Module-1:Viscous Flow

NUMERICALS
(Part-3)

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1) An oil of viscosity 9 poise and specific gravity 0.9 is flowing through a horizontal pipe of 60 mm diameter. If pressure drop in 100 m of the pipe is 1800 KN/m^2, determine,

~~(i)~~ Rate of flow of oil, ~~(ii)~~ Centreline velocity

(iii) velocity gradient at pipe wall

(iv) velocity and shear stress at 8 mm from the pipe wall

Soln

Given

$\mu = 9$ poise $= 0.9\ Ns/m^2$, $D = 60\ mm = 0.06\ m$

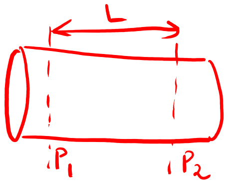
Specific gravity $= 0.9 \Longrightarrow \rho_{oil} = 900\ kg/m^3$, $R = 0.03\ m$

$L = 100\ m$, $P_1 - P_2 = 1800 \times 10^3\ N/m^2$

① Discharge (Q):

$$Q = u_{avg} \times Area$$

$$u_{avg} = \frac{1}{8\mu}\left(-\frac{\partial P}{\partial x}\right) \cdot R^2 ,$$



pressure gradient $= -\frac{\partial P}{\partial x} = \frac{P_1 - P_2}{L}$

$$= \frac{1800 \times 10^3}{100} = 18 \times 10^3 \ \underline{N/m^2}/m$$

$$= \frac{1}{8 \times 0.9} \times (18 \times 10^3) \times (0.03)^2$$

$$u_{avg} = 2.25 \ m/s$$

* $Q = u_{avg} \times \text{Area} = 2.25 \times \frac{\pi}{4}(0.06)^2 = 0.00636 \ m^3/s$

$$\boxed{Q = 6.36 \ L/s} \ \#$$

* <u>Calculate Reynold's number</u>:

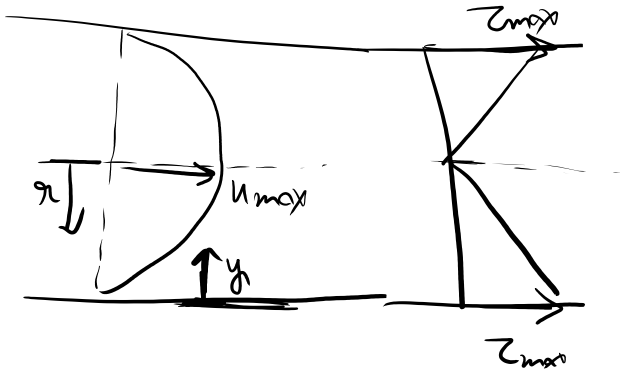
$$Re = \frac{\rho V D}{\mu} = \frac{900 \times 2.25 \times 0.06}{0.9}$$

$$= 135 < 2000$$

flow is Laminar. #

(ii) Centerline velocity: (U_{max})

→ (*) For laminar flow in circular pipe



$$\frac{U_{max}}{U_{avg}} = 2$$

$$\Rightarrow \boxed{U_{max} = 4.5 \text{ m/s}}$$

(iii) Velocity gradient at pipe wall: $\left[\frac{\partial u}{\partial y}\right]_{y=0}$

$$\tau = \mu\left(\frac{\partial u}{\partial y}\right)$$

$$\Rightarrow \tau_{max} = \mu \left[\frac{\partial u}{\partial y}\right]_{y=0} \qquad \Rightarrow \left[\frac{\partial u}{\partial y}\right]_{y=0} = \frac{\tau_{max}}{\mu}$$

Now,

$$\tau_{max} = \left(-\frac{\partial P}{\partial x}\right) \cdot \frac{R}{2}$$

$$= 18\times10^{3} \times \frac{0.03}{2}$$

$$\tau_{max} = 270 \quad N/m^2$$

$$\tau = -\frac{\partial P}{\partial x} \cdot \frac{r}{2}$$

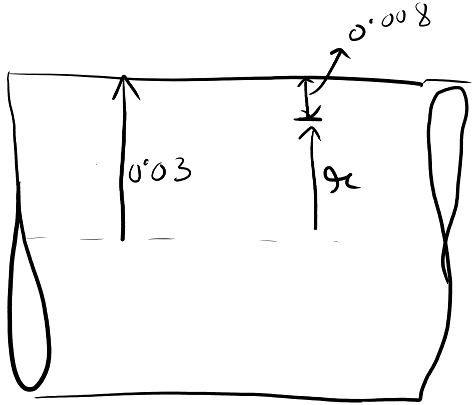
$$\tau_{max} = \left(-\frac{\partial P}{\partial x}\right) \cdot \frac{R}{2}$$

$$\left[\frac{\partial u}{\partial y}\right]_{y=0} = \frac{270}{0.9} = 300 \ \frac{ms^{-1}}{m} \quad \boxed{= 300 \ s^{-1}}$$

