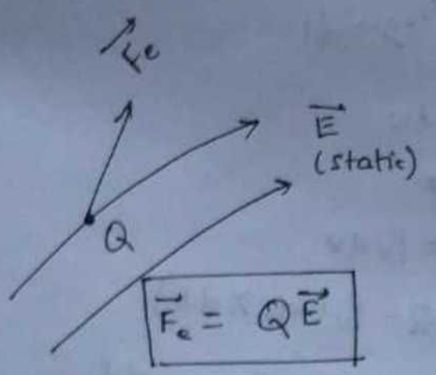
Magnetic force

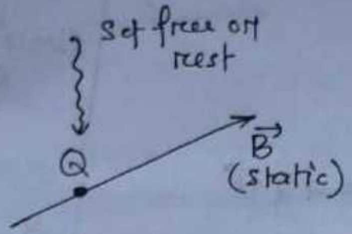
⇒ Force due to magnetic field $\vec{B}$ (steady magnetic field)

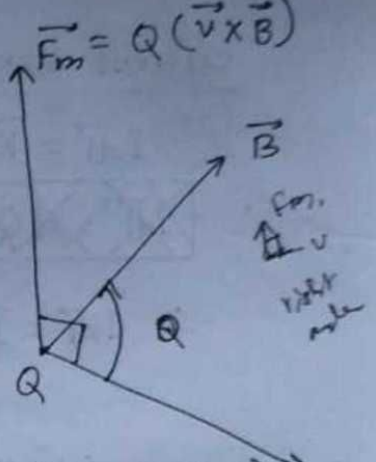
Three ways.

- moving charge in $\vec{B}$ field
- on current element on external $\vec{B}$
- Between two current elements.

Force on a charge particle







+ Q ≈ set free or moving
  - --→ accelerated in the direction of $\vec{E}$, due to electric force $\vec{F_e}$.
  - * $\vec{F_e}$ does some work

+ Q ≈ Remains at rest
  - --→ nothing happens
  - * $\vec{F_m} = 0$ (does not work)

+ Q ≈ move with v velocity $\vec{v}$
  - --→ experience a force at right angle to its velocity $\vec{v}$

Fm, v, right rule

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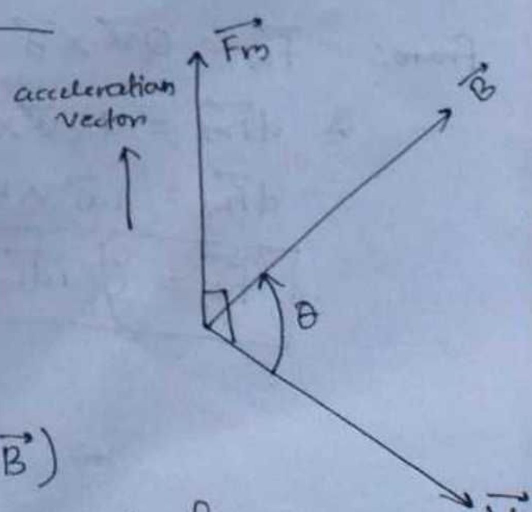
moving charge in $\vec{B}$

Magnitude of $\vec{F_m}$:

$$\boxed{F_m = QvB\sin\theta}$$

$$\boxed{\vec{F_m} = Q\vec{v} \times \vec{B}}$$



case:1 Moving charge in both $\vec{E}$ & $\vec{B}$

Total force $\vec{F} = \vec{F_e} + \vec{F_m}$

$= Q\vec{E} + Q(\vec{v} \times \vec{B})$

$$\boxed{\vec{F} = Q(\vec{E} + \vec{v} \times \vec{B})} \rightarrow \text{Lorentz force equation.}$$

Eg: Application

① Electron orbit in magnetron

② Proton path in the cyclotron etc.

