Module-1:Viscous Flow

Viscous flow between two parallel plates

- Couette Flow
- Numericals (Part-2)

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(*) Water at 15° C flows betⁿ two large (↑ fixed) parallel plates kept 1·6 mm apart. Determine (i) Maxᵐ velocity (ii) Pressure drop per unit length $\left[\frac{P_1 - P_2}{L} = \left(-\frac{\partial P}{\partial x}\right)\right]$ (iii) Shear stress at the walls of the plates

if the avg velocity is 0·2 m/s. Viscosity of water is 0·01 poise.

Solⁿ

$U_{avg} = 0.2 \ m/s$, $\mu = 0.01$ poise $= 0.001 \ Ns/m^2$, $t = 1.6 \ mm = 0.0016 \ m$

$\boxed{1 \text{ poise} = 0.1 \ Ns/m^2}$

$$U_{max} = 1.5 \ U_{avg} = 0.3 \ m/s$$

Again,

$$U_{avg} = \frac{1}{12\mu}\left(-\frac{\partial P}{\partial x}\right)t^2$$

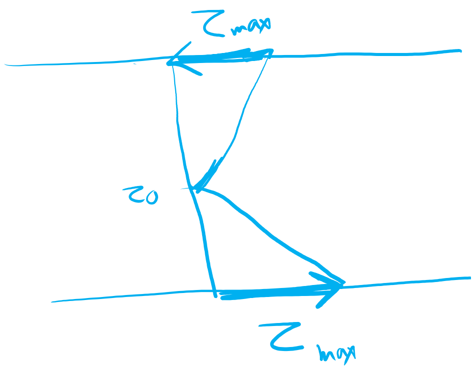
$$\Rightarrow -\frac{\partial P}{\partial x} = \frac{12\mu \ U_{avg}}{t^2} = \frac{12 \times 0.001 \times 0.2}{0.0016^2} = 937.5 \ \frac{N/m^2}{m}$$

$$\tau_{max} = \frac{1}{2}\left(-\frac{\partial p}{\partial x}\right) \cdot t$$

$$= \frac{1}{2} \times \cancel{0}\,37.5 \times 0.0016$$

$$\frac{N/m^2}{\cancel{m}} \times \cancel{m}$$

$$\boxed{\tau_{max} = 0.75 \ N/m^2}$$

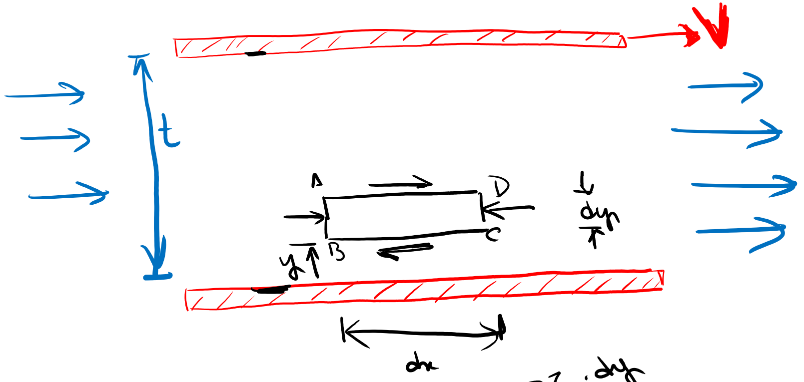


* Couette flow:

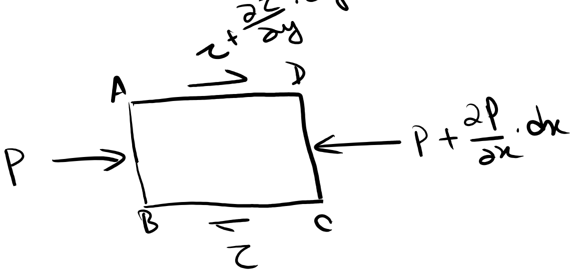
↳ flow between two parallel plates when one plate is moving relative to the other.

Consider two plates of unit width placed at 't' distance apart.

