chapter! 3.

The static Magnetic field
OR
Magnetostatic Fields

(1) charges moving with constant velocity i.e constant current (or Direct current) in current carrying wires.

(2) Permanent magnets.

} Produces steady magnetic field (i.e constant with time)

Steady Magnetic field governed by

↓

Biot-Savart's law

- General
- applycable for any current Distribution.
- Similiar to coulomb's law in electrostatics

Ampere's Circuit law

- For only symmetrical current distribution.
- Similiar to Gauss's law in Electrostatics.

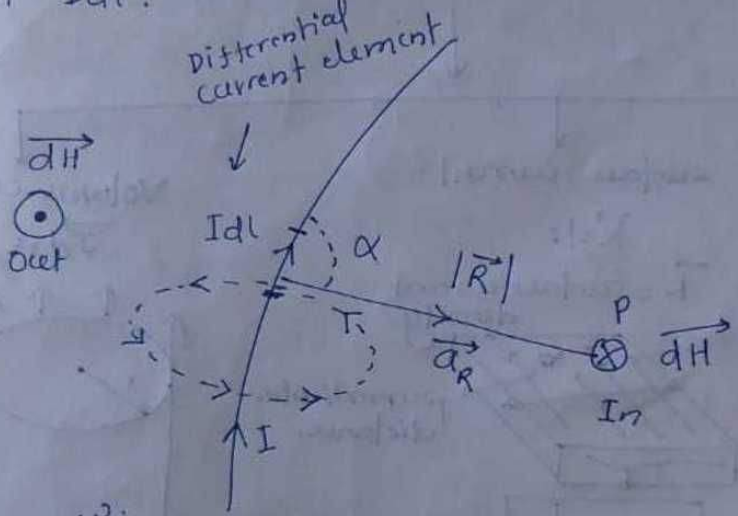
Applications :

1. Development of motors
2. Microphone
3. Telephone ringers
4. Transformers
5. High speed Velocity Device
6. Particle accelerators like cyclotrons.
7. Electromagnetic pump & so on...

Note: Magnetic field Intensity $\vec{H} \approx \vec{E}$ electric field intensity.
(wb/m^2) Magnetic flux density $\vec{B} \approx \vec{D}$ Electric flux Density (c/m^2)

Biot Savart's law

states that "The differential magnetic field Intensity $|\vec{dH}|$ produced at a point P by the differential current element $|I\vec{dl}|$ is proportional to the product Idl & sine of the angle between the element & the line joining P to the element & is inversely proportional to the square of the distance R between P & the element Idl.



A/ Biot savart's law.

current element = Idl ... (magnitude)

so, $$dH \propto \frac{Idl \sin\alpha}{R^2} \quad \text{- (magnitude)}$$

$$dH = \frac{K\, Idl \sin\alpha}{R^2}$$

where; K = constant of proportionality

$$K = \frac{1}{4\pi} \text{ in S.I unit}$$

$$\therefore\ dH = \frac{Idl \sin\alpha}{4\pi R^2} \quad \text{.... (magnitude form)}$$

Note:

Direction Using Right hand Rule

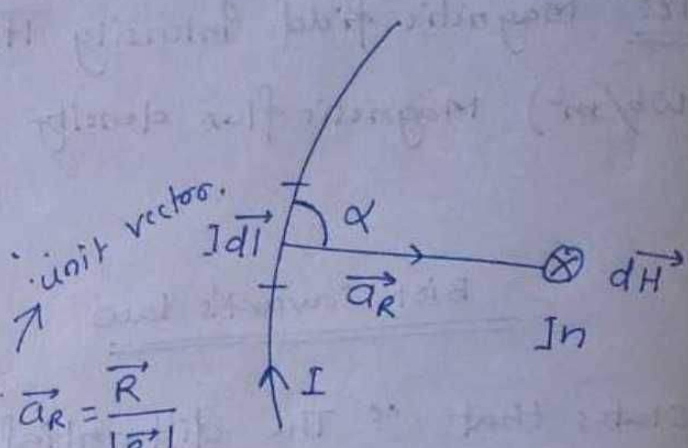
$\vec{H}$ ⊙ out  ↑ I ⊗ $\vec{H}$ in

⊙ $\vec{H}$ (o.I) is out   ⊗ $\vec{H}$ (o.I) is IN

(conventional Representation)

