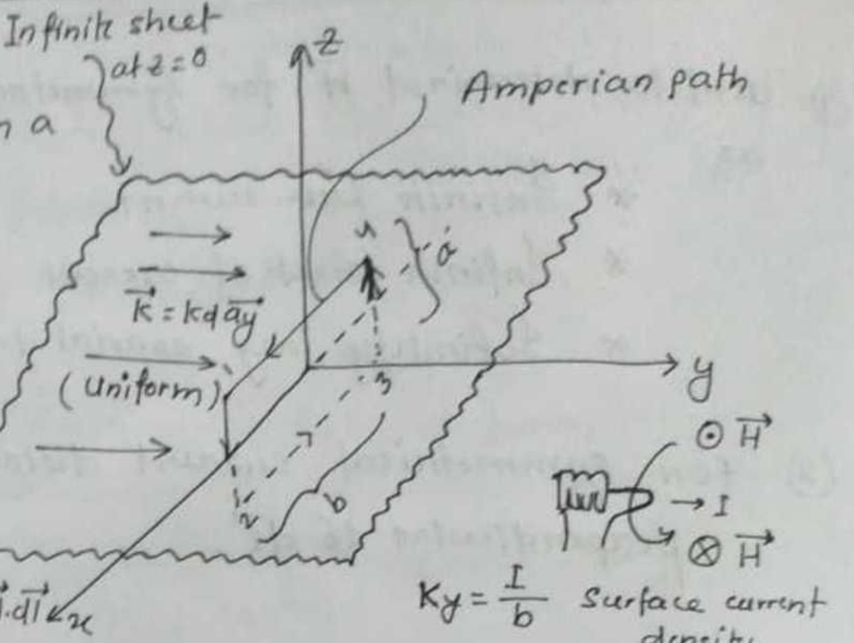
※ Infinite sheet of Current :



⇒ Apply Ampere's circuit law in a rectangular closed path.
i.e 1-2-3-4 (Amperian path)
(using Right hand Rule)

$K_y = \frac{I}{b}$ Surface current density (magnitude)

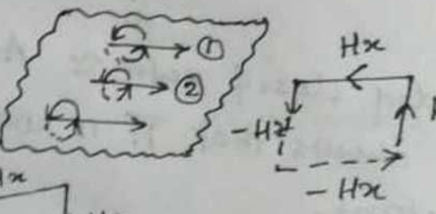
$$\oint_L \vec{H}\cdot\vec{dl} = I = K_y b$$

$$\oint_L \vec{H}\cdot\vec{dl} = \left(\int_1^2 + \int_2^3 + \int_3^4 + \int_4^1\right)\vec{H}\cdot\vec{dl}$$

$$= (-H_z)(-a) + (-H_x)(-b) + (H_z)(a) + (H_x)(b)$$

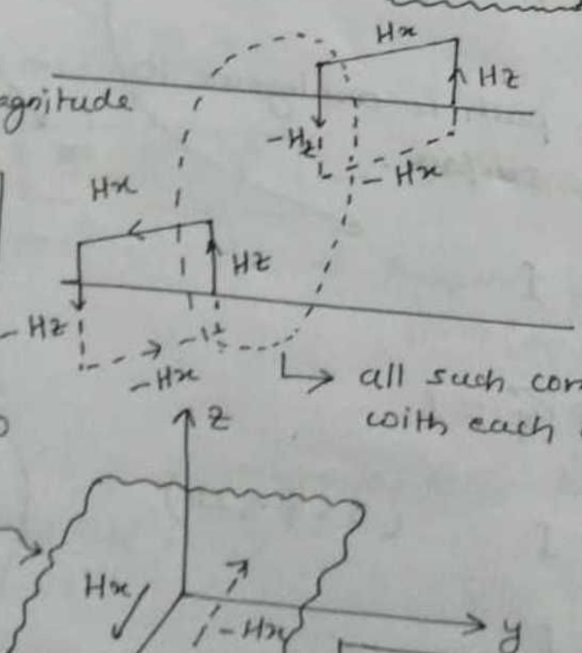
$$= H_x b + H_x b \quad \because H_z = -H_z$$

