ME181403: Fluid Mechanics - I

Module 2: Pressure and Fluid Statics

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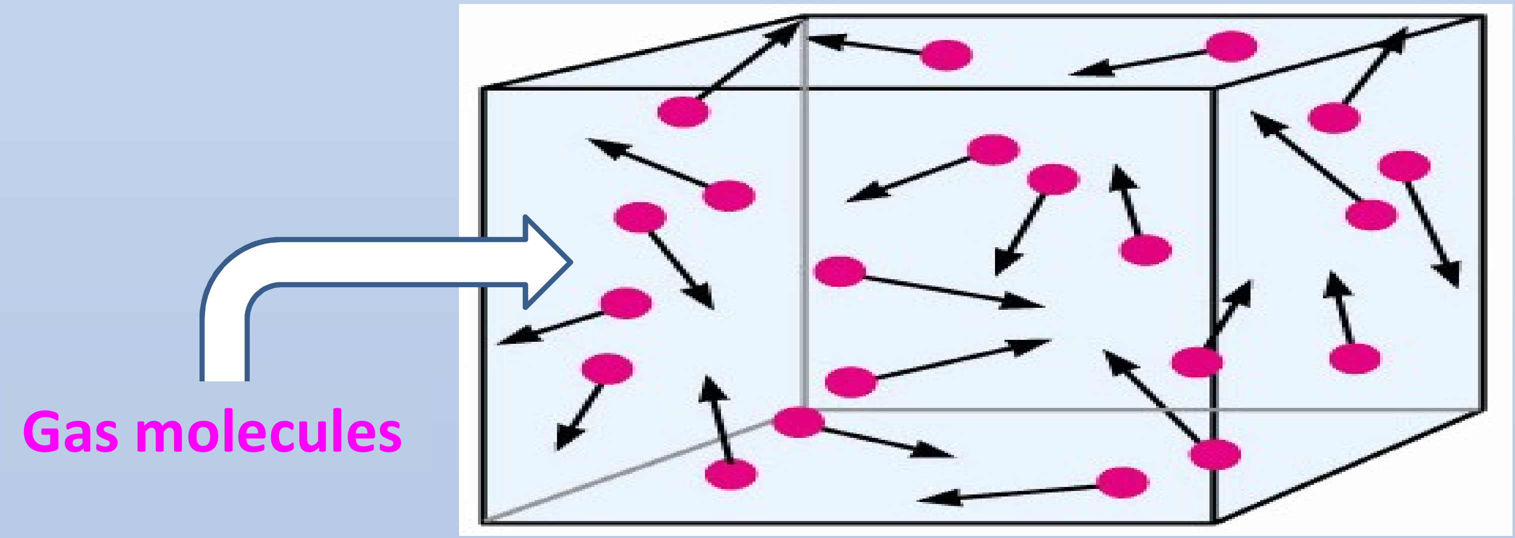
Module 2: Pressure and Fluid Statics

- Define pressure
- Pressure at a point
- The manometer, other pressure measuring devices
- Hydrostatic forces on submerged plane and curved surfaces
- Buoyancy
- Stability of floating and submerged bodies

❖ Pressure

- At the molecular level, pressure in a fluid at any point represents the impact of force on the surface due to molecular collisions per unit area.



- For example, how is the pressure created inside a container (see the Figure)? The gas molecules inside the container will collide with the inside wall of the container. Every collision between a gas molecule and the inside of the wall of the container creates a tiny force pushing on the wall in the outward direction.
- What do you think about the pressure if there are no gas molecules inside the container?

Pressure (cont.)

- If F denotes the compressive or normal force exerted by a fluid on an area A, then pressure P is given by

$$P = \frac{F}{A}$$

- Pressure has the unit of Newton per square metre (N/m^2), which is known as Pascal (Pa), i.e.,

$$1\ N/m^2 = 1\ Pa.$$

- Other commonly used units of pressure in practice are *bar* and *standard atmosphere* and they are related to Pa given by

$$1\ \text{bar} = 10^5 \text{Pa}$$

And, $1\ atm = 101325\ Pa = 1.01325\ bar$

Pressure (cont.)

