$\frac{1}{\infty} = 0$

$V_{AB} = V_B - V_A$

↳ reference. reference.

↳ Potential differ at B w.r.t A

$r_A \rightarrow \infty$, $V_A \rightarrow 0$

$\therefore V_{AB} = V_B$

$$\therefore \boxed{V_{AB} = \frac{Q}{4\pi\epsilon_0 r}}$$

⇒ without origin point change.

$$V_{AB} = \frac{Q}{4\pi\epsilon_0 |r - r'|}$$

$A \longrightarrow B$ distribution

⇒ for n^{th} number of point charge

$$V_{AB} = \frac{Q_1}{4\pi\epsilon_0 |r - r_1|} + \frac{Q_2}{4\pi\epsilon |r - r_2|} + \cdots \frac{Q_n}{4\pi\epsilon |r - r_n|}$$

$$V_{AB} = \sum_{k=1}^{k=n} \frac{Q_k}{4\pi\epsilon_0 |r - r_k|}$$

⇒ For line charge

$$V_{AB} = \int \frac{\rho_l \, dl}{4\pi\epsilon_0 |r - r'|}$$

⇒ For sheet charge

$$V_{AB} = \int \frac{\rho_s \, ds}{4\pi\epsilon_0 |r - r'|}$$

⇒ For Volume charge

$$V_{AB} = \int \frac{\rho_v \, dv}{4\pi\epsilon |r - r'|}$$

E & charge distributed.

$V = -\int_A^B E \cdot dl$

(B)

$\Rightarrow$ ~~A=∞~~ if $A=\infty$

$$\boxed{V=-\int_{\infty}^{B} E\cdot dl}$$

$\Rightarrow$ if $A\neq\infty$

$$\boxed{V=-\int_{A}^{B} E\cdot dl + C}$$

$$\boxed{V_{AB}=\frac{Q}{4\pi\epsilon r}+C}$$

Problem!

1. Two point charges $-3\mu C$ & $6\mu C$ are located at $(2,-1,3)$ & $(0,4,-2)$ respectively. find the potential at $(1,0,1)$. Assuming zero potential at infinity.

Soln:

$$V=\frac{Q}{4\pi\epsilon_0|r-r_1|}+\frac{Q_2}{4\pi\epsilon_0|r-r_2|}+C$$

Given, $r=a_x+a_z$

$r_1=2a_x+a_y+3a_z$

$r-r_1=-a_x+a_y-2a_z$

$|r-r_1|=\sqrt{1^2+1^2+2^2}=\sqrt{6}$.

⇒ Relationship between electrical field intensity & Electric potential (E & V)

$$V_{AB} = -V_{BA} \Rightarrow V_{AB} + V_{BA} = 0 \rightarrow \text{close loop. (KVL)}$$

$$V = -\int E \cdot dl \longrightarrow \text{open.}$$

⇒ $\oint E \cdot dl = 0 \longrightarrow$ maxwell's 2nd equation.

⇒ Net work in moving a charge in a closed path in static electric field 'E' is alway zero.

applying stoke's theorem.

$$\oint A \cdot dl = \int (\nabla \times A) \cdot ds$$

$$\rightarrow \quad \oint E \cdot dl = \int (\nabla \times E) \cdot ds = 0$$

$$\boxed{\nabla \times E = 0} \rightarrow \text{Maxwell's 2nd eq}^n$$

— Irrotational

⟶ conservative property.

According to formula.

$$V = -\int E \cdot dl$$

differentiating

$$dV = -E \cdot dl$$

$$E = E_x a_x + E_y a_y + E_z a_z$$
$$dl = dx a_x + dy a_y + dz a_z$$

$$E \cdot dl = -E_x dx - E_y dy - E_z dz \quad — (1)$$

$$dV = -E_x dx - E_y dy - E_z dz$$

$$dV = \frac{\partial V}{\partial x} dx + \frac{\partial V}{\partial y} dy + \frac{\partial V}{\partial z} dz$$

15.