$$\oint_L \vec{H}\cdot\vec{dl} = 2H_x b$$



$$2H_x b = K_y b$$

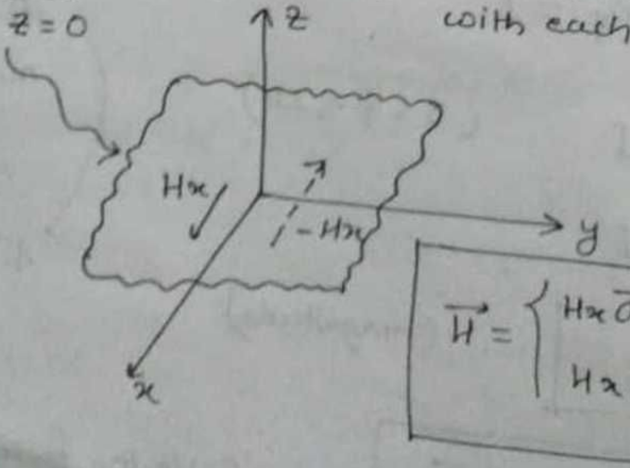
$$\boxed{H_x = \frac{K_y}{2}}$$ - magnitude



$$\boxed{\vec{H} = \begin{cases} \frac{1}{2} K_y \vec{a_x}, & z > 0 \\ \frac{1}{2} K_y (-\vec{a_x}), & z < 0 \end{cases}}$$

↳ all such components are cancel with each other.

$\because x \perp y$
$\sin\alpha = 1$
$(\alpha = 90°)$



$$\boxed{\vec{H} = \begin{cases} H_x \vec{a_x}, & z > 0 \\ H_x (-\vec{a_x}), & z < 0 \end{cases}}$$

In general

$$\boxed{\vec{H} = \frac{1}{2}\, \vec{K} \times \vec{a_n}}$$

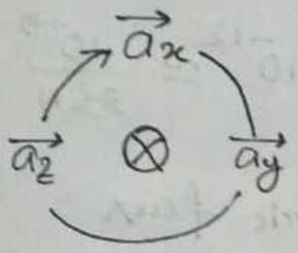
Here $\vec{a_n} = \vec{a_z}$

so,

$\vec{K} = K_y \vec{a_y}$

& $\vec{K} \times \vec{a_n} = K_y (\vec{a_y} \times \vec{a_z})$

$\vec{K} \times \vec{a_n} = K_y \vec{a_x}$



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Magnetic

Magnetic flux Density - Maxwell's equation

$\vec{B} = \mu_0 \vec{H}$ (wb/m²) $\approx$ $\vec{D} = \epsilon_0 \vec{E}$ (C/m²)

μ_0 = constant permeability of free space (H/m)
$= 4\pi \times 10^{-7}$ H/m

ϵ_0 = constant permittivity of free space. (F/m)
$= 8.854 \times 10^{-12} \simeq \frac{10^{-9}}{36\pi}$ F/m

magnetic flux

$$\psi = \int_s \vec{B} \cdot \vec{ds} \text{ .... webers (wb)}$$

unit of $\vec{B}$: (wb/m²)