(iv) Velocity and shear stress at 8 mm from pipe wall;

$$R = 0.03\ m \quad , \quad y = 0.008\ m$$

$$r = R - y = 0.022\ m$$



$$u = \frac{1}{4\mu}\left(-\frac{\partial P}{\partial x}\right)\left(R^2 - r^2\right) = \frac{1}{4 \times 0.9} \times 18 \times 10^3 \times \left[0.03^2 - 0.022^2\right]$$

$$\boxed{u = 2.08\ m/s}$$

$$\tau = \left(-\frac{\partial P}{\partial x}\right) \cdot \frac{r}{2} = 18 \times 10^3 \times \frac{0.022}{2}$$

$$\boxed{\tau = 1980\ N/m^2 = 1.98\ kN/m^2}$$

- Two parallel plates kept 100mm apart have laminar flow of oil (of viscosity 24.5 poise) between them with a maximum velocity of 1.5 m/s, Calculate:
  - (i) Discharge per meter width, (ii) Shear stress at the plates
  - (iii) Difference in pressure between two points 20 m apart
  - (iv) Velocity gradient at the plates
  - (v) Velocity at 20 mm from the plate.

Soln

$t = 100\,mm = 0.1\,m$, $\mu = 24.5$ poise $= 2.45\ Ns/m^2$, $U_{max} = 1.5\ m/s$

① Discharge/width: $\left(\frac{Q}{B}\right)$

$Q = U_{avg} \cdot A$ & , $A = B \cdot t$

$\Rightarrow Q = U_{avg} \cdot B \cdot t$

$\Rightarrow \boxed{\frac{Q}{B} = U_{avg} \cdot t} \longrightarrow ①$

For laminar flow between parallel plates, $\frac{U_{max}}{U_{avg}} = 1.5$

$\Rightarrow U_{avg} = 1\ m/s$

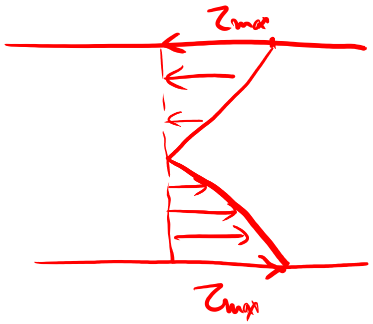
From ①,

$\frac{Q}{B} = 1 \times 0.1 = 0.1\ m^2/s$

$Q/B = 0.1\ m^3/s$ per 'm'

(ii) Shear stress at the plates:

$$\tau = \frac{1}{2}\left(-\frac{\partial P}{\partial x}\right)(t - 2y)$$



$$\tau_{max} = \left(-\frac{\partial P}{\partial x}\right) \cdot \frac{t}{2}$$

Now, we know, $U_{avg} = \frac{1}{12\mu}\left(-\frac{\partial P}{\partial x}\right) \cdot t^2$

$$\Rightarrow \left(-\frac{\partial P}{\partial x}\right) = \frac{12\,\mu \cdot U_{avg}}{t^2} = \frac{12 \times 2.45 \times 1}{0.1^2}$$

$$\left(-\frac{\partial P}{\partial x}\right) = 2940 \ N/m^3$$

$$\therefore \ \tau_{max} = 2940 \times \frac{0.1}{2} = 147 \ N/m^2$$

(iii) Diff in pressure betⁿ two points 20 m apart :

