Module: 02
FLUID MECHANICS

By, Mr. Birajit Basumatary
Assistant Professor (C)
Department Of Physics
BBEC, Kokrajhar

Outlines:

- Numerical Discussion on Fluid mechanics

WORKED EXAMPLES

I—Fluid motion

Example 15.1. A horizontal pipe of a non-uniform bore has water flowing through it such that the velocity of flow is 40 cm/sec at a point where the pressure is 2 cm of mercury column. what is the pressure at a point where the velocity of flow is 60 cm/sec? (Take g = 980 cm/sec² and density of water = 1 gm/c.c.).

Solution. Here, $p_1 = 2$ cm *of mercury column* $= 2 \times 13.6 \times 980$ dynes/cm², $\rho_1 = \rho_2 = \rho = 1$ gm/c.c., $v_1 = 40$ cm/sec and $h_1 = h_2 = h$.

In accordance with *Bernoulli's equation*,

$$p_1/\rho + gh + \frac{1}{2}v_1^2 = p_2/\rho + gh + \frac{1}{2}v_2^2,$$

where p_2 is the pressure at the second point.

Or, $\frac{1}{2}(v_2^2 - v_1^2) = (p_1/\rho - p_2/\rho) = p_1 - p_2.$ $[\because \quad \rho = 1 \text{ gm/c.c.}]$

So that, $\frac{1}{2}(60^2 - 40^2) = 2 \times 13.6 \times 980 - p_2.$ Or, $1000 = 26650 - p_2,$

whence, $p_2 = 26650 - 1000 = 25650$ dynes/cm²

$= 25650/13.6 \times 980 = 1.925$ cm *of mercury column.*

Example 15.2. Calculate the velocity of efflux of kerosene oil from a tank in which the pressure is 50 lb wt per square inch above the atmospheric pressure. The density of kerosene is 48 lb per c. ft. ***(Bombay)***

Solution. As we know, the *velocities of efflux* $v = \sqrt{2gh}$, where h is the height of the liquid surface from the axis of the orifice.

Now, *pressure due to kerosene at the level of the axis of the orifice*

$= h\rho g$ *poundals/ft²* $= h\rho$ *lb wt/ft².*

But this is given to be 50 *lb wt/(inch)²* $= 50 \times 144$ *lb wt/ft²*

$\therefore$ $hg\rho = 50 \times 144$. Or, $h = 50 \times 144/\rho = 50 \times 144/48$ *ft.*

Hence, *velocity of efflux,* $v = \sqrt{2gh} = \sqrt{2 \times 32 \times 50 \times 144/48}$

$= 97.97$ or 98 *ft/sec.*

Example 15.3. A water main of 20 cm diameter has a Pitot tube fixed into it and the pressure difference indicated by the gauge is 5 cm of water column. Calculate the rate of flow of water through the main. (Take $g = 980$ cm/sec² and ρ for water = 1 gm/cc).

Solution. Here, *radius (r) of the main* $= 20/2 = 10$ cm and, therefore,its *area of cross section* $a = \pi r^2 = \pi(10)^2 = 100\pi$ sq cm.

Since, *loss of kinetic energy per unit mass on stoppage of flow* $= \frac{1}{2}v^2 =$ *gain in pressure energy per unit mass* $= p/\rho = p/1 = p = 5 \times 1 \times 980$

Or, $v^2 = 10 \times 980$, whence, $v = \sqrt{9800} = 99.0$ cm/sec

$\therefore$ *rate of flow of water through the main*

= *velocity of flow × area of cross section of the main*

= 99 × 100p = 31100 c/c./sec or 31.1 litres/sec

Example 15.4. The diameters of a water main where a venturimeter is connected to it are 20 cm and 10 cm. What is the rate of water flow if the water levels in the two piezometer tubes differ by 5 cm? ($g = 980$ cm/sec^2).

Solution. As we know, the *rate of flow of water through the main* is given by the relation

$$Q = a_1 v_1 = a_1 a_2 \sqrt{2hg/(a_1^2 - a_2^2)}$$

Here, $a_1 = \pi r_1^2 = \pi(20/2)^2 = 100\pi$ sq cm, $a_2 = \pi r_2^2 = \pi(12/2)^2 = 36\pi$ sq cm and $h = 5$ cm. We, therefore, have

rate of flow of water through the main, i.e., $Q = 100\pi \times 36\pi \sqrt{\dfrac{2 \times 5 \times 980}{(100\pi)^2 - (36\pi)^2}}$

$$= 3600\pi \sqrt{\frac{9800}{8704}} = 11920 \text{ cc/sec or } 11.92 \text{ or } 12 \text{ litres/sec.}$$

Example 15.5. Calculate the speed at which the velocity head of a stream of water is equal to 0.50 m of Hg.

