<u>*Module-1:Viscous Flow*</u>

(Viscous flow through circular pipe)

- Numericals (Part-1)

Arindom Das

Assistant Professor

BBEC Kokrajhar

(*) A laminar flow is taking place inside a pipe of diameter 200 mm. The max^m velocity is 15 m/s. Find out the mean velocity & radius where the mean velocity will occur. Also calculate the velocity at a distance of 4 cm from the pipe boundary.

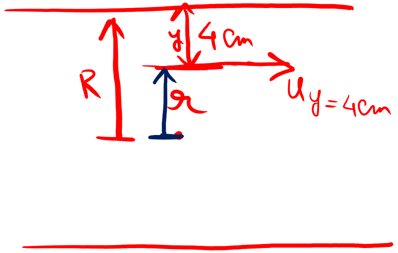
Sol^n

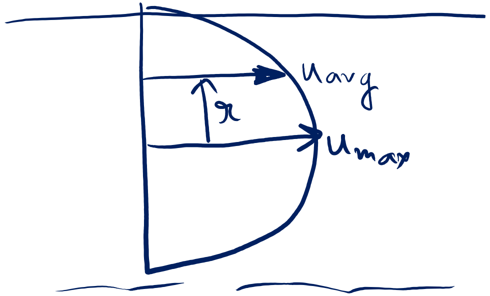
$D = 200\ mm = 0.2\ m$, $\quad R = 0.1\ m$

$u_{max} = 15\ m/s$,

$\boxed{u_{avg} = ?}$ $\quad r_{u_{avg}} = ?$

$u_{y=4cm} = ?$





(i) We know that, for laminar flow,

$$U_{max} = 2 \cdot U_{avg}$$

$$\Rightarrow U_{avg} = \frac{U_{max}}{2} = \frac{15}{2} = 7.5 \text{ m/s}$$

(ii) $r_{u_{avg}}$: $\left(r_{u_{avg}} = r\right)$

$$u = \frac{1}{4\mu}\left(-\frac{\partial P}{\partial x}\right) \cdot \left(R^2 - r^2\right)$$

[If we replace $u = U_{avg}$, then $r = r_{u_{avg}}$]

$$\Rightarrow u = \underbrace{\left[\frac{1}{4\mu}\left(-\frac{\partial P}{\partial x}\right) R^2\right]}_{U_{max}} \left(1 - \frac{r^2}{R^2}\right)$$

$$\boxed{u = u_{max} \cdot \left(1 - \frac{r^2}{R^2}\right)}$$

$$\Rightarrow 7.5 = 15 \left(1 - \frac{r^2}{R^2}\right)$$

$$\Rightarrow 1 - \frac{r^2}{0.01} = \frac{1}{2}$$

$$\Rightarrow \frac{r^2}{0.01} = 0.5$$

$$\Rightarrow r = 0.0707 \text{ m}$$

$$= 7.07 \text{ mm}$$

* Hence the avg velocity will occur at a dist of <u>7.07 mm</u> from center.

#

(iii) $y = 4\ cm = 0.04\ m$

$R = 0.1\ m$

$r = R - y = 0.1 - 0.04 = 0.06\ m$

$$\therefore\ u = \frac{1}{4\mu}\left(-\frac{\partial p}{\partial x}\right)\cdot\left(R^2 - r^2\right)$$

$$\boxed{u = U_{max}\left(1 - \frac{r^2}{R^2}\right)}$$

$$\Rightarrow u = 15\left(1 - \frac{(0.06)^2}{(0.1)^2}\right) = 9.6\ m/s$$

#

(*2) An oil of viscosity 0.1 Ns/m^2 and relative density 0.9 is flowing through a circular pipe of diameter 50 mm and length 300 m. The rate of flow through the pipe is 3.5 l/s. Find the pressure drop in a length 300 m and also calculate shear stress at pipe boundary.