$L = 20 m$, $P_1 - P_2 = ?$

$$P_1 - P_2 = \frac{12\, \mu . U_{avg} . 20}{t^2} = \frac{12 \times 2.45 \times 1 \times 20}{0.1^2}$$

$$P_1 - P_2 = 58800 \; N/m^2 = 58.8 \; kN/m^2$$

$\boxed{OR}$

$$-\frac{\partial P}{\partial x} = 2940 \; N/m^3$$

$$\Rightarrow \frac{P_1 - P_2}{L} = 2940$$

$$\Rightarrow P_1 - P_2 = 58.8 \; kN/m^2$$

(iv) Velocity gradient at plate: $\left[\frac{\partial u}{\partial y}\right]_{y=0}$

$$\tau_{max} = \mu \cdot \left(\frac{\partial u}{\partial y}\right)_{y=0}$$

$$\Rightarrow \frac{\partial u}{\partial y} = \frac{\tau_{max}}{\mu}$$

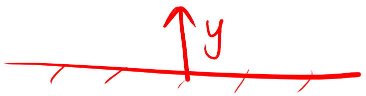
$$= \frac{147}{2.45}$$

$$\left[\frac{\partial u}{\partial x}\right]_{y=0} = 60 \; s^{-1}$$

$\divideontimes$

(v) Velocity at a dist 20 mm from the plate: $y = 20\,mm = 0{\cdot}02\,m$

$$u = \frac{1}{2\mu}\left(-\frac{\partial p}{\partial x}\right)\cdot\left(ty - y^2\right)$$



$$= \frac{1}{2 \times 2{\cdot}45} \times 2940 \times \left(0{\cdot}1 \times 0{\cdot}02 - 0{\cdot}02^2\right)$$

$$u = 0{\cdot}96\ m/s$$ #

- Flow rate of fluid (density 1000 kg/m^3) in a small diameter tube is 800 mm^3/sec. The length and diameter of the tube are 2 m and 0.5 mm. The pressure drop in 2 m length is 2 MPa. Find the viscosity of fluid.

Soln

$\rho = 1000 \ kg/m^3$, $Q = 800 \ mm^3/s = 800 \times 10^{-9} \ \frac{m^3}{s} = 8 \times 10^{-7} \ \frac{m^3}{s}$

$L = 2m$ $D = 0.5 \ mm = 0.0005 \ m$

$R = 0.00025 \ m$

$\Delta P = P_1 - P_2 = 2 \ MPa$, $\mu = ?$

$$\Delta P = P_1 - P_2 = \frac{32 \mu u_{avg} L}{D^2}$$

$$\Rightarrow \mu = \frac{\Delta P \cdot D^2}{32 u_{avg} \cdot L}$$

$$u_{avg} = \frac{Q}{A} = \frac{8 \times 10^{-7}}{\frac{\pi}{4} \times (0.0005)^2} = 4.07 \ m/s$$

$$\Delta P = 2 \ MPa = 2 \times 10^6 \ Pa = 2 \times 10^6 \ N/m^2$$

$$\mu = \frac{\Delta P \cdot D^2}{32 \ u_{avg} \ L} = \frac{2 \times 10^6 \times 0.0005^2}{32 \times 4.07 \times 2}$$

$$= \underline{\underline{0.00192}} \ Ns/m^2$$

Couette flow

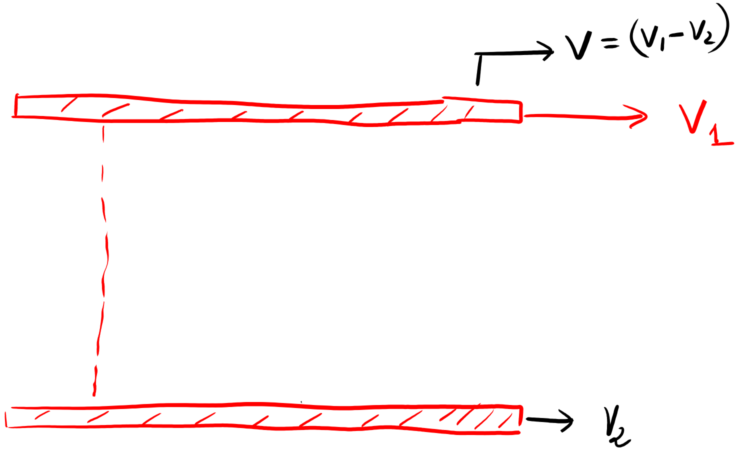
$$\frac{du}{dy} = 0$$

$\hookrightarrow y = A$

$$[u]_{(y=A)} = U_{max}$$

Case I $V_1 > V_2$

$V = V_1 - V_2$



Case II $V_1 = V_2$

Case III $(V_1 < V_2)$