In Vector form:

$$\boxed{\vec{dH} = \frac{I\vec{dl} \times \vec{a}_R}{4\pi R^2}}$$

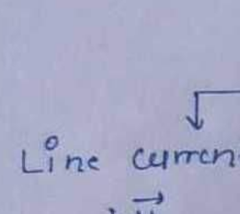


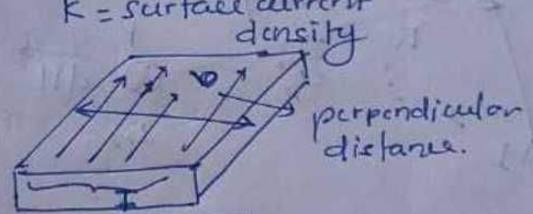
$$\vec{a}_R = \frac{\vec{R}}{|\vec{R}|}$$

$$\boxed{\vec{dH} = \frac{I\vec{dl} \times \vec{R}}{4\pi R^3}}$$

※ current Distribution in Magnetostatics

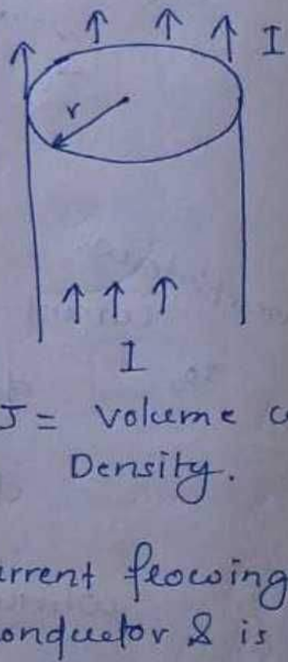
- Line current: $I\vec{dl}$
- surface current: $\vec{K}ds$, $\vec{K}$ = surface current density
- Volume current: $\vec{J}dv$





$$\boxed{K = \frac{I}{b}}$$

b = perpendicular to direction of current



J = Volume current Density.

Note: Surface current Density (K) = for current flowing on the surface of a conductor & is defined for uniform current density

$$\boxed{I = Kb}$$

For Non uniform Surface current density:

⇒ $\boxed{I = \int K dN}$ where dN = Differential element of path across which current is flowing.

⇒ current element can be expressed in terms of :-

$$\vec{I}\vec{dl} \equiv \vec{K}\,ds \equiv \vec{J}\,dv$$

——————— x ———————

⇒ <u>According to</u> <u>Biot-savart's law</u> :

$$\vec{H} = \int_L \frac{I\vec{dl} \times \vec{a_R}}{4\pi R^2} = \int_L \frac{I\vec{dl} \times \vec{R}}{4\pi R^3} \quad ---- \text{(line current)}$$

$$\vec{H} = \int_s \frac{\vec{K}ds \times \vec{a_R}}{4\pi R^2} = \int_s \frac{\vec{K}ds \times \vec{R}}{4\pi R^3} \quad ---- \text{(Surface current)}$$

$$\vec{H} = \int_v \frac{J dv \times \vec{a_R}}{4\pi R^2} = \int_v \frac{\vec{J}dv \times \vec{a_R}}{4\pi R^3} \quad --- \text{(Volume Current)}$$

⇒ Magnetic field due to straight conductor

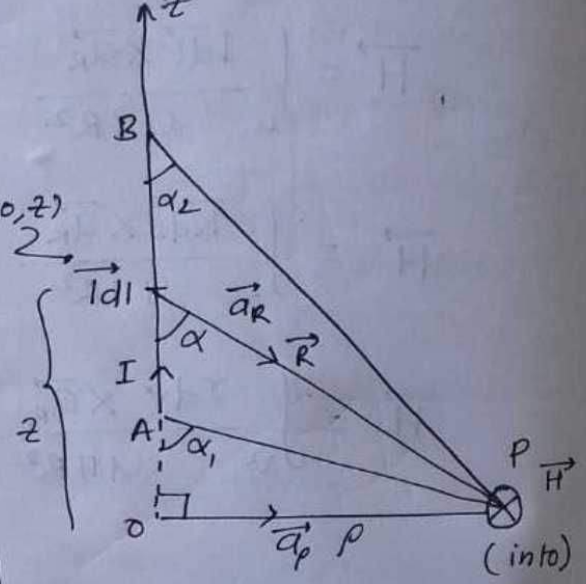
⇒ Field at point P Due to a straight filamentary conductor: –

consider AB finite length of conductor.

