Module-1:Viscous Flow

Viscous flow between two parallel plates

(when both plates are in rest)

- To understand the ***shear stress distribution***.
- To understand the ***velocity distribution*** and find out the maximum and average velocity.
- To find out the ***pressure drop*** for a particular length.

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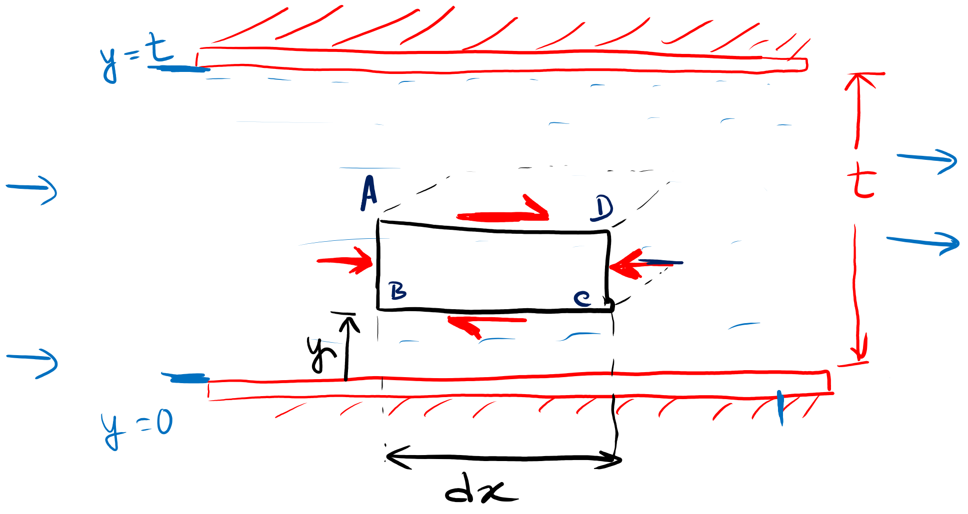
Consider two fixed plates which are at a distance 't' from each other.

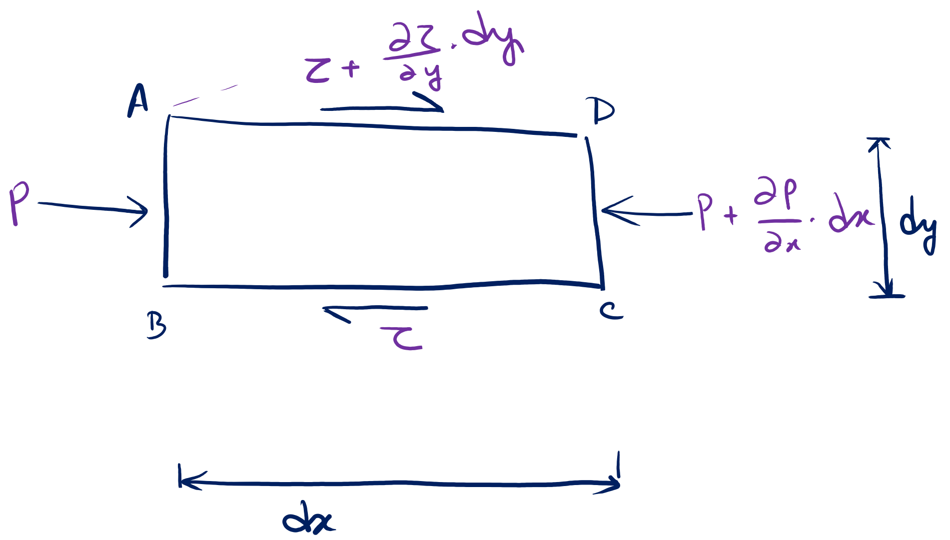
Consider an element (ABCD) of unit width (=1) at a distance 'y' from the bottom plate.

