Numerical

Module-4: Centre of Gravity and Moment of Inertia

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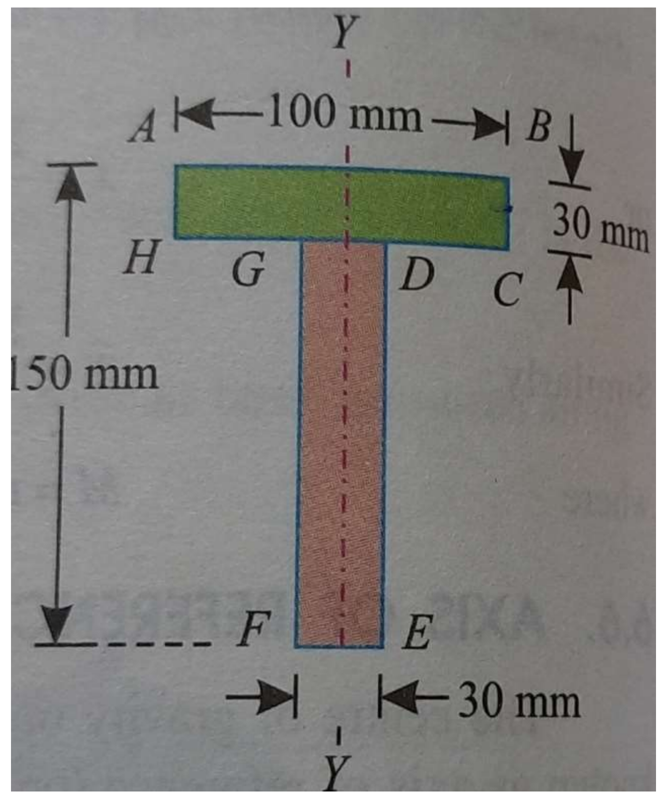
Q.1. Find the centre of gravity of a 100 $mm \times$ 150 mm $\times$ 30 mm T-section as shown in the following figure.

Sol: The given T-section is symmetry about the Y-Y axis as seen from the figure, therefore the centre of gravity lies on this axis.

We can split the section into two rectangles ABCH and EFGD.

Let the bottom line EF be the reference axis.



a) Considering the rectangle ABCH:

Area of ABCH, $A_1 = 100 \times 30 = 3000 mm^2$

Location of the centre of gravity with respect to the reference axis EF, $y_1 = (150 - 30/2)\ mm$

$= 135\ mm$

b) Considering the rectangle EFGD:

Area of EFGD, $A_2 = 120 \times 30 = 3600\ mm^2$

Location of the centre of gravity with respect to the reference axis EF, $y_2 = (150\text{-}30)/2 = 60\ mm$