$i\vec{dl}$ = small current element

current I flows from A; $(\alpha = \alpha_1)$

to B; $(\alpha = \alpha_2)$



from Biot-Savart's law

$$d\vec{H} = \frac{I\vec{dl} \times \vec{R}}{4\pi R^3}$$

from Fig: $\vec{dl} = dz\,\vec{a_z}$

$\vec{R} = \rho\vec{a_\rho} - z\vec{a_z}$

so, $\vec{dl} \times \vec{R} = (dz\,\vec{a_z}) \times (\rho\vec{a_\rho} - z\vec{a_z})$

$= \rho\, dz\, \vec{a_\phi}$

$$d\vec{H} = \frac{I\rho\, dz\, \vec{a_\phi}}{4\pi\rho^3 \operatorname{cosec}^3\alpha}$$

To get total

$$\vec{H} = \int \frac{I\rho(-\rho\operatorname{cosec}^2\alpha)\, d\alpha}{4\pi(\rho^3\operatorname{cosec}^3\alpha)}\,\vec{a_\phi}$$

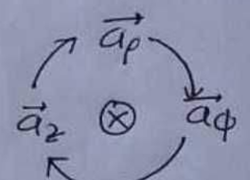
$$= \frac{-I}{4\pi}\int_{\alpha=\alpha_1}^{\alpha_2} \frac{\rho^2\operatorname{cosec}^2\alpha\, d\alpha}{\rho^3\operatorname{cosec}^3\alpha}\,\vec{a_\phi}$$

$$= -\frac{I}{4\pi}\int_{\alpha_1}^{\alpha_2} \frac{1}{\rho\operatorname{cosec}\alpha}\, d\alpha\,\vec{a_\phi}$$

$$\boxed{\vec{H} = -\frac{I}{4\pi\rho}\left[\cos\alpha_2 - \cos\alpha_1\right]\vec{a_\phi}}$$

$z\vec{a_z} + \vec{R} = \rho\vec{a_\rho}$

$\vec{R} = \rho\vec{a_\rho} - z\vec{a_z}$



$\tan\alpha = \frac{\rho}{z}$

$z = \rho\frac{1}{\tan\alpha} = \rho\cot\alpha$

$dz = -\rho\operatorname{cosec}^2\alpha\, d\alpha$

$R^2 = (\rho^2 + (-z)^2)$

$R^2 = (\rho^2 + \rho^2\cot^2\alpha)$

$R^2 = \rho^2(1 + \cot^2\alpha)$

$= \rho^2\left(1 + \frac{\cos^2\alpha}{\sin^2\alpha}\right)$

$R^2 = \rho^2\left(\frac{\sin^2\alpha + \cos^2\alpha}{\sin^2\alpha}\right)$

$R^2 = \rho^2\frac{1}{\sin^2\alpha} = \rho\operatorname{cosec}^2\alpha$

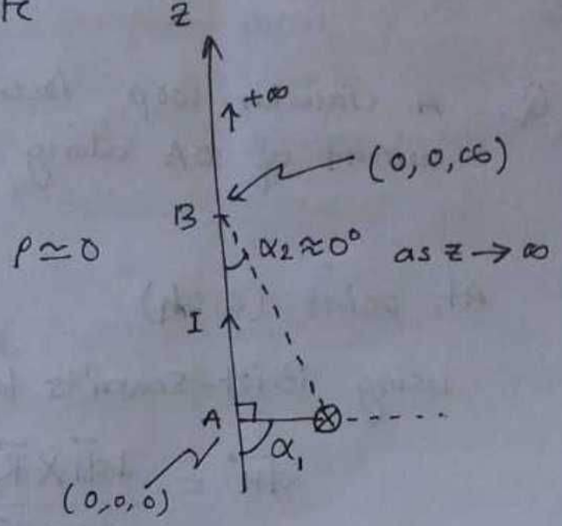
or $R = \rho\operatorname{cosec}\alpha$.