$dx \rightarrow$ length of element

$dy \rightarrow$ Thickness of element



Now, the forces acting on the element are,

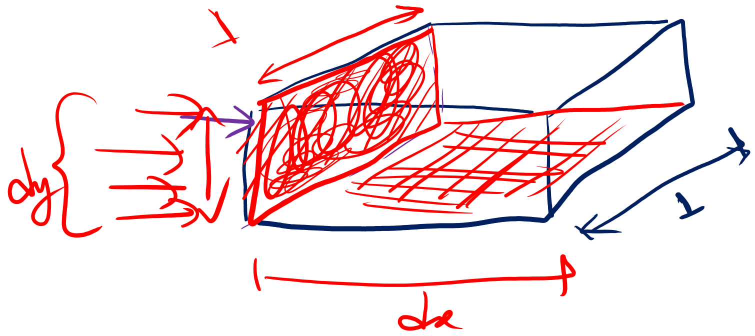


(i) ON AB $= P \cdot dy \cdot 1$ (+ve)

(ii) CD $= \left(P + \frac{\partial P}{\partial x} \cdot dx\right) \cdot dy \cdot 1$ (−ve)

(iii) BC $= \tau \cdot dx \cdot 1$ (−ve)

(iv) AD $= \left(\tau + \frac{\partial \tau}{\partial y} \cdot dy\right) dx \cdot 1$ (+ve)



$$P\cdot dy - \left(P + \frac{\partial P}{\partial x}dx\right)\cdot dy - \tau\cdot dx + \left(\tau + \frac{\partial \tau}{\partial y}\cdot dy\right)\cdot dx = 0$$

$$\Rightarrow \quad -\frac{\partial P}{\partial x}dx\,dy + \frac{\partial \tau}{\partial y}\cdot dx\,dy = 0$$

$$\Rightarrow \quad \boxed{\frac{\partial \tau}{\partial y} = \frac{\partial P}{\partial x}}$$

① Velocity distribution

$$\boxed{\tau = \mu \cdot \frac{\partial u}{\partial y}}$$

$$\frac{\partial \tau}{\partial y} = \frac{\partial p}{\partial x}$$

$$\Rightarrow \frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right) = \frac{\partial p}{\partial x}$$

$$\Rightarrow \frac{\partial^2 u}{\partial y^2} = \frac{1}{\mu} \frac{\partial p}{\partial x}$$

Integrating,

$$\Rightarrow \frac{\partial u}{\partial y} = \frac{1}{\mu} \frac{\partial p}{\partial x} \cdot y + C_1$$

$$\Rightarrow \boxed{u = \frac{1}{\mu} \cdot \frac{\partial p}{\partial x} \cdot \frac{y^2}{2} + C_1 y + C_2} \longrightarrow ①$$

Applying boundary condition,

$$y = 0,\quad u = 0,\quad ① \Rightarrow 0 = C_2$$

$$y = t,\quad u = 0,\quad ① \Rightarrow 0 = \frac{1}{\mu}\frac{\partial p}{\partial x}\cdot\frac{t^2}{2} + c_1\cdot t + 0$$

$$\Rightarrow c_1 = \frac{1}{2\mu}\left(-\frac{\partial p}{\partial x}\right)\cdot t$$

Hence,

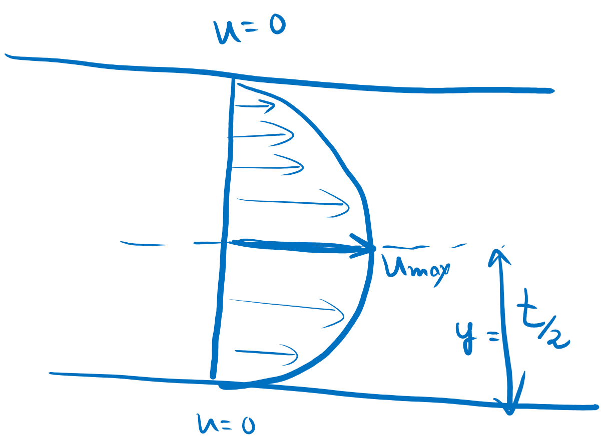
$$① \Rightarrow \quad u = \frac{1}{2\mu}\frac{\partial p}{\partial x}\cdot y^2 + \underline{\frac{1}{2\mu}\left(-\frac{\partial p}{\partial x}\right)}\cdot t\cdot y$$

$$\boxed{u = \frac{1}{2\mu}\left(-\frac{\partial p}{\partial x}\right)\left[ty - y^2\right]}$$