The top plate is moving with a velocity 'V'



$$P \cdot dy - \left(P + \frac{\partial P}{\partial x} \cdot dx\right) dy - \tau \cdot dx + \left(\tau + \frac{\partial \tau}{\partial y} \cdot dy\right) dx = 0$$



$$\Rightarrow \left(-\frac{\partial P}{\partial x} + \frac{\partial \tau}{\partial y}\right) dx\, dy = 0$$

$$\Rightarrow \frac{\partial \tau}{\partial y} = \frac{\partial P}{\partial x}$$

$$\frac{\partial}{\partial y}\tau = \frac{\partial P}{\partial x}$$

$$\Rightarrow \frac{\partial}{\partial y}\left(\mu \cdot \frac{du}{dy}\right) = \frac{\partial P}{\partial x}$$

$$\Rightarrow \frac{\partial^2 u}{\partial y^2} = \frac{1}{\mu}\frac{\partial P}{\partial x}$$

$$\Rightarrow \frac{\partial u}{\partial y} = \frac{1}{\mu}\cdot\frac{\partial P}{\partial x}\cdot y + C_1$$

$$\Rightarrow u = \frac{1}{\mu}\frac{\partial P}{\partial x}\cdot\frac{y^2}{2} + C_1\cdot y + C_2 \longrightarrow (1)$$

Apply boundary conditions:

$$y = 0, \quad u = 0$$

$$(1) \Rightarrow C_2 = 0$$

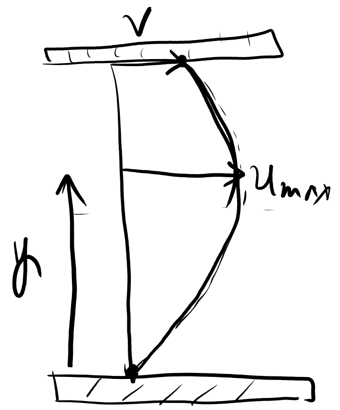
$$y = t, \quad u = V$$

$$\Rightarrow V = \frac{1}{\mu}\cdot\frac{\partial P}{\partial x}\cdot\frac{t^2}{2} + C_1\cdot t$$

$$\Rightarrow C_1 = \frac{V}{t} + \frac{1}{\mu}\left(-\frac{\partial P}{\partial x}\right)\cdot\frac{t}{2}$$

$$① \Rightarrow u = \frac{1}{2\mu} \cdot \frac{\partial P}{\partial x} \cdot y^2 + \frac{V}{t} \cdot y + \frac{1}{2\mu}\left(-\frac{\partial P}{\partial x}\right) \cdot ty$$

$$\boxed{u = \frac{Vy}{t} + \frac{1}{2\mu}\left(-\frac{\partial P}{\partial x}\right)\left[ty - y^2\right]} \longrightarrow ①$$



***Max^m velocity*:**

$$\frac{\partial u}{\partial y} = 0 \quad \text{and find } y = ?$$

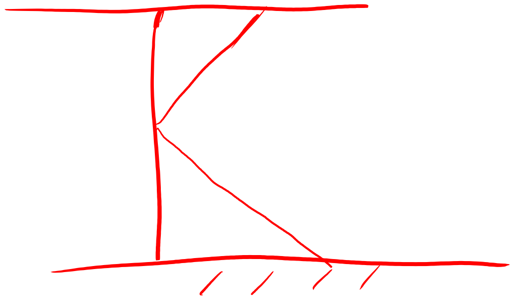
$$u_{max} = \text{by substituting } y \text{ in eq}^n ①$$

$$\tau = \mu \cdot \frac{\partial u}{\partial y} = \mu \cdot \left[\frac{1}{\mu} \cdot \frac{\partial P}{\partial x} \cdot y + \frac{V}{t} - \frac{1}{\mu} \left(-\frac{\partial P}{\partial x} \right) \cdot \frac{t}{2} \right]$$

$$\boxed{\tau = \frac{V \cdot \mu}{t} - \left(-\frac{\partial P}{\partial x} \right) \left(y + \frac{t}{2} \right)}$$

* For simple couette flow, $\left[-\frac{\partial P}{\partial x} = 0 \right]$

$\hookrightarrow u = \frac{V \cdot y}{t}$



*

- Water flows through a 100 mm diameter pipe with a velocity of 0.015 m/sec. If the kinematic viscosity of water is $1.13X10^{-6}\ m^2$/sec, find the friction factor. [(A) 0.0015 (B) 0.032 (C) 0.050 (d) 0.048] options [GATE 2009]

Soln

$D = 0.1\ m$, $u_{avg} = 0.015\ m/sec$, $\nu = 1.13 \times 10^{-6}\ m^2/s$, $f = ?$

$\rho = 1000\ kg/m^3$,

$$Re = \frac{\rho V \cdot D}{\mu} = \frac{VD}{\nu} = \frac{0.015 \times 0.1}{1.13 \times 10^{-6}}$$

$h_f =$

$$f = \frac{64}{Re}$$

$$Re = 1327.4$$

Hence friction factor $= \dfrac{64}{Re} = 0.048$ #

Try Solving some more numerical to improve yourself.

Thank you.