CASE-I conductor in Semi-Infinite

$\alpha_1 = 90^\circ \quad \alpha_2 \simeq 0^\circ$

$$\boxed{\vec{H} = \frac{I}{4\pi\rho}\,\vec{a}_\phi}$$

$\rho \simeq 0$

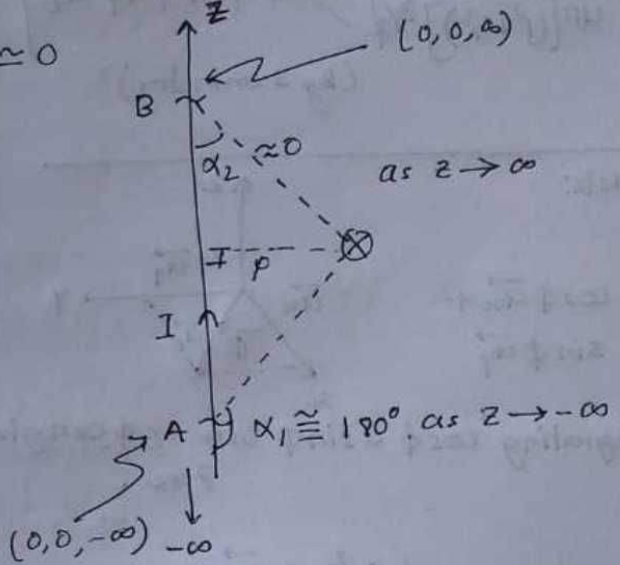


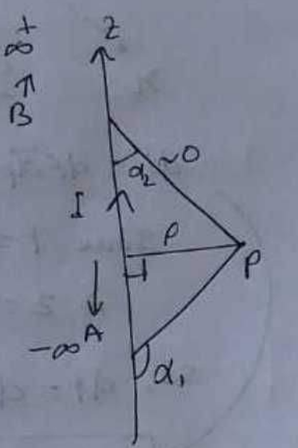
CASE-II Infinite line

$A(0,0,-\infty)$ & $B(0,0,\infty)$

$\alpha_1 \simeq 180^\circ ; \alpha_2 \simeq 0^\circ$

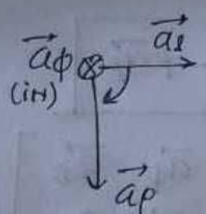
$$\boxed{\vec{H} = \frac{I}{2\pi\rho}\,\vec{a}_\phi} \quad \rho \simeq 0$$

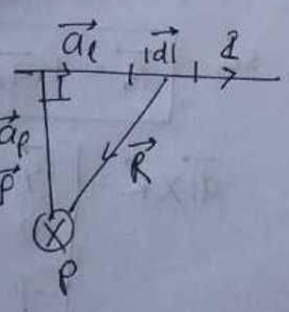




Note: Simple formula to find $\vec{a}_\phi$

$$\boxed{\vec{a}_\phi = \vec{a}_l \times \vec{a}_\rho}$$





$\vec{a}_l$ = unit vector along line current

$\vec{a}_\rho$ = unit vector ⊥ along the line current to the field point.

⇒ Magnetic field due to circular loop

Q. A circular loop located on $x^2+y^2=9$, $z=0$ carries a direct current of 10A along $\vec{a_\phi}$. Determined $\vec{H}$ at $(0,0,4)$ & $(0,0,-4)$

At, point $(0,0,h)$

using Biot-Savart's law

$$d\vec{H} = \frac{I\,d\vec{l} \times \vec{R}}{4\pi R^3}$$

$$d\vec{H} = \frac{I}{4\pi[(\rho^2+h^2)]^{3/2}} \left\{ \underbrace{\cancel{\rho h\, d\phi\, \vec{a_\rho}}}_{\to 0} + \rho^2 d\phi\, \vec{a_z} \right\}$$

(By summetry)

Note:

$\vec{a_\rho} = \cos\phi\,\vec{a_x} + \sin\phi\,\vec{a_y}$

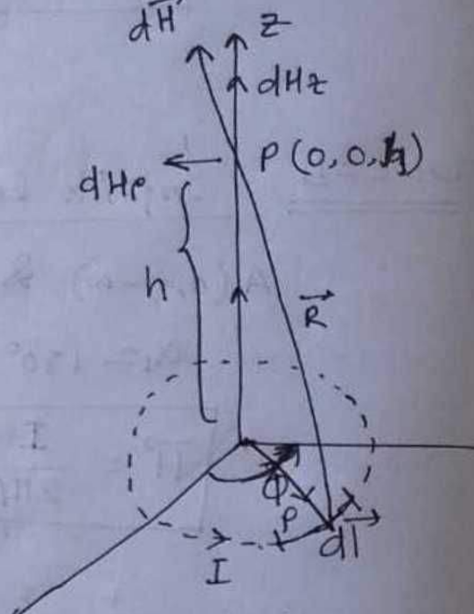
Integrating $\cos\phi$ & $\sin\phi$ over $0 \le \phi \le 2\pi$ gives zero.

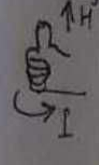
$$\therefore \vec{H} = \int d\vec{H}\,\vec{a_z} = \int \frac{I\rho^2 d\phi\, \vec{a_z}}{4\pi[\rho^2+h^2]^{3/2}} =$$

$$= \frac{I\rho^2 \vec{a_z}}{4\pi[\rho^2+h^2]^{3/2}} \int_0^{2\pi} d\phi$$

$$\vec{H} = \frac{I\rho^2 (2\pi)\,\vec{a_z}}{4\pi[\rho^2+h^2]^{3/2}}$$

$$\boxed{\text{or} = \vec{H} = \frac{I\rho^2 (2\pi)\,\vec{a_z}}{2\pi\, 2[\rho^2+h^2]^{3/2}}}$$





cylindrical c.s

$d\vec{l} = d\rho\,\vec{a_\rho} + \rho\, d\phi\,\vec{a_\phi} + dz\,\vec{a_z}$

since ρ = constant

& $z = 0$ for given loop

so, $d\rho = dz = 0$

$$\boxed{d\vec{l} = \rho\, d\phi\, a_\phi}$$

From Fig:

$\rho\vec{a_\rho} + \vec{R} = h\vec{a_z}$

so, $\boxed{\vec{R} = h\vec{a_z} - \rho\vec{a_\rho}}$

$$d\vec{l} \times \vec{R} = \begin{bmatrix} \vec{a_\rho} & \vec{a_\phi} & \vec{a_z} \\ 0 & \rho\, d\phi & 0 \\ -\rho & 0 & h \end{bmatrix}$$

$$\boxed{d\vec{l} \times \vec{R} = \rho h\, d\phi\, \vec{a_\rho} + \rho^2 d\phi\, \vec{a_z}}$$

$$\boxed{R = \sqrt{\rho^2+h^2}}$$

$$\boxed{\vec{H} = \frac{I\rho^2 \vec{a_z}}{2[\rho^2+h^2]^{3/2}}}$$ → Magnetic field intensity at point (0, 0, h) due to circular loop (at z = 0) having radius ρ.

(a) At (0, 0, 4)

put I = 10A, h = 4 To find $\rho \Rightarrow$ $x^2+y^2 = \rho^2$

$x^2+y^2 = (3)^2$ ... given

so, $\rho = 3$

$$\vec{H}(0,0,4) = \frac{10(3)^2 \vec{a_z}}{2[9+16]^{3/2}}$$

$$= 0.36\, \vec{a_z} \text{ A/m}$$

(b) At (0, 0, −4), h = −4

$$\vec{H} = \frac{I\rho^2 \vec{a_z}}{2(\rho^2+h^2)^{3/2}}$$ I = 10A $\rho = 3$

$$H = \frac{10(3)^2 \vec{a_z}}{2(9+(-4)^2]^{3/2}}$$

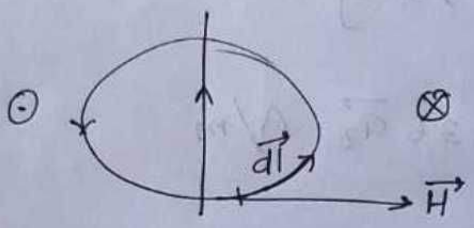
$$= 0.36\, \vec{a_z} \text{ A/m.}$$

※ Ampere's Circuit law – OR int. of Maxwell's equation.