Max^m velocity

$y = \frac{t}{2},$

$$U_{max} = \frac{1}{2\mu}\left(-\frac{\partial p}{\partial x}\right)\cdot\left[t\cdot\frac{t}{2} - \left(\frac{t}{2}\right)^2\right]$$



$$\boxed{U_{max} = \frac{1}{8\mu}\left(-\frac{\partial p}{\partial x}\right)t^2}$$

Avg. velocity ($\bar{u}$ or u_{avg})

$$dQ = u\cdot(dy\cdot 1)$$

$$= \frac{1}{2\mu}\left(-\frac{\partial p}{\partial x}\right)\cdot\left[ty - y^2\right]\cdot dy$$

$$\int_0^Q dQ = \frac{1}{2\mu}\left(-\frac{\partial P}{\partial x}\right)\int_0^t \left(t y - y^2\right) dy$$

$$= \frac{1}{2\mu}\cdot\left(-\frac{\partial P}{\partial x}\right)\cdot\left[\frac{t \cdot y^2}{2} - \frac{y^3}{3}\right]_0^t$$

$$Q = \frac{1}{12\mu}\left(-\frac{\partial P}{\partial x}\right)\cdot t^3$$

$$U_{avg} = \frac{Q}{A} = \frac{Q}{t}$$

$$\boxed{U_{avg} = \frac{1}{12\mu}\cdot\left(-\frac{\partial P}{\partial x}\right)\cdot t^2}$$

Ratio of max & avg velocity

$$\frac{u_{max}}{u_{avg}} = \frac{1/8}{1/12} = 1.5$$

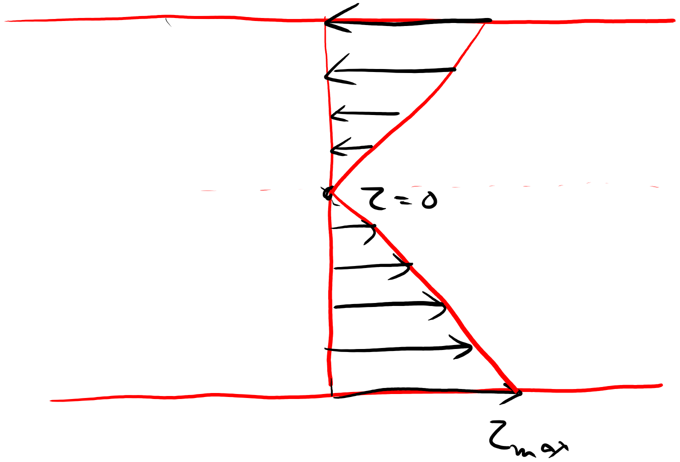
$$\boxed{u_{max} = 1.5\, u_{avg}}$$

Shear stress

$$\tau = \mu \cdot \frac{\partial u}{\partial y}$$

$$= \mu \cdot \frac{\partial}{\partial y}\left[\frac{1}{2\mu}\left(-\frac{\partial p}{\partial x}\right)\left(ty - y^2\right)\right]$$

$$\boxed{\tau = \frac{1}{2}\left(-\frac{\partial p}{\partial x}\right)(t-2y)} \longrightarrow \underline{\underline{\text{Linear}}}\ \text{variation of shear stress}$$



$y = 0$, $\boxed{\tau_{max} = \frac{1}{2}\left(-\frac{\partial p}{\partial x}\right) . t}$

$y = t$, $\boxed{\tau_{max} = \frac{1}{2}\left(\frac{-\partial p}{\partial x}\right) . (-t)}$