Case: 2 **Force on Differential current element**

$$\vec{F_m} = Q(\vec{v} \times \vec{B})$$

& $\vec{J} = \rho_v \vec{v}$

current elements in magnetostatic

$$I\vec{dl} \equiv \vec{K}ds \equiv J dv$$

$\therefore$ ~~$I\vec{dl} = dQ\, d\vec{v}$~~

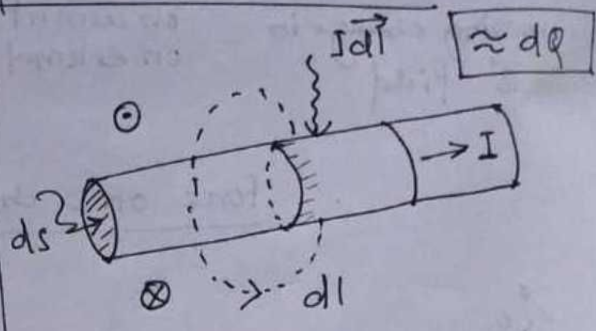
$\therefore I\vec{dl} = \vec{J}dv$

$$= \rho_v \vec{v} dv = dQ\vec{v}$$

$\because dQ = \rho_v dv$

$$\boxed{I\vec{dl} = dQ\vec{v}}$$

→ Elemental change dQ moving with velocity $\vec{v}$ is equivalent to the conduction current element $I\vec{dl}$.



$\therefore dQ = \rho_v dv$

since,

$$dQ = \rho_v dv$$

$$dQ = \rho_v (ds \times dl)$$

& $dI = \frac{dQ}{dt} = \rho_v ds \frac{dl}{dt}$

$$dI = \rho_v ds dv$$

$$J = \frac{dI}{ds} = \rho_v dv$$

$$\vec{J} = \frac{I}{s} = \rho_v \vec{v}$$

$$\boxed{\vec{J} = \rho_v \vec{v}}$$

from: $\vec{F_m} = Q\vec{v} \times \vec{B}$

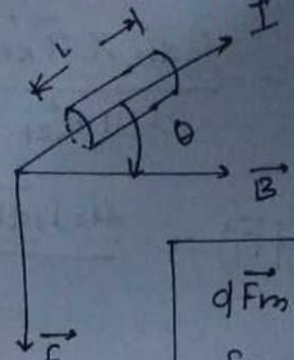
& $d\vec{F_m} = dQ\vec{v} \times \vec{B}$

$$d\vec{F_m} = I\vec{dl} \times \vec{B}$$

$$\boxed{\vec{F_m} = \oint_L I\vec{dl} \times \vec{B}}$$

In general

$$\boxed{\vec{F_m} = I\vec{L} \times \vec{B}}$$



$L \approx$ total length of conductor

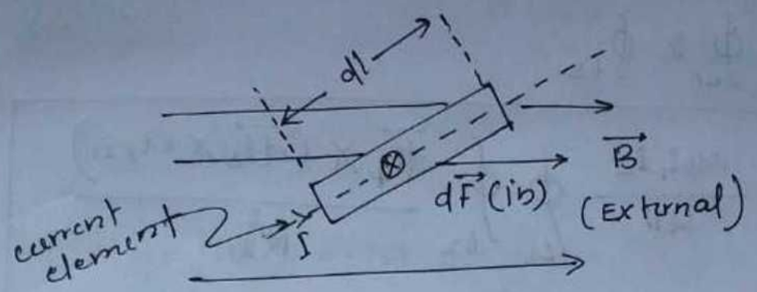
$$\boxed{|\vec{F_m}| = F_m = BIL\sin\theta}$$

$$d\vec{F_m} = \vec{K}ds \times \vec{B}$$

$$f_m = \int_S \vec{K}ds \times \vec{B}$$

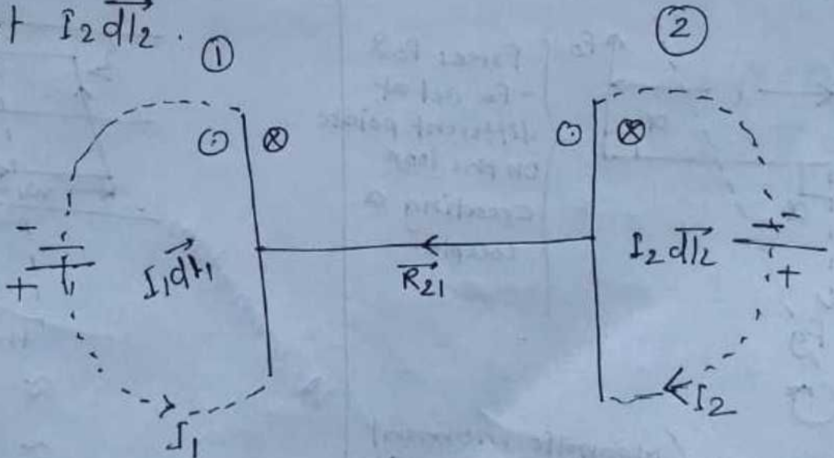
$$\vec{f_m} = \int_V \vec{J}dv \times \vec{B}$$