$\Rightarrow E_x = -\frac{\partial V}{\partial x}$

$\Rightarrow E_y = -\frac{\partial V}{\partial y}$

$\Rightarrow E_z = -\frac{\partial V}{\partial z}$

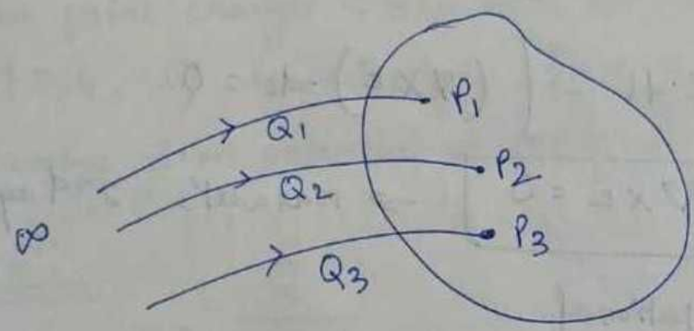
Putting this equatⁿ in eqⁿ ① we get

$$\therefore \quad E = -\frac{\partial V}{\partial x} a_x - \frac{\partial V}{\partial y} a_y - \frac{\partial V}{\partial z} a_z$$

$$E = -\left[\nabla \cdot V\right]$$

$$\boxed{E = -\nabla V}$$

$\Rightarrow$ Energy Density in Electrostatic field



$\infty \to P_1 \quad W = \frac{Q}{V}$

Charge free region so, $E = 0$.

$W_1 = 0$

$W_2 = Q_2 V_{21}$

$V_{21} \to$ voltage at point 2 due to Q_1

$W_3 = Q_3 (V_{31} + V_{32})$

$\therefore$ Total work done

$$W = 0 + Q_2 V_{21} + Q_3 (V_{31} + V_{32}) \quad \text{—— (i)}$$

$$W = Q_1 (V_{12} + V_{13}) + Q_2 (V_{23}) + 0 \quad \text{—— (ii)}$$

adding both the equation

$$2W = Q_2 (V_{21} + V_{23}) + Q_1 (V_{12} + V_{13}) + Q_3 (V_{31} + V_{32})$$

$$W = \frac{1}{2}\left[Q_1V_1 + Q_2V_2 + Q_3V_3\right]$$

For n^{th} number of point change 'Q'

$$W_{o} = \frac{1}{2}\sum_{k=1}^{n} Q_k V_k$$

⇒ In case of continuous charge Distribution we can write :— (line charge/ sheet charge/ volume charge)

$$W = \frac{1}{2}\int \rho_l V dl \rightarrow \text{For line charge}$$

$$W = \frac{1}{2}\int_s \rho_s V ds \longrightarrow \text{sheet charge.}$$

$$W = \frac{1}{2}\int_v \rho_v V dv \longrightarrow \text{Volume charge density.}$$

⇒ From volum charge density formula.

$$W = \frac{1}{2}\int \rho_v V dv \quad \text{—(1)}$$

$$W = \frac{1}{2}\int (\nabla \cdot D) V dv \rightarrow \text{From maxwell's } 1^{st} \text{ eq}^n.$$

As we know, -

ex! - $\nabla \cdot (VA) = V(\nabla \cdot A) + A \cdot \nabla V.$

Similiarly we can write eq^n 1

$$\Rightarrow W = \frac{1}{2}\int \left(\nabla \cdot (VD) - D \cdot \nabla V\right) dv$$

$$\Rightarrow W = \frac{1}{2}\int_v \nabla \cdot (VD) dv - \frac{1}{2}\int D \cdot \nabla V \, dv.$$

As we know from divergence theorem

$$\oint A \cdot ds = \int (\nabla \cdot A) dv$$

(17)

$\therefore$ From divergence theorem we can write above equation —

$$W = \cancel{\frac{1}{2}\oint V D\cdot ds}^{\,0} - \frac{1}{2}\int D\cdot\nabla V\, dv$$

$\therefore$ From eq^n $\boxed{E = -\nabla V.}$ ~~Relation bet^n~~ we can write.

putting this eq^n in above

$\therefore D = \epsilon E$

$$W = \frac{1}{2}\int D\cdot E dv$$

$$W = \frac{1}{2}\epsilon\int E\cdot E\, dv$$

$$W = \frac{1}{2}\epsilon\int |E|^2 dv$$

Differentiating both side w.r.t v $\rightarrow$ volume we get

$$\boxed{\frac{dw}{dv} = \frac{1}{2}\epsilon E^2}$$

J/m^3