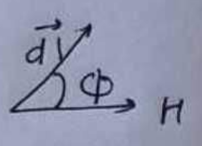
" Ampere's Circuit law, states that the line integral of $\vec{H}$ around a closed path is the same as the net current I_{enc} enclosed by the path "

$$\boxed{\oint_L \vec{H}\cdot d\vec{l} = I_{enc}}$$ – Integral form

→ " The Circulation of $\vec{H}$ equals I_{enc} ;



direction – RHR



using stokes theorem. $\oint_L \approx \oint_s$

$$\int_s (\nabla \times \vec{H})\cdot d\vec{s} = I_{enc}$$

$\nabla \rightarrow$ curl

Also, $I_{enc} = \int_s \vec{J}\cdot d\vec{s}$

$$\boxed{\nabla \times \vec{H} = \vec{J}}$$ → 3rd maxwell's equatⁿ

→ Ampere's law in differential form (or point form)

Note: since, $\nabla \times \vec{H} \neq 0$; a magnetostatic field is not conservative.

Application's of Amperes law

(1) used to determined $\vec{H}$ for symmetrical current distribution such as

- Infinite line current
- Infinite sheet of current
- Infinitely long coaxial transmission line.

} Apply $\oint_L H \cdot dl = I_{enc}$

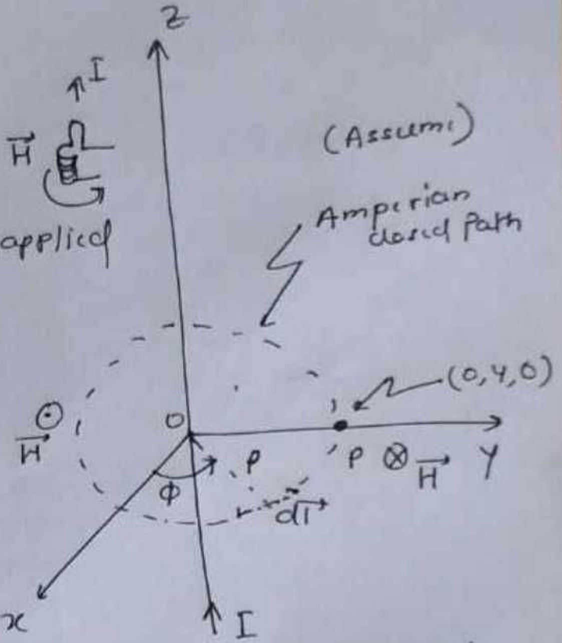
(2) For symmetrical current distribution, $\vec{H}$ is either parallel or perpendicular to $d\vec{l}$.

Infinite line current

To determined $\vec{H}$, Ampere's Circuit law is applied

→ consider closed path ≈ Amperian path
→ which shows that $\vec{H}$ is constant as ρ is constant.

Note: Amperian path is analogous to Gaussian surface.



Cylindrical coordinate system.
(ρ, ϕ, z)

$$\vec{H} = H_\rho \vec{a}_\rho + H_\phi \vec{a}_\phi + H_z \vec{a}_z$$

$$\vec{dl} = d\rho\, \vec{a}_\rho + \rho\, d\phi\, \vec{a}_\phi + dz\, \vec{a}_z$$

Also, ρ = constant
$z = 0$ for point P
$\therefore d\rho = dz = 0$

$$\vec{dl} = \rho\, d\phi\, \vec{a}_\phi$$

$$\oint_L \vec{H} \cdot \vec{dl} = I$$

$$\oint_L H_\phi\, \vec{a}_\phi \cdot \rho\, d\phi\, \vec{a}_\phi = I$$

$$H_\phi \rho \oint_0^{2\pi} d\phi = I \qquad (0 \le \phi \le 2\pi)$$

$$H_\phi \rho (2\pi) = I$$

$$\boxed{H_\phi = \frac{I}{2\pi\rho}} \quad \text{... (magnitude)}$$

$$\boxed{\vec{H} = \frac{I}{2\pi\rho}\, \vec{a}_\phi} \quad \text{.... (Vector form)}$$