Module-1:Viscous Flow

(Viscous flow through circular pipe)

- To understand the ***shear stress distribution*** inside a circular pipe.
- To understand the ***velocity distribution*** and find out the maximum and average velocity.
- To find out the ***pressure drop*** inside the pipe for a particular length.

Arindom Das

Assistant Professor

BBEC Kokrajhar

Shear stress distribution

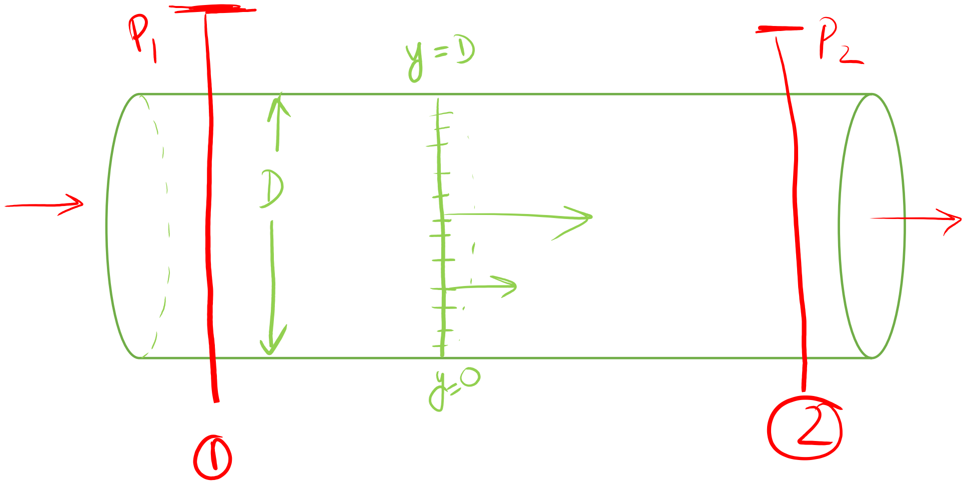
Velocity distribution

Pressure drop

diameter

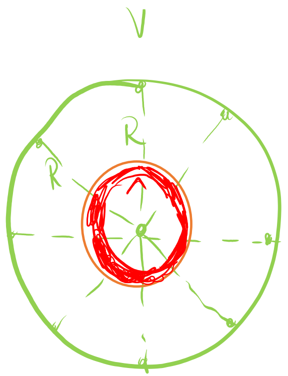
$h_1 - h_2$

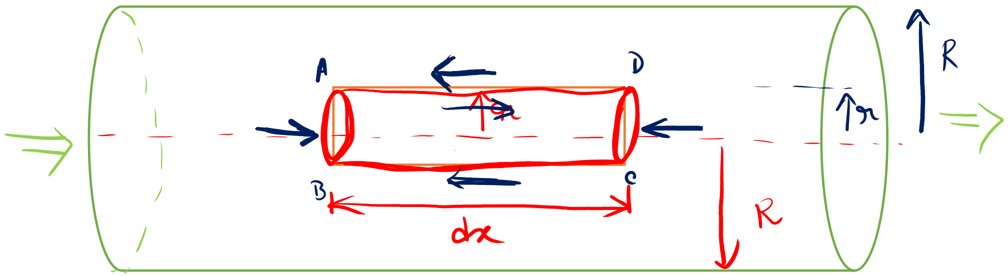
$P_1 - P_2$



* Consider a pipe of radius R & diameter D, $D = 2R$

→ Consider a cylindrical (hollow) element inside the (flow) pipe such that the internal radius is 'r' thickness dr

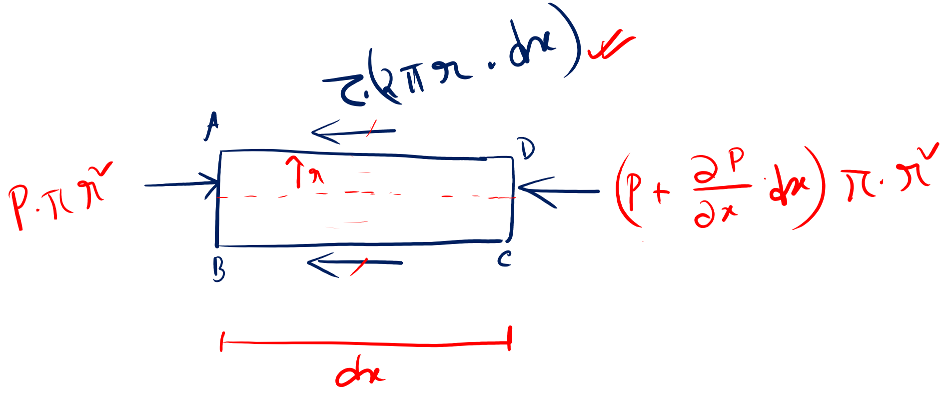




Total radius of the element $= r + dr$

The length of the element is 'dx'