- Absolute pressure: It is the actual pressure at a given location. It is measured from the absolute zero pressure or no pressure. Absolute zero pressure is also called absolute vacuum.

- Atmospheric pressure: It is the pressure due to the atmosphere of the earth. The pressure at the sea level is considered as the standard atmospheric pressure and its value is 101325 *Pa*.

- Gage pressure: It is the pressure measured in a pressure gage which is the difference between the absolute pressure and atmospheric pressure. This is what we see in most of the pressure measuring devices (e.g.: measuring air pressure inside a tire) which are calibrated to read the atmospheric pressure as zero.

Pressure (cont.)

- The relation among the gage pressure, absolute pressure and atmospheric pressure can be expressed as

$$P_{gage} = P_{abs} - P_{atm}$$

where,

P_{gage} - gage pressure

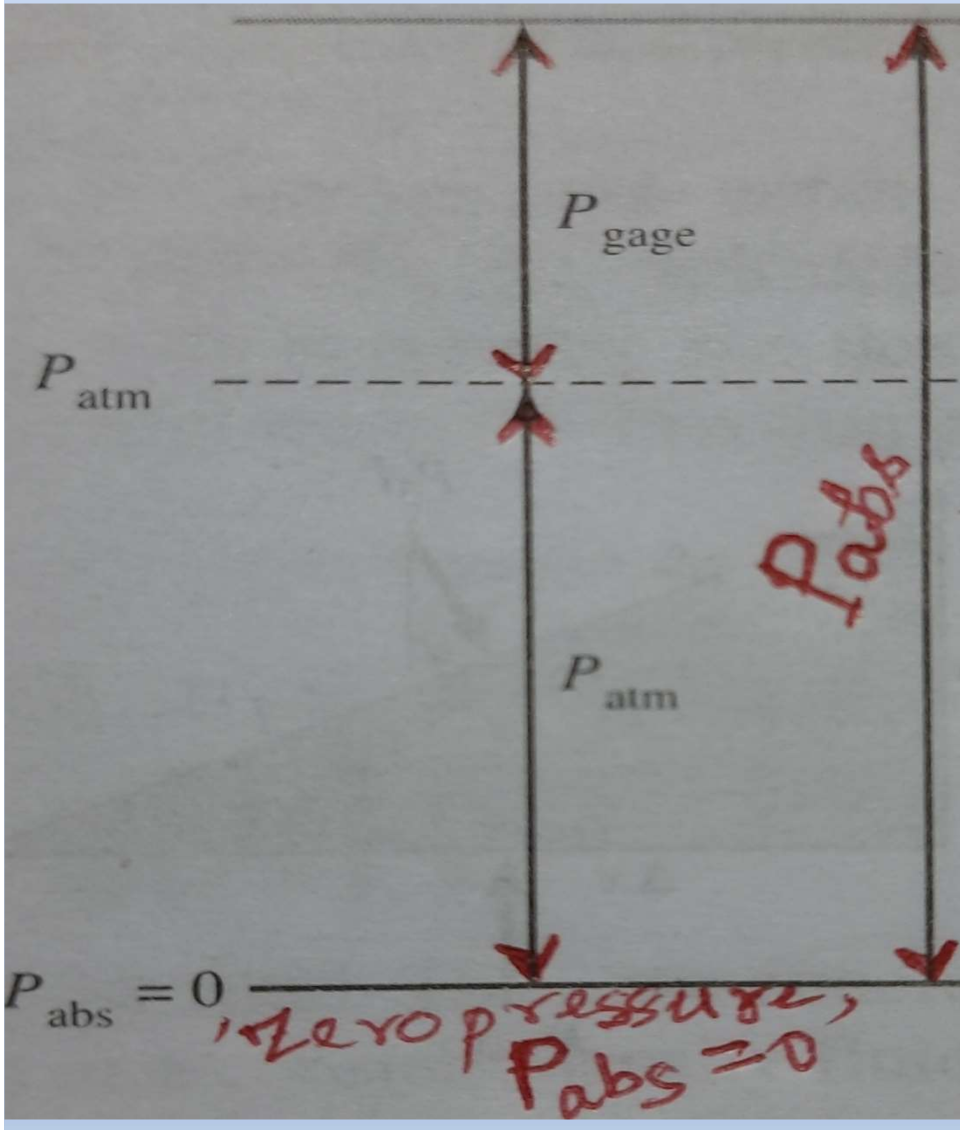
P_{abs} - absolute pressure

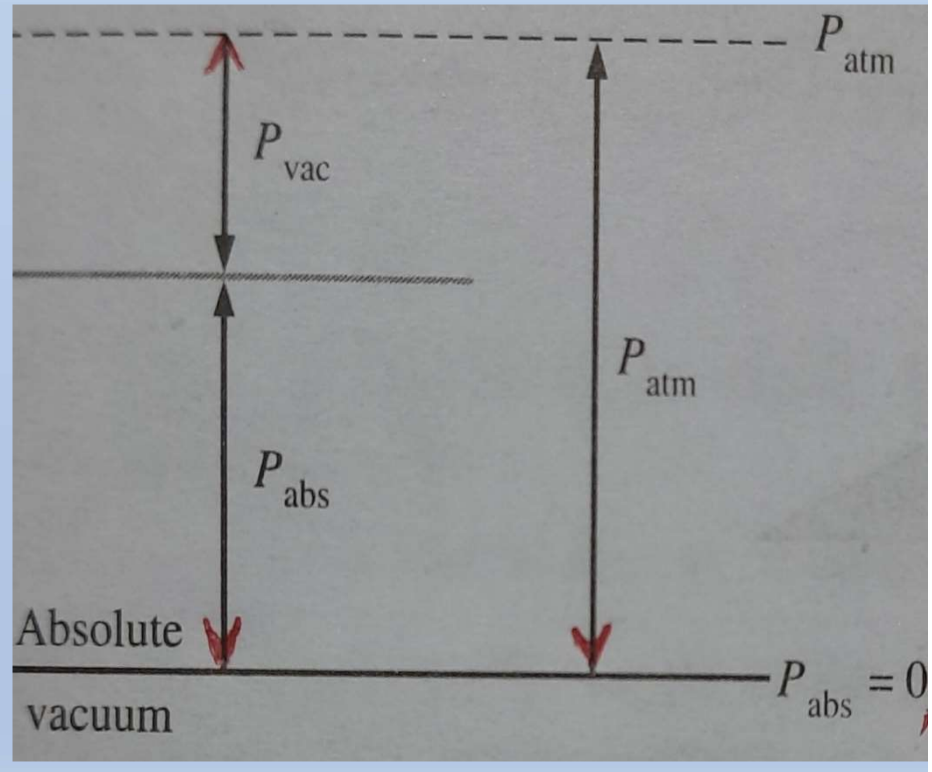
and, P_{atm} - atmospheric pressure

- Vacuum gage pressure (P_{vac}): It is the pressure of vacuum measured in a pressure gage which is the difference between the atmospheric pressure and absolute pressure. This pressure is always below the atmospheric pressure. Their relation can be expressed as

$$P_{vac} = P_{atm} - P_{abs}$$

Pressure (cont.)





