \Rightarrow T_2 = \sqrt{T_3 T_1}$$

Similarly,

$$T_3 = T_4 (R_C)^{\gamma-1}$$

$$\Rightarrow T_3 = T_4 \left(\frac{T_3}{T_1}\right)^{\frac{\gamma-1}{2(\gamma-1)}}$$

$$\Rightarrow \frac{T_3^2}{T_4^2} = \frac{T_3}{T_1}$$

$$\Rightarrow T_4^2 = T_3 T_1$$

$$\Rightarrow T_4 = \sqrt{T_3 T_1}$$

$$T_2 = T_4 = \sqrt{T_3 T_1}.$$

Hence proved.

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