$$\therefore \vec{B} = \frac{\text{Force}}{\text{current element}}$$



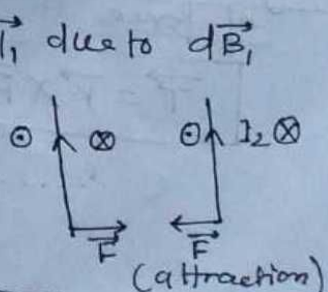
Case: 3 Force Between two current elements.

Let $d\vec{B_2}$ is the differential magnetic field for loop 2 by the current element $I_2 d\vec{l_2}$.



$d\vec{F_{11}} \approx$ differential force on current element $I_1 d\vec{l_1}$ due to $d\vec{B_1}$ i.e. loop 1.

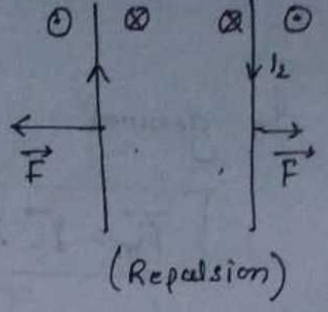
Now, The force on current element $Id\vec{l}$ due to $d\vec{B}$ is given as



$$d(d\vec{F_1}) = I_1 d\vec{l_1} \times \underbrace{d\vec{B_2}}_{\mu_0 d\vec{H_2}}$$

$$\boxed{\because d\vec{F} = Id\vec{l} \times \vec{B}}$$

Using Biot-Savart law :-



$$d\vec{H}_2 = \frac{I_2 \vec{dl_2} \times \vec{a}_{R21}}{4\pi R_{21}^2}$$

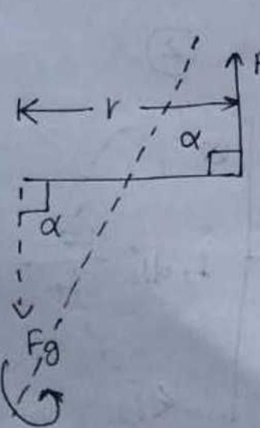
$$\therefore d(d\vec{F}_1) = \frac{\mu_0 I_1 \vec{dl_1} \times I_2 \vec{dl_2} \times \vec{a}_{R21}}{4\pi R_{21}^2}$$

$\therefore$ To determined total force, Integrate two times,

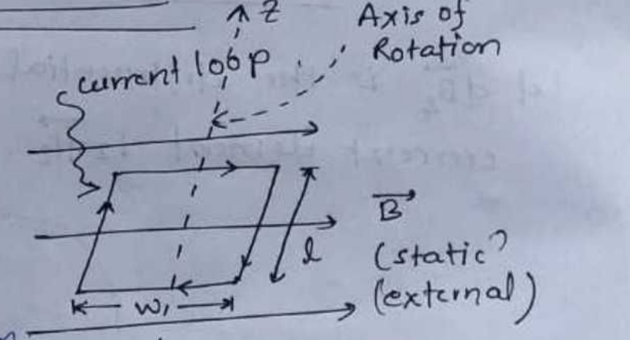
$$\oint_{L1} \& \oint_{L2}$$

$$\boxed{\vec{F}_1 = \frac{\mu_0 I_1 I_2}{4\pi} \oint_{L1} \oint_{L2} \frac{\vec{dl_1} \times (\vec{dl_2} \times a_{R21})}{R_{21}^2}}$$

Magnetic torque and moment



Forces F_0 & $-F_0$ act at different points on the loop creating a couple.