(a) $P_{gage} = P_{abs} - P_{atm}$

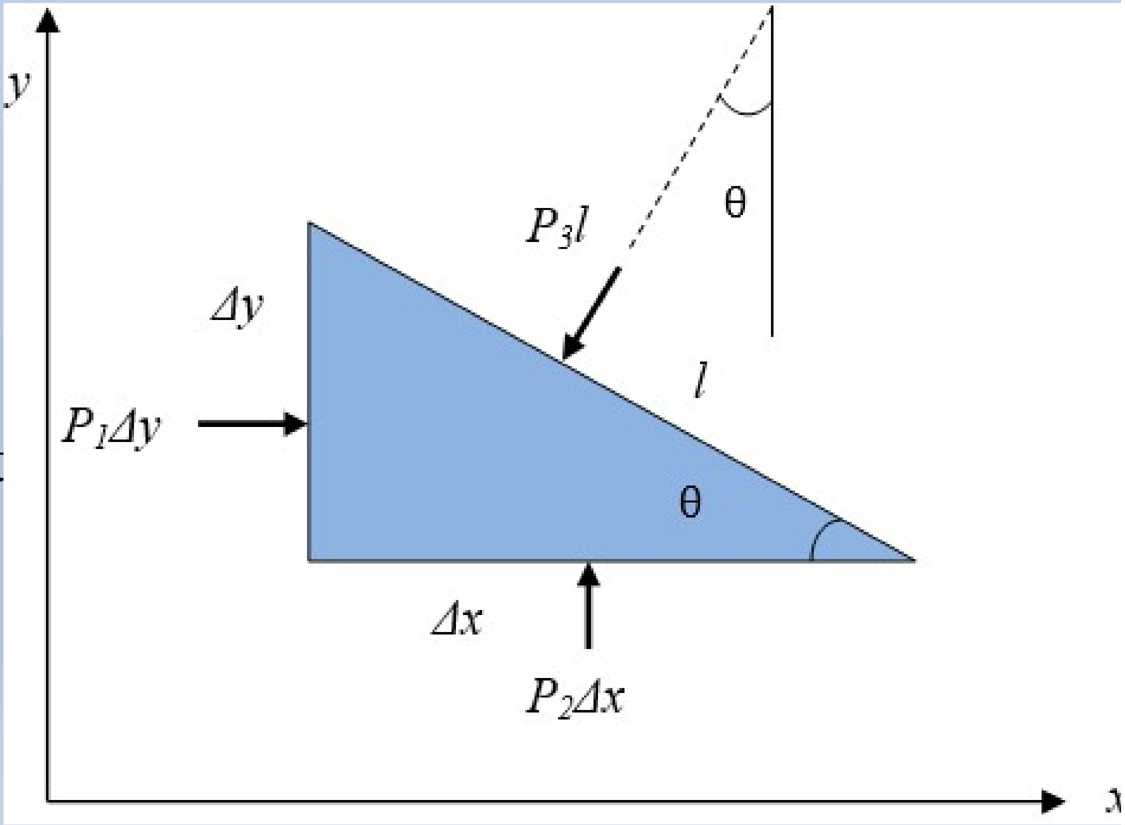
(b) $P_{vac} = P_{atm} - P_{abs}$

❖ **Pressure at a point**

- Pressure at any point in a fluid is the same in all directions. That is, it has magnitude but not a specific direction and thus it is a scalar quantity.
- This can be demonstrated by considering a small wedge-shaped fluid element of unit depth ($\Delta z = 1\ unit$) having density ρ in equilibrium as shown in the figure.

Let, the mean pressures at the three surfaces are P_1, P_2, and P_3.

Let, $W = mg = \rho g \Delta x \Delta y/2$ be the weight of the fluid element where m is the mass of the fluid element.



Pressure at a point (cont.)

The force acting on a surface is the product of mean pressure and the corresponding surface area. At equilibrium doing force balance in the *x* and *y* directions give,

$$\sum F_x = 0 \Rightarrow P_1 \Delta y - P_3 l \sin\theta = 0$$

$$\sum F_y = 0 \Rightarrow P_2 \Delta x - P_3 l \cos\theta - \frac{1}{2}\rho g \Delta x \Delta y = 0$$

The wedge is a right angled triangle, therefore we have $\Delta x = l \cos\theta$ and $\Delta y = l \sin\theta$. Substituting these geometric relations and dividing the above two equations by Δy and Δx respectively, we get

$$P_1 - P_3 = 0 \qquad (1)$$

$$P_2 - P_3 - \frac{1}{2}\rho g \Delta y = 0 \qquad (2)$$

The last term in Eq. (2) drops out as $\Delta y \rightarrow 0$, $\Delta x \rightarrow 0$ and the wedge becomes infinitesimal and thus the fluid element shrinks to a point. Thus,