P τ



C/s area of element $= \pi r^2$

The forces acting on the cylindrical element (ABCD)

(i) ON AB pressure force $= P \cdot \pi r^2$

(ii) ON CD " " $= \left(P + \frac{\partial P}{\partial x} \cdot dx\right) \pi r^2$

(iii) On the surface shear stress $= \tau \cdot 2\pi r \cdot dx$

From Newton's 2nd Law,

$$F = m \cdot a \quad , \quad a = 0$$

$$\Rightarrow p \cdot \pi r^2 - \left(p + \frac{\partial p}{\partial x} \cdot dx\right) \pi r^2 - \tau \cdot 2\pi r \cdot dx = 0$$

$$\Rightarrow - \frac{\partial p}{\partial x} \cdot \pi r^2 \, dx - \underline{\underline{\tau}} \cdot 2\pi r \cdot dx = 0$$

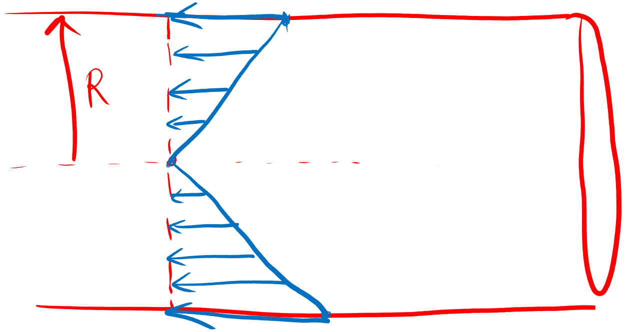
$$\Rightarrow \boxed{\tau = - \frac{\partial p}{\partial x} \cdot \frac{r}{2}}$$

Distribution of Shear stress

$$\tau = -\frac{\partial p}{\partial x} \cdot \frac{r}{2}$$

At center, $r = 0$, $\tau_{min} = 0$

At boundary, $r = R$, $\tau_{max} = -\frac{\partial P}{\partial x} \cdot \frac{R}{2}$



The variation will be linear.

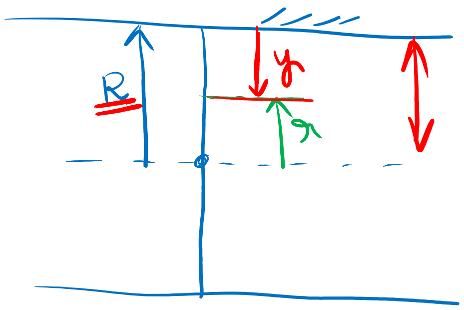
* Shear stress distribution diagram

The max^m shear stress occurs at boundary & decreases linearly and becomes zero at the centre of pipe.

Velocity distribution

$$\tau = -\frac{\partial p}{\partial x} \cdot \frac{r}{2} \longrightarrow (\mathrm{I})$$

As per Newton's Law of viscosity, $\tau = \mu \cdot \frac{du}{dy}$



$$y = R - r \Rightarrow dy = -dr$$

Hence,

$$\tau = -\mu \cdot \frac{du}{dr} \longrightarrow (\mathrm{II})$$

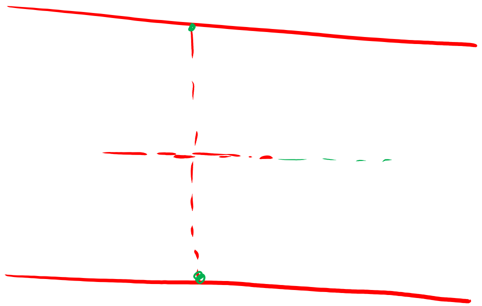
(I) & (II),

$$\mu \frac{du}{dr} = \frac{\partial p}{\partial x} \cdot \frac{r}{2}$$

$$\Rightarrow \int du = \frac{1}{2\mu} \cdot \frac{\partial p}{\partial x} \int (r \cdot dr)$$