Electric flux

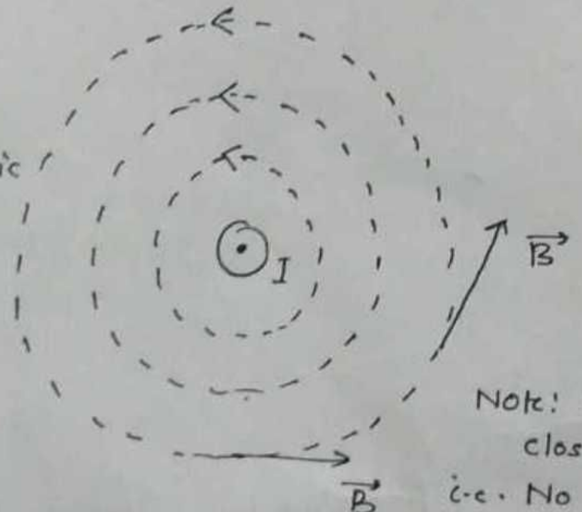
$$\psi = \int_s \vec{D} \cdot \vec{ds} \text{ ... coulomb (C)}$$

unit of $\vec{D}$: (C/m²)

⇒ Magnetic flux lines due to straight wire carrying current comings out

Magnetic flux lines

$\vec{B} \approx$ Tangential at every point on the magnetic flux lines.



Note: Each flux line is closed.
i.e. No begining or end.

In Magnetostatic field

Not possible to have isolated magnetic change (or magnetic poles)

Note: Isolated magnetic charge does not exist

$$\boxed{\oint_s \vec{B} \cdot \vec{ds} = 0}$$ → Total flux through a closed surface is zero.

In Electrostatic field

$$\psi = \oint_s \vec{D} \cdot \vec{ds} = Q_{enc}$$

Gauss law

Isolated charge

* Electric flux lines are not necessarily closed. they have some source point and end point (sink point)

⇒ law of conservation of magnetic flux or,
Gauss's law for magnetostatic field.

Ampere's law

$\nabla \times \vec{H} = \vec{J}$ .... Maxwell's eq^n

$\oint_s \vec{B} \cdot d\vec{s} = 0$ .... (No Isolated magnetic charge)

→ Non conservative nature of $\vec{H}$

$\nabla \times \vec{E} = 0$ .. → conservative nature of $\vec{E}$

as $\oint_L \vec{E} \cdot d\vec{l} = 0$

$\oint_s \vec{D} \cdot d\vec{s} = Q_{enc}$ (Isolated electric charge)

Note: $\oint_s \vec{B} \cdot d\vec{s} = \int_v \nabla \cdot \vec{B}\, dv = 0$.

so, $\boxed{\nabla \cdot \vec{B} = 0}$ → 4th equation of Maxwell's

→ This equation shows that magnetostatic fields have no sources or sinks.

→ Magnetic field lines are always continuous.

Scalar Magnetic potential

'V_m'.

$\because \vec{E} \simeq \vec{H}$

$\vec{D} \simeq \vec{B}$

$\because \nabla \times \vec{E} = 0$

As in Electrostatic, $\vec{E} = -\nabla V$

so, in Magnetostatic, $\boxed{\vec{H} = -\nabla V_m}$

since, $\nabla \times \vec{H} = \vec{J}$ → Ampere's law

$\nabla \times \vec{H} = \nabla \times (-\nabla V_m)$

$\nabla \times \vec{H} = 0$

$\therefore \vec{J} = 0$ — current density.

using identity
$\nabla \times (\nabla V) = 0$
$\nabla \cdot (\nabla \times \vec{A}) = 0$

Note: Magnetic scalar potential V_m is only defined in a region where $\vec{J} = 0$

$$\boxed{\vec{H} = -\nabla V_m}$$ This is possible only when $\vec{J} = 0$.

⇒ The scalar potential satisfies laplace's equation : $\boxed{\nabla^2 V_m = 0}$

$(\vec{J} = 0)$

Since, $\nabla \cdot \vec{B} = 0$ ....→ Non-existance of magnetic monopole.

$\mu_0 \nabla \cdot \vec{H} = 0$ $\quad \because \vec{B} = \mu_0 \vec{H}$ ... free space

$\mu_0 \nabla \cdot (-\nabla V_m) = 0$ $\quad \because \vec{H} = -\nabla V_m$

$$\boxed{\nabla^2 V_m = 0}$$ only for $J = 0$