$$P_1 = P_2 = P_3 = P$$

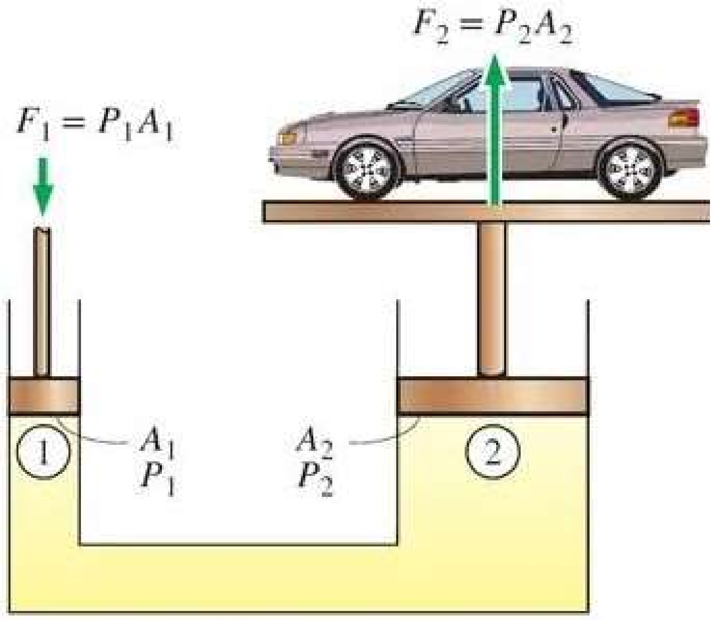
∴ Pressure at a point in a fluid has the same magnitude in all directions.

Pressure at a point (cont.)

Thus, we can conclude that at a point in a static fluid, the force per unit area is independent of the angular orientation of the surfaces. Therefore, pressure is a scalar quantity.

- A balance of forces in the horizontal direction on a fluid element shows that pressures are the same in the horizontal at the same depth. This fact is expressed by *Pascal's law* which states as that all points in a resting fluid medium (connected by the same fluid) are at the same pressure if they are at the same depth. For example,

as shown figure, the car can be lifted easily using the principle of Pascal's law. Reason: Pressures at point 1 and 2 are the same.