$$\Rightarrow \quad u = \frac{1}{2\mu} \frac{\partial p}{\partial x} \cdot \int r\,dr$$

$$u = \frac{1}{4\mu} \frac{\partial p}{\partial x} \cdot r^2 + C_1 \longrightarrow \text{(A)}$$



when, * $r = R$ (at boundary), No slip condition $u = 0$,

$$\text{(A)} \Rightarrow 0 = \frac{1}{2\mu} \cdot \frac{\partial p}{\partial x} \cdot \frac{R^2}{2} + C_1$$

$$\Rightarrow C_1 = -\frac{1}{4\mu} \cdot \frac{\partial p}{\partial x} \cdot R^2$$

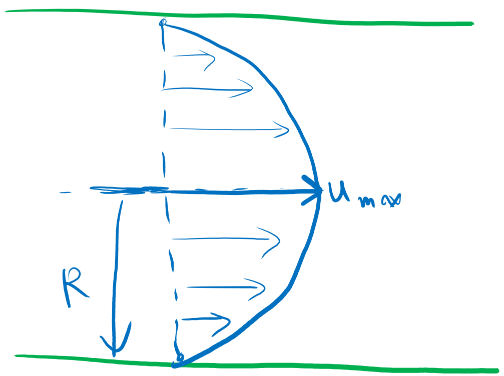
(A) $\Rightarrow u = \frac{1}{4\mu} \cdot \frac{\partial p}{\partial x} \cdot r^2 - \frac{1}{4\mu} \cdot \frac{\partial p}{\partial x} \cdot R^2$

$$\boxed{u = \frac{1}{4\mu}\left(-\frac{\partial p}{\partial x}\right)\left[R^2 - r^2\right]}$$

→ Hence the velocity distribution curve will have parabolic variation

The max^m velocity will be at the center, $r = 0$

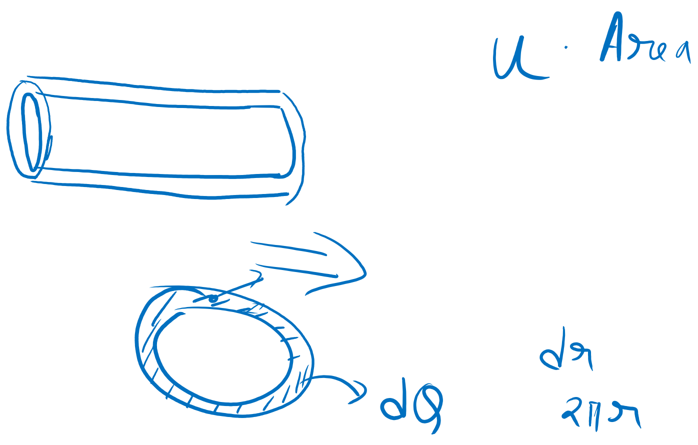
$$u_{max} = \frac{1}{4\mu}\left(-\frac{\partial p}{\partial x}\right) \cdot R^2$$



V. D. D

Avg velocity

$dQ \rightarrow$ discharge through the elementry strip



$$dQ = u \cdot \text{area}$$

$$dQ = \frac{1}{4\mu} \cdot \left(-\frac{\partial p}{\partial x}\right) \cdot \left(R^2 - r^2\right) \left(2\pi r \cdot dr\right)$$

Total discharge

$$Q = \int_0^Q dQ = \int_0^R \frac{2\pi}{4\mu} \left(-\frac{\partial p}{\partial x}\right) \left[R^2 \cdot r - r^3\right] dr$$

$$Q = \frac{\pi}{2\mu} \cdot \left(-\frac{\partial p}{\partial x}\right) \cdot \int_0^R \left(R^2 r - r^3\right) dr$$

$$= \frac{\pi}{2\mu} \cdot \left(-\frac{\partial p}{\partial x}\right) \cdot \left[R^2 \cdot \frac{r^2}{2} - \frac{r^4}{4}\right]_0^R$$

$$= \frac{\pi}{2\mu} \cdot \left(-\frac{\partial p}{\partial x}\right) \left[\frac{R^4}{2} - \frac{R^4}{4}\right]$$

$$= \frac{\pi}{2\mu} \cdot \left(-\frac{\partial p}{\partial x}\right) \cdot \frac{R^4}{4}$$

$$Q = \frac{\pi}{8\mu}\left(-\frac{\partial p}{\partial x}\right)R^4$$

Total area of pipe $= \pi R^2$

$\therefore$ Average velocity, $\bar{u} = \boxed{u_{avg} = \frac{1}{8\mu}\left(-\frac{\partial p}{\partial x}\right) \cdot R^2}$

$$\boxed{u_{max} = \frac{1}{4\mu} \cdot \left(-\frac{\partial p}{\partial x}\right) R^2}$$