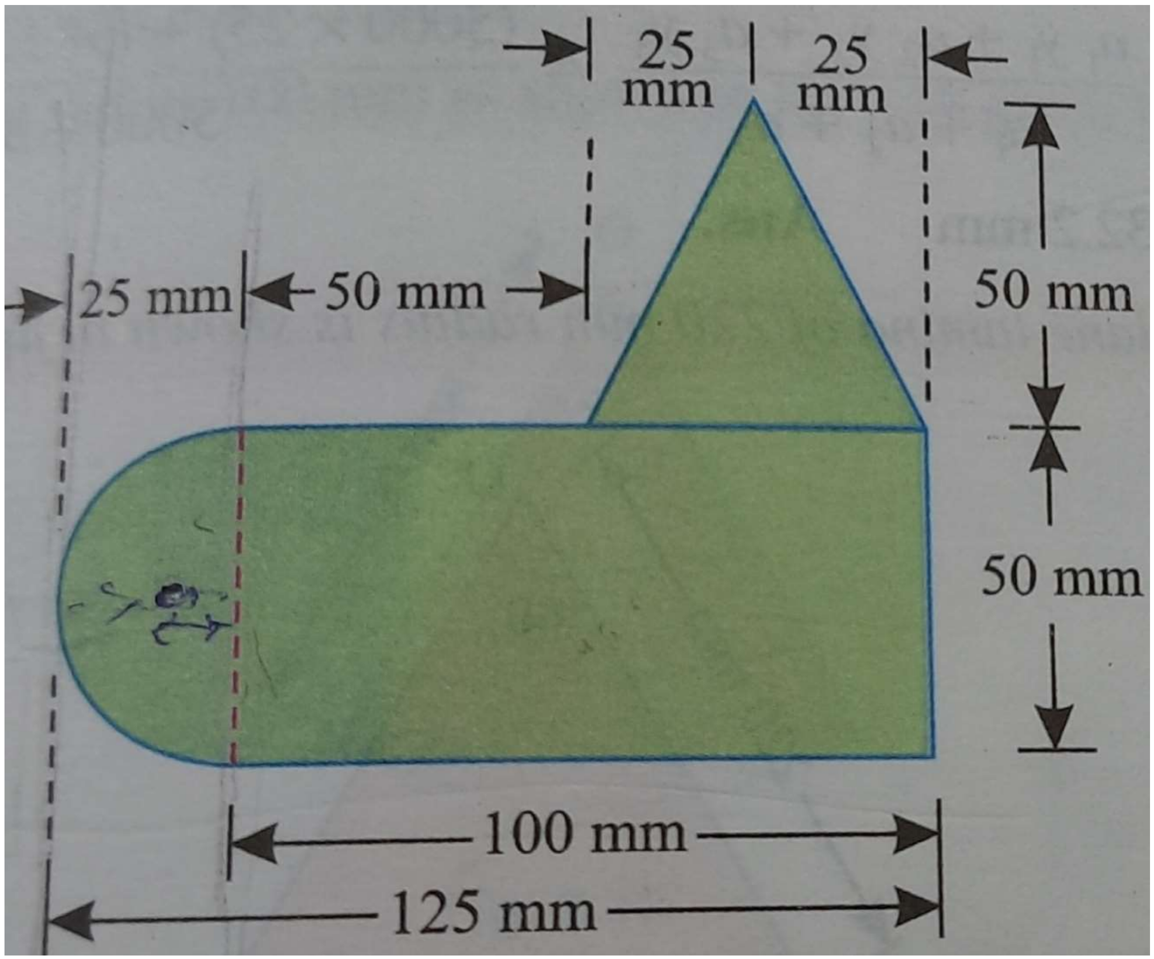
Centre of gravity of the T-section (combined rectangles) about the reference axis EF,

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2}$$

$$= \frac{(3000 \times 135) + (3600 \times 60)}{3000 + 3600} = 94.10\ mm$$

Remark: For symmetrical section, the centre of gravity will always lie in the axis about which the section is symmetry and have to find out only one unknown (either x or y coordinate).

Q.2. A uniform lamina as shown in the following figure consists of a rectangle, a circle and a triangle. Determine the centre of gravity of the lamina.



Sol: Here the section is not symmetrical about any axis, therefore we have to find out both the values of $\bar{x}$ and $\bar{y}$ for the centre of gravity of the lamina.

Let, the left edge of circular portion and bottom edge of the rectangular portion be the axes of references.

a) Considering the rectangular part:

Area of the rectangle, $A_1 = 100 \times 50 = 5000\ mm^2$

And, $x_1 = 25 + 100/2 = 75\ mm$

$y_1 = 50/2 = 25\ mm$

b) Considering the semicircular portion:

Area of the semicircular, $A_2 = (\pi r^2)/2 = (\pi \times 25^2)/2 = 982\ mm^2$

And, $x_2 = 25 - (4r)/(3\pi) = (4\times25)/(3\times \pi) = 14.4\ mm$

$y_2 = 50/2 = 25\ mm$

Note: Distance of the centre of gravity of a semicircle from the reference axis as the diameter (2*r* or base) is 4*r*/(3 π).

c) Considering the triangular part:

Area of the rectangle, $A_3 = 1/2 \times 50 \times 50 = 1250\ mm^2$

And, $x_3 = 25 + 50 + 25 = 100\ mm$

$y_3 = 50 + 50/3 = 66.7\ mm$

Distance between the centre of gravity of the section with respect to the reference axis as the left edge of the semicircle,

$$\bar{x} = \frac{A_1x_1 + A_2x_2 + A_3x_3}{A_1 + A_2 + A_3}$$

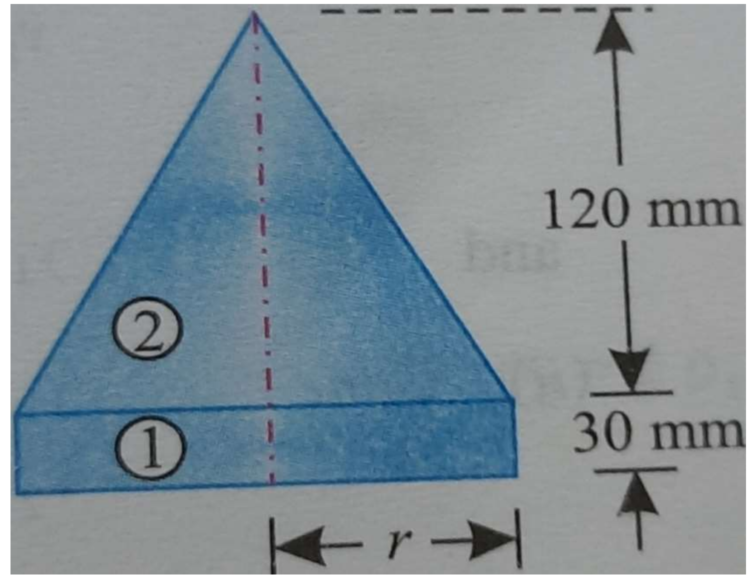
$$= \frac{(5000 \times 75) + (982 \times 14.4) + (1250 \times 100)}{5000 + 982 + 1250} = 71.10\ mm$$

Similarly, distance between the centre of gravity of the section with respect to the reference axis as the bottom of the rectangle,

$$\bar{y} = \frac{A_1y_1 + A_2y_2 + A_2y_2}{A_1 + A_2 + A_3}$$

$$= \frac{(5000 \times 25) + (982 \times 25) + (1250 \times 66.7)}{5000 + 982 + 1250} = 32.2\ mm$$

Q.3. A solid body as shown in the following figure is formed by joining the base of a right circular cone of height *h* to the equal base of a right circular cylinder of height *H*. Calculate the distance of the centre of mass of the solid from its plane face if $h = 120\ mm$ and $H = 30\ mm$.



Sol: The body is symmetric about the vertical axis hence the centre gravity of it lies in this axis.

Let *r* denotes the radius of the cylinder base In *mm* and the base of the cylinder be the reference axis of the solid body.

a) Considering the cylinder: Volume of the cylinder,

$$V_1 = \pi r^2 H = 30\pi r^2\ mm^2$$

And, $$y_1 = 30/2 = 15\ mm$$

b) Considering the right circular cone: Volume of the right circular cone,

$$V_2 = \frac{h}{3}\pi r^2$$

$$= (120\ \pi r^2)/3 = 40\pi r^2\ mm^2$$

And, $$y_1 = 30 + 120/4 = 60\ mm$$

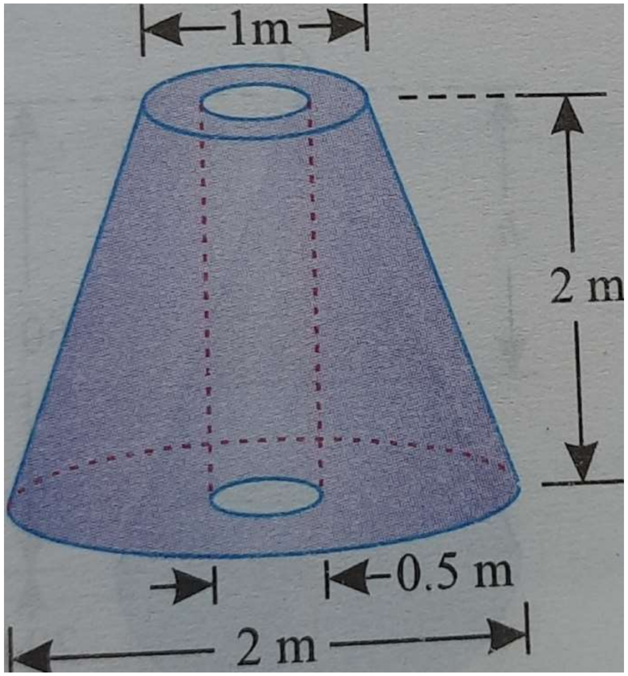
We know that the distance between the centre of gravity of the body and the reference as the base of the cylinder,