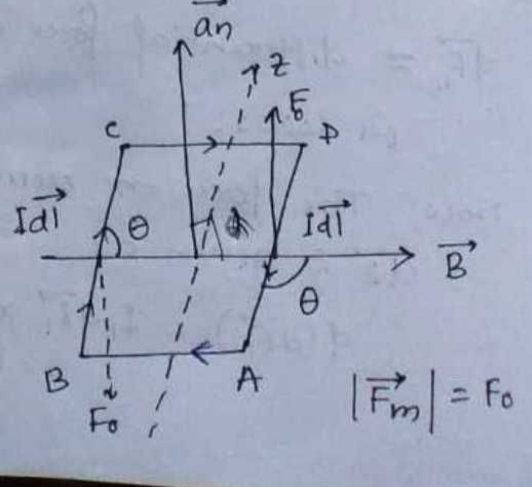
w = momentum

Loop experience a force that tends to rotate it
≈ Torque in $\vec{B}$
≈ Magnetic torque

Magnetic torque/Magnetic moment of force

$$\boxed{\vec{T} = \vec{r} \times \vec{F}} \text{ N.m.}$$

$\vec{F}$ → Force
$\vec{r}$ → Moment arm.



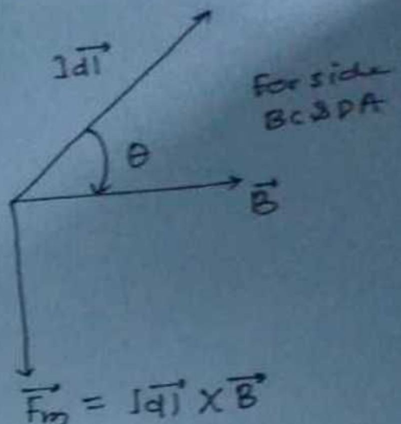
$|\vec{F_m}| = F_0$

Thus total force

$$\vec{F} = I\int_B^C \vec{dl}\times\vec{B} + I\int_B^A \vec{dl}\times\vec{B}$$

$$= I\int_0^l \vec{dl}\times\vec{B} + \int_l^0 \vec{dl}\times\vec{B}$$

$$= I\left\{\int_0^l \vec{dl}\times\vec{B} - \int_0^l \vec{dl}\times B\right\}$$



$\vec{F} = 0$ – when side BC & AB are $\perp$ to $\vec{B}$; $\theta = 90°$

$\therefore$ F_0 is equal and opposite on side BC and DA

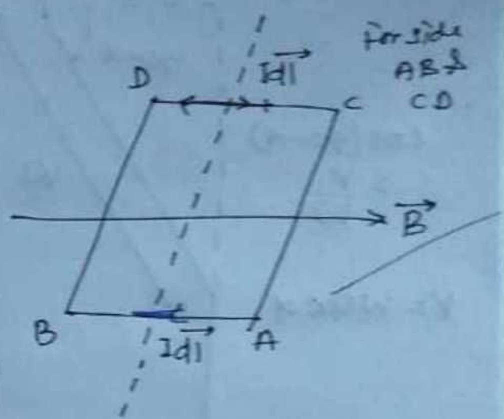
$\vec{T} = \vec{r}\times\vec{F} = |\vec{r}||\vec{F}|\sin\alpha$

$\vec{T} = w \times IBl \times \sin\alpha$

$\vec{T} = BIlw \quad \because \alpha = 90°$

$\boxed{\vec{T} = BIA} \quad \because A = \text{Area of loop} = l \times w$

(maximum torque)



$I\vec{dl} \parallel \vec{B}$, $\theta = 0°$

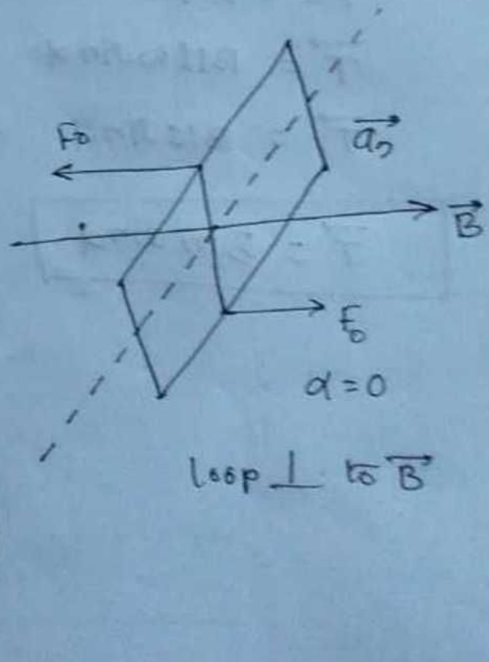
$\therefore \vec{F_m} = I\vec{dl}\times\vec{B} = 0$