Ratio:

$$\frac{u_{max}}{u_{avg}} = 2 \Rightarrow \boxed{u_{max} = 2\,u_{avg}}$$

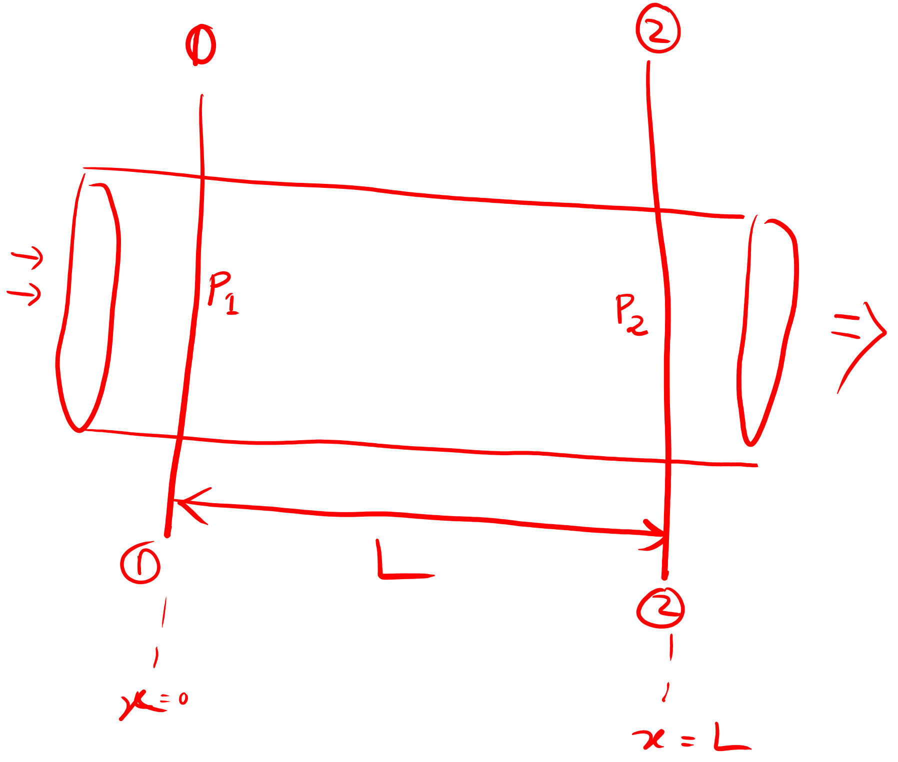
* Pressure drop for a given length (L)

$$\bar{u} = \frac{1}{8\mu} \cdot \left(-\frac{\partial P}{\partial x}\right) \cdot R^2$$

$$\Rightarrow -\frac{\partial P}{\partial x} = \frac{8\mu\bar{u}}{R^2}$$

$$\Rightarrow -\int_{P_1}^{P_2} \partial P = \frac{8\mu\bar{u}}{R^2} \cdot \int_0^L dx$$

$$\Rightarrow -\left[P\right]_{P_1}^{P_2} = \frac{8\mu\bar{u}}{R^2} \cdot \left[x\right]_0^L$$