Soln

$\mu = 0.1\ Ns/m^2$, Relative density = 0.9

$D = 50\ mm = 0.05\ m$, $Q = 3.5\ l/s = 3.5 \times 10^{-3}\ m^3/s$

$L = 300\ m$

, $R = 0.025\ m$

$P_1 - P_2 = ?$

$\tau_{max} = \tau_{boundary} = ?$

u_{avg}

* **Type of flow:**

$$Re = \frac{\rho u_{avg} D}{\mu} = \frac{900 \times 1.783 \times 0.05}{0.1} = 802.3 < 2000$$

$\mu = 0.1\ Ns/m^2$

$D = 0.05\ m$

$\rho_{oil} = 0.9 \times 1000$

$= 900\ kg/m^3$

$$u_{avg} = \frac{Q}{A} = \frac{3.5 \times 10^{-3}}{\frac{\pi}{4} \times (0.05)^2}$$

$= 1.783\ m/s$

* Hence the flow is Laminar *

$$\frac{P_1 - P_2}{\rho g} = \frac{32\,\mu\,.\,\bar{u}\,.\,L}{\rho g\,D^2}$$

$$\Rightarrow \boxed{P_1 - P_2 = \frac{32\,\mu\,\bar{u}\,L}{D^2}} \rightarrow \text{Laminar flow}$$

$$= \frac{32 \times 0{\cdot}1 \times 1{\cdot}783 \times 300}{(0{\cdot}05)^2} = 684672 \ N/m^2$$

$$= 684{\cdot}67 \ KN/m^2$$

Now, Pressure gradient,

$$-\frac{\partial P}{\partial x} = \frac{P_1 - P_2}{L} = 2282{\cdot}2 \ (N/m^2)/m$$

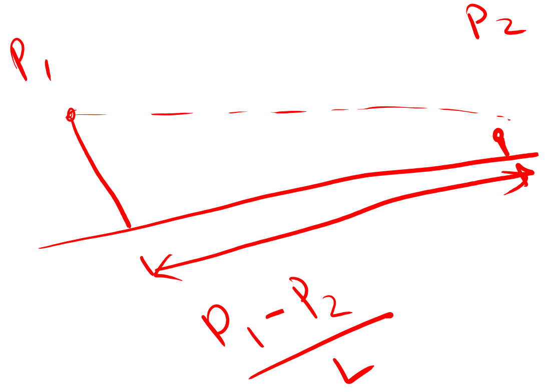
$$\tau_{boundary} = \left(-\frac{\partial p}{\partial x}\right)\frac{R}{2}$$

$$= 2282.2 \times \frac{0.025}{2}$$

$$\tau_{max} = \tau_{bound} = 28.53 \ N/m^2 \quad \#$$

$$\tau = \left(-\frac{\partial p}{\partial x}\right)\frac{r}{2}$$

$\left(-\frac{\partial p}{\partial x}\right) \rightarrow$ pressure gradien.



* Flow rate of a fluid ($\rho = 1000\ kg/m^3$) in a small diameter tube is $800\ mm^3/s$. The length and diameter of the tube is 2 m and 0.5 mm respectively. The pressure drop in 2 m length is 2 MPa. Find the viscosity of fluid:

(Gate - 2007)

Soln

$\rho = 1000\ kg/m^3$, $Q = 800\ mm^3/s = 800 \times 10^{-9}\ \frac{m^3}{s} = 8 \times 10^{-7}\ \frac{m^3}{s}$

$L = 2m$ $D = 0.5\ mm = 0.0005\ m$

$R = 0.00025\ m$

$\Delta P = P_1 - P_2 = 2\ MPa$, $\mu = ?$

$$\Delta P = P_1 - P_2 = \frac{32\ \mu\ u_{avg} L}{D^2}$$

$$\Rightarrow \mu = \frac{\Delta P \cdot D^2}{32\ u_{avg} L}$$

$$U_{avg} = \frac{Q}{A} = \frac{8 \times 10^{-7}}{\frac{\pi}{4} \times (0.0005)^2} = 4.07 \ m/s$$

$$\Delta P = 2 \ MPa = 2 \times 10^6 \ Pa = 2 \times 10^6 \ N/m^2$$

$$\mu = \frac{\Delta P \cdot D^2}{32 \ U_{avg} \ L} = \frac{2 \times 10^6 \times 0.0005^2}{32 \times 4.07 \times 2}$$

$$= 0.00192 \ Ns/m^2$$

Try Solving some more numericals to improve yourself.

Thank you.