$\boxed{\vec{H} = -\nabla V_m}$ This is possible only when $\vec{J} = 0$

⇒ The scalar potential satisfies laplace's equation : $\boxed{\nabla^2 V_m = 0}$

($\vec{J} = 0$)

Since, $\nabla \cdot \vec{B} = 0$ ......⇒ Non-existance of magnetic monopole.

$\mu_0 \nabla \cdot \vec{H} = 0$ $\because \vec{B} = \mu_0 \vec{H}$ ... free space

$\mu_0 \nabla \cdot (-\nabla V_m) = 0$ $\because \vec{H} = -\nabla V_m$

$\boxed{\nabla^2 V_m = 0}$ only for $J = 0$

Directly defined as $V_m = -\int_A^B \vec{H} \cdot \vec{dl}$ ... (unit: Ampere)

$\vec{H} \approx A/m$

$\vec{dl} \approx m$.

⇒ Vector magnetic potential

↳ Exist where $\vec{J}$ is present

" Magnetic vector potential is defined in such a way that its curl gives the magnetic flux density.

Using curl because curl of vector ≈ Vector quantity

$\boxed{\vec{B} = \nabla \times \vec{A}}$

where $\vec{A} \approx$ magnetic Vector potential (wb/m)

Since, $\nabla \times \vec{H} = \vec{J}$ ---- Ampere's circuit law.

$\vec{B} = \mu_0 \vec{H}$ ---- Free space

$\vec{H} = \frac{\vec{B}}{\mu_0}$

$\nabla \times \vec{B} = \mu_0 \vec{J}$

Since, $\vec{B} = \nabla \times \vec{A}$

$$\nabla \times \vec{B} = \nabla \times \nabla \times \vec{A} = \mu_0 \vec{J}$$

using Laplacian of Vector
i.e $\boxed{\nabla^2 \vec{A} = \nabla . (\nabla \cdot \vec{A}) - \nabla \times \nabla \times \vec{A}}$

$$\nabla \times \nabla \times \vec{A} = \nabla(\nabla . \vec{A}) - \nabla^2 \vec{A}$$

$$\therefore \nabla(\nabla . \vec{A}) - \nabla^2 \vec{A} = \mu_0 \vec{J}$$

$= 0$

Since, For D.C. current only

$$\nabla . \vec{A} = 0$$

(because magnetic field is continuous and it has no divergence.)

$$-\nabla^2 \vec{A} = \mu_0 \vec{J}$$

or $\boxed{\nabla^2 \vec{A} = -\mu_0 \vec{J}}$

↳ Vector poisson's equation.

$\nabla^2 \vec{A} = -\mu_0 \vec{J}$ ----- for (x, y, z) -- Cartesian condinate system.

$$\nabla^2 (A_x \vec{a_x} + A_y \vec{a_y} + A_z \vec{a_z}) = -\mu_0 (J_x \vec{a_x} + J_y \vec{a_y} + J_z \vec{a_z})$$

$$\therefore \left\{ \begin{aligned} \nabla^2 A_x &= -\mu_0 J_x \\ \nabla^2 A_y &= -\mu_0 J_y \\ \nabla^2 A_z &= -\mu_0 J_z \end{aligned} \right\}$$

These are the poisson's equation in magnetostatic.

$$A_x = \mu_0 J_x \left[\frac{1}{4\pi} \int_v \frac{dv}{r} \right]$$

$$A_x = \frac{\mu_0}{4\pi} \int \left(\frac{J_x}{r} \right) dv$$

$$A_y = \frac{\mu_0}{4\pi} \int_v \left(\frac{J_y}{r} \right) dv$$

$$A_z = \frac{\mu_0}{4\pi} \int_v \left(\frac{J_z}{r} \right) dv$$

⇒ In case of Poisson's equation in Electrostatics

$$\nabla^2 V = -\frac{\rho_v}{\epsilon_0}$$

volume charge density

$$V = \frac{Q}{4\pi \epsilon_0 r}$$

$\because Q = \int \rho_v dv$

$$V = \frac{1}{4\pi\epsilon_0} \int_v \frac{\rho_v dv}{r}$$

$$= \frac{\rho_v}{\epsilon_0} \left[\frac{1}{4\pi} \int_v \frac{dv}{r} \right]$$

In general

$$\boxed{\vec{A} = \frac{\mu_0}{4\pi}\int_V \frac{\vec{J}}{r}\,dv}$$

with $\vec{B} = \nabla \times \vec{A}$

$\nabla \cdot \vec{B} = 0$

$\therefore \nabla \cdot (\nabla \times \vec{A}) = 0$

$\Rightarrow$ $\vec{A}$ in terms of three standard current configuration

i.e $\boxed{I\vec{dl} = \vec{K}ds = \vec{J}dv}$ ----> Unit (Ampere - meter)

so, $\vec{A} = \frac{\mu_0}{4\pi}\int_V \frac{\vec{J}dv}{r}$ .... Volume current

$\vec{A} = \frac{\mu_0}{4\pi}\int_S \frac{\vec{K}ds}{r}$ --- Surface current

$I =$ Ampere

$\vec{dl} =$ meter

$\vec{K} = \frac{I}{b}$ (A/m)

$ds =$ meter^2.

$J = \frac{I}{s}$ (A/m^2)

$dv = m^3$