$$\Rightarrow -(P_2 - P_1) = \frac{8\mu\bar{u}}{R^2}\cdot(L-0)$$

$$\Rightarrow P_1 - P_2 = \frac{8\mu\bar{u}L}{R^2}.$$

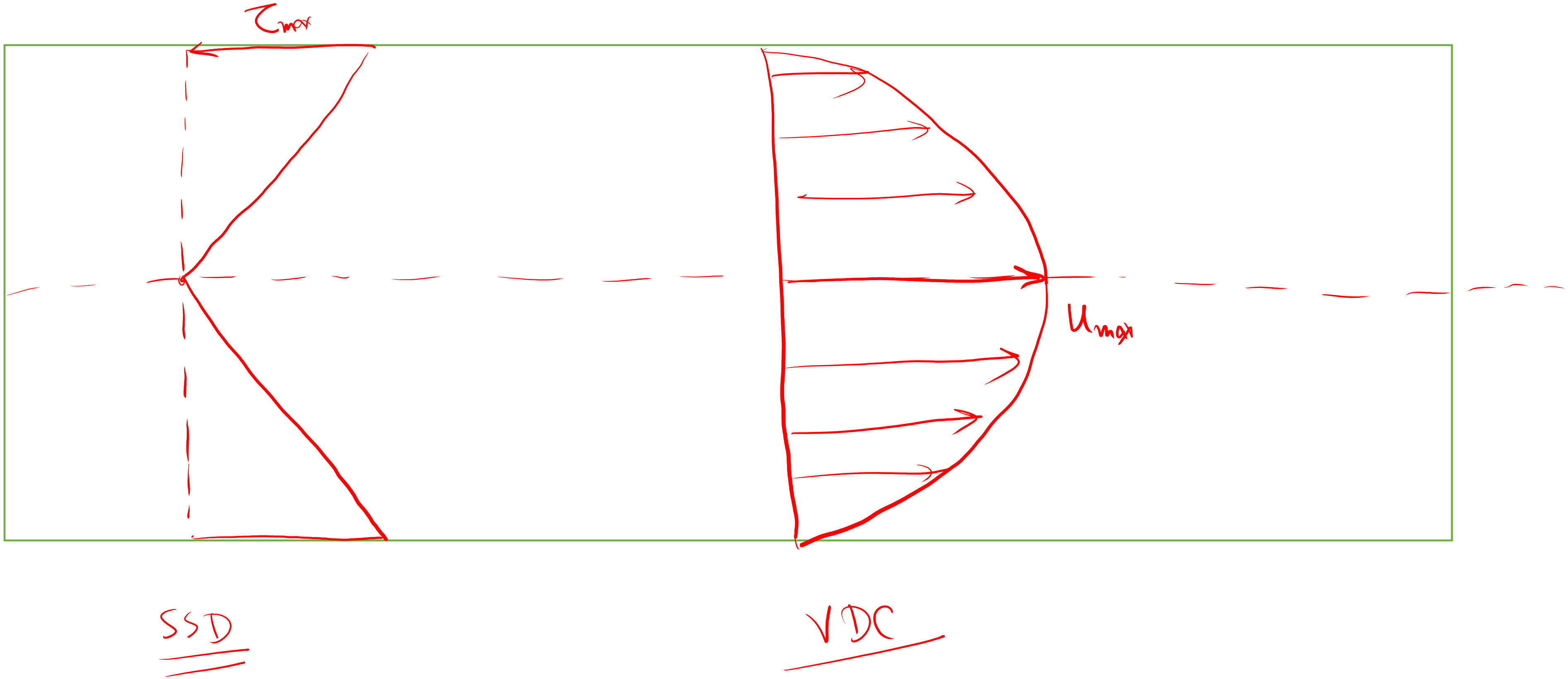
$$R = \frac{D}{2}$$

$$R^2 = \frac{D^2}{4}$$

$$\boxed{P_1 - P_2 = \frac{32\mu\bar{u}L}{D^2}}$$

Loss in head, $\boxed{h_L = \frac{P_1 - P_2}{\rho g} = \frac{32\mu\bar{u}L}{D^2\cdot\rho g}.}$

$\rightarrow$ {Hagen Poiseuille Eqn}**



τmax
Umax
SSD
VDC

$$\tau = \left(-\frac{\partial P}{\partial x}\right) \cdot \frac{r}{2}$$

$$\tau_{max} = -\frac{\partial P}{\partial x} \cdot \frac{R}{2}$$

$$u = \frac{1}{4\mu}\left(-\frac{\partial P}{\partial x}\right)\left(R^2 - r^2\right)$$

$$U_{avg} = \frac{1}{8\mu}\left(-\frac{\partial P}{\partial x}\right) \cdot R^2$$

$$U_{max} = \frac{1}{4\mu}\left(-\frac{\partial P}{\partial x}\right) R^2$$

$$\frac{P_1 - P_2}{\rho g} = \frac{32 \mu \bar{u} L}{\rho g D^2}$$

Try Solving some numerical to improve yourself.

Thank you.