$$\bar{y} = \frac{V_1 y_1 + V_2 y_2}{V_1 + V_2}$$

$$= \frac{(30\pi r^2 \times 15) + (40\pi r^2 \times 60)}{30\pi r^2 + 40\pi r^2}$$

$$= 40.71\ mm$$

Q.4. A frustum of a solid right circular cone has an axial hole of 50*cm* diameter as shown in the figure. Determine the centre of gravity of the body.



Sol: The body is symmetric about the vertical axis, therefore its centre of gravity will lie on this axis.

For the sake of simplicity, let us consider a right circular cone OCD from which the right circular cone OAB is cut out as shown in the following figure.

Let, the base of the base of the right circular cone be the axis of reference.

a) Considering the right circular cone OCD as shown in the figure: Volume of OCD,

$$V_1 = \frac{H}{3}\pi r^2 = \frac{4}{3} \times \pi \times 1^2 = \frac{4\pi}{3}\ m^3$$

$y_1 = h/4 = 4/4 = 1\ m$

b) Consider right circular cone OAB: Volume of OAB

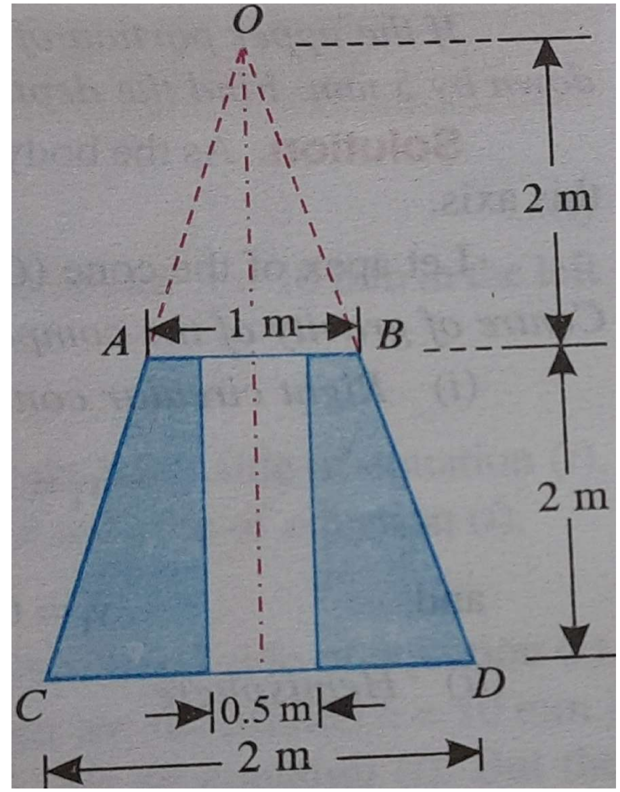
$$V_2 = \frac{H}{3}\pi r^2 = \frac{2}{3} \times \pi \times (0.5)^2 = \frac{\pi}{6}\ m^3$$

$y_2 = 2 + 2/4 = 2.5\ m$

c) Considering the circular hole:

$$V_3 = \frac{\pi}{4} d^2 h = \frac{\pi}{4}(0.5)^2 \times 2 = \frac{\pi}{8}\ m^3$$

$y_3 = 2/2 = 1\ m$



The centre of gravity of the body as shown in the above figure is found out from the following equation when the base of the cone is the reference axis,

$$\bar{y} = \frac{V_1 y_1 - V_2 y_2 - V_3 y_3}{V_1 - V_2 - V_3}$$

$$= \frac{\frac{4\pi}{3} \times 1 - \left(\frac{\pi}{6} \times 2.5\right) - \left(\frac{\pi}{8} \times 1\right)}{\frac{4\pi}{3} - \frac{\pi}{6} - \frac{\pi}{8}}$$

$$= \frac{\frac{4}{3} - \frac{2.5}{6} - \frac{1}{8}}{\frac{4}{3} - \frac{1}{6} - \frac{1}{8}}$$

$$= 0.76\ m$$

Thank You