<u>Case: 2</u> current loop normal direction

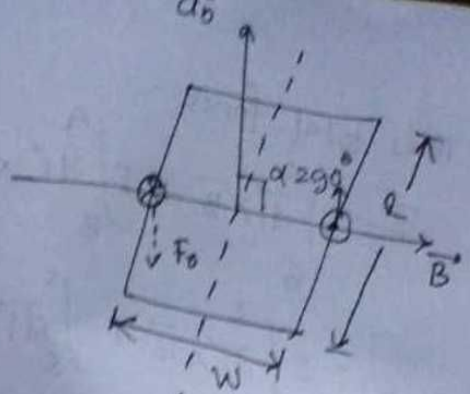
$T = \vec{r}\times\vec{F}$

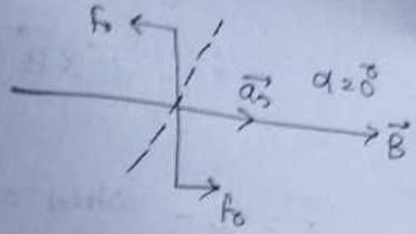
$= |\vec{r}||\vec{F}|\sin 0°$

$\boxed{\vec{T} = 0}$

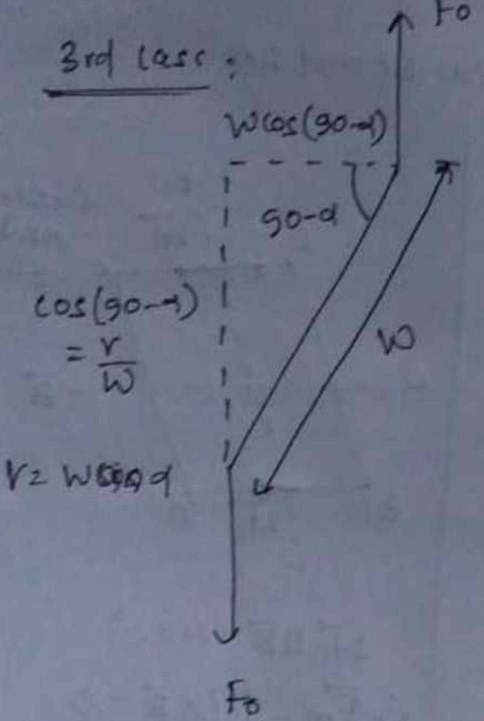


loop $\perp$ to $\vec{B}$

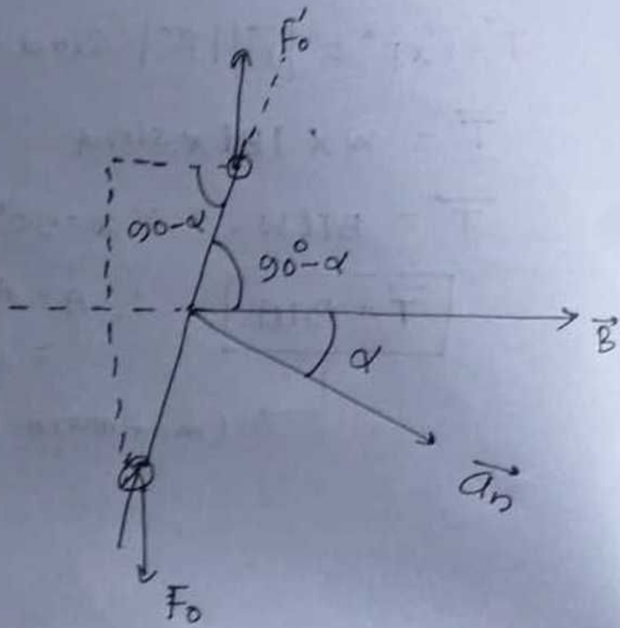




3rd case:



3rd case



$\vec{T} = \vec{r} \times \vec{F}$

$\vec{T} = BIlw\sin\alpha$

$\vec{T} = BIS\sin\alpha \qquad S = l \times W$

↳ Area of loop

$\boxed{\vec{T} = Bm\sin\alpha}$

where: $m = IS$

$m = IS\vec{a}_n$

↳ magnetic dipole moment ($A\text{-}m^2$)