$y = \frac{t}{2}$, $\tau_{min} = \underline{\underline{0}}$

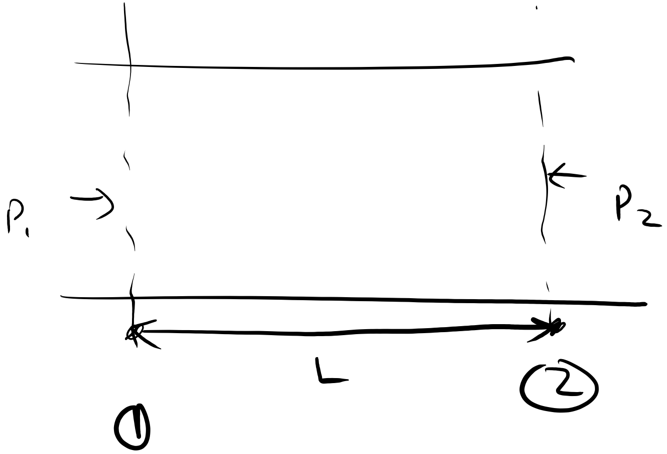
⊛ Pressure drop betⁿ two points at a dist 'L' apart:

$$U_{avg} = \frac{1}{12\mu}\left(-\frac{\partial P}{\partial x}\right) \cdot t^2$$

$$\Rightarrow \left(-\frac{\partial P}{\partial x}\right) = \frac{12 \cdot \mu \cdot U_{avg}}{t^2}$$

$$\Rightarrow \int_{P_1}^{P_2} -\partial P = \frac{12 \mu U_{avg}}{t^2} \int_0^L \partial x$$

$$\Rightarrow \boxed{P_1 - P_2 = \frac{12 \mu \cdot U_{avg} \cdot L}{t^2}}$$



$$\boxed{\frac{P_1 - P_2}{\rho g} = \frac{12 \mu \cdot U_{avg} \cdot L}{\rho g t^2}}$$

Q.3 Calculate (i) pressure gradient along the flow, (ii) average velocity and discharge of oil of viscosity 0.02 N. s/m2 flowing between two parallel stationary plates 1 m wide maintained 10 mm apart. Velocity at midway between the plates is 2 m/s

Sol^n

$\mu = 0.02 \; Ns/m^2$

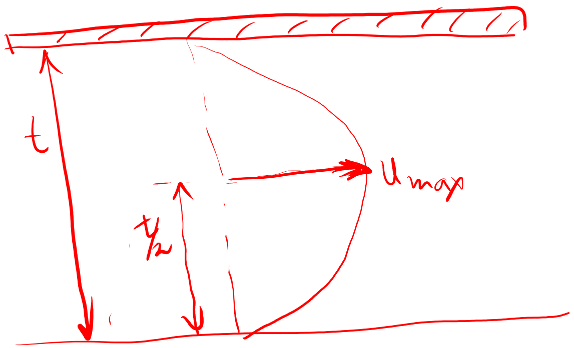
$t = 10 \; mm = 0.01 \; m \;, \quad B = 1 \; m$

$U_{max} = 2 \; m/s$

$\left(-\frac{\partial p}{\partial x}\right) = ?$

$U_{avg} = ?$ ✓

$Q = ?$ ✓



(i)

$$\frac{U_{max}}{U_{avg}} = 1.5$$

$$\Rightarrow U_{avg} = \frac{2}{1.5} = \frac{4}{3} \; m/s = 1.33 \; m/s$$

(ii)

$$Q = U_{avg} \times A = \frac{4}{3} \times (0.01 \times 1) = 0.0133 \; m^3/s$$

$$u_{avg} = \frac{1}{12\mu}\left(-\frac{\partial p}{\partial x}\right) \cdot t^2$$

$$\Rightarrow -\frac{\partial p}{\partial x} = \frac{12\mu \cdot u_{avg}}{t^2} = \frac{12 \times 0.02 \times \frac{4}{3}}{1^2} = 0.32 \ N/m^3$$

$$= 0.32 \ \frac{N/m^2}{m}$$

Try Solving some more numericals to improve yourself.

Thank you.