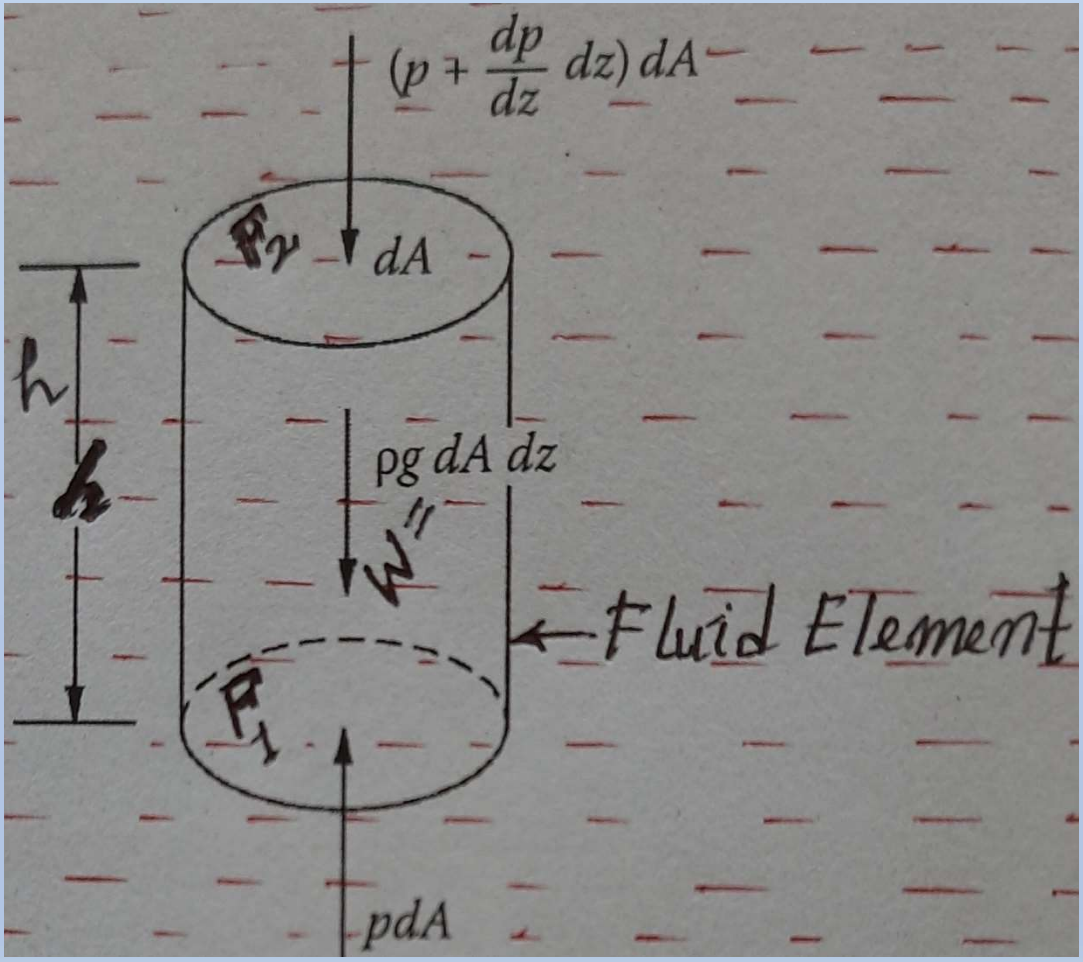
$$\therefore P_1 = P_2$$

$$\Rightarrow \frac{F_1}{A_1} = \frac{F_2}{A_2}$$

$\Rightarrow F_1 = \frac{A_1}{A_2} F_2$ Since area $A_2 >> A_1$, therefore, the car is easily lifted.

❖ Variation of pressure with depth

Let us consider a fluid element having height *dz* and cross sectional area *dA* at rest as shown in the following figure.



Variation of pressure with depth (cont.)

Let, F_1 and F_2 denote the forces acting on the bottom and top of the fluid element respectively. Let p be the pressure at the bottom of the fluid element. Therefore, pressure at the top of the fluid element will be $\left(p + \frac{dp}{dz}dz\right)$.

$$\therefore F_1 = pdA$$

And, $$F_2 = \left(p + \frac{dp}{dz}dz\right)dA$$

Weight of the fluid element, $W = \rho\ dz\ dA\ g$

At equilibrium,

$$F_1 - F_2 - W = 0$$

Substituting the values of F_1, F_2, and W in the above equations, give

$$pdA - \left(p + \frac{dp}{dz}dz\right)dA - \rho\ g\ dz\ dA = 0$$

Variation of pressure with depth (cont.)

After simplification, we get

$$\frac{dp}{dz} = -\rho g$$

Suppose, the variation of the pressure is from p_1 to p_2 with the corresponding heights z_1 to z_2. Then, integrating the above equation in the given limit, yields

$$\int_{p_1}^{p_2} dp = -\rho g \int_{z_1}^{z_2} dz$$

$$\Rightarrow p_2 - p_1 = -\rho g(z_2 - z_1)$$

$$\Rightarrow p_1 = p_2 + \rho g(z_2 - z_1)$$

Or,

$$\Rightarrow p_1 = p_2 + \rho g h$$

Thus, pressure of a fluid at rest increases with the increase in depth. We can experience it easily when we dive into a swimming pool, as we go deeper and deeper, we will feel the increase of pressure in our ear drum. Interestingly, the above relation may be used to find the height of a mountain and depth of a sea.

Text/Reference Books:

1. Introduction to Fluid Mechanics & Fluid Machines – S K Som, Gautam Biswas and Suman Chakraborty
2. Fluid Mechanics – Yunus A Cengel and John M Cimbala
3. Fluid Mechanics and Machines – Victor L Streeter
4. Fluid Mechanics - A K Jain
5. Fluid Mechanics – J Dalal

Thank You