Solution. Velocity head = $\dfrac{v^2}{2g}$ metres of Hg

Given: Velocity head = 0.50 m of Hg

= 0.50 × 13.6 m of water

$$\therefore \quad \frac{v^2}{2g} = 0.5 \times 13.6$$

$$v^2 = 2 \times 9.8 \times 0.5 \times 13.6 = 9.8 \times 13.6$$

$$v = \sqrt{9.8 \times 13.6} = \mathbf{11.54 \text{ m/s}}$$

Example 15.6. A railway engine is fitted with a tube whose one end is inside a reservoir of water is between the rails. The other end of the tube is 4 m above the surface of water in the reservoir. Calculate the speed of with which the water rushes out of the upper end, of the engine is moving with a speed of 108 km/hr.

Solution. Applying Bernoulli's theorem,

$$mgh_1 + \frac{1}{2}mv_1^2 = mgh_2 + \frac{1}{2}mv_2^2$$

$$gh_1 + \frac{1}{2}v_1^2 = gh_2 + \frac{1}{2}v_2^2$$

$$\frac{1}{2}v_1^2 = g(h_2 - h_1) + \frac{1}{2}v_2^2$$

$$v_1 = \sqrt{2g(h_2 - h_1) + v_2^2}$$

Here $(h_1 - h_2) = 4$ m; $g = 9.8$ m/s^2; $v_2 = 108$ km/hr = 30 m/s

$$v_1 = \sqrt{2 \times 9.8 \times (-4) \times (30)^2}$$

$$v_1 = \sqrt{900 - 78.4}$$

$$v_1 = \mathbf{28.66 \text{ m/s}}$$

Example 15.7. Water flows through a horizontal pipe line of varying cross-section. At a point where the pressure of water is 0.5 m of mercury the velocity of flow is 0.25 m/s. Calculate the pressure at another point where velocity of flow is 0.4 m/s. Density of water = 10^3 kg/m^3.

(Nag. U., 2001)

Solution. Here, $P_1 = 0.5$ m of Hg $= 0.05 \times 13.6 \times 10^3 \times 9.8$

$$= 6.664 \times 10^3 \text{ Nm}^{-2}$$

$$v_1 = 0.25 \text{ ms}^{-1}, \quad v = 0.4 \text{ ms}^{-1} \quad P_2 = ?$$

As the pipe is horizontal, according to Bernoulli's theorem

$$\frac{P_1}{\rho} + \frac{v_1^2}{2} = \frac{P_2}{\rho} + \frac{v_2^2}{2}$$

or

$$\frac{6.664 \times 10^3}{10^3} + \frac{0.25^2}{2} = \frac{P_2}{10^3} + \frac{0.4^2}{2}$$

or

$$\frac{P_2}{10^3} = 6.664 + \frac{0.25^2}{2} - \frac{0.4^2}{2}$$

or

$$P = (6.664 + 0.03125 - 0.08)10^3 = 6.61525 \times 10^3 \text{ Nm}^{-2}$$

$$= \frac{6.61525 \times 10^3}{13.6 \times 10^3 \times 9.8} = \mathbf{0.0496 \text{ m of mercury}}$$

Example 15.8. A pipe is running full of water. At a certain point *A* it tapers from 0.6 m diameter to 0.2 m diameter at *B*. The pressure difference between *A* and *B* is 1 m of water column. Find the rate of flow of water through the pipe.

Solution. Let p_1, v_1 be the pressure and velocity at A and p_2 and v_2 the corresponding values at B, then,

$$p_1 - p_2 = 1 \times 10^3 \times 9.8 \text{ Nm}^{-2}$$

Rate of flow at $A = v_1 \times \text{area} = \pi \times 0.3^2 \times v_1 = 0.09\pi\, v_1$

Rate of flow at $B = v_2 \times \text{area} = \pi \times 0.1^2 \times v_2 = 0.01\pi\, v_2$

For a steady flow $0.01\, \pi v_2 = 0.09 \pi v_1$

or $v_2 = 9 v_1$

As the height remains the same, according to Bernoulli's theorem, we have

$$\frac{p_1}{\rho} + \frac{v_1^2}{2} = \frac{p_2}{\rho} + \frac{v_2^2}{2}$$

or

$$\frac{p_1 - p_2}{\rho} = \frac{1}{2}(v_2^2 - v_1^2)$$

or

$$\frac{10^3 \times 9.8}{10^3} = \frac{1}{2}(81 v_1^2 - v_1^2)$$

$\therefore$

$$v_1 = \sqrt{\frac{9.8}{40}} = 0.495 \text{ ms}^{-1}$$

$\therefore$ Rate of flow $= \pi \times 0.09 \times 0.495 = \mathbf{0.14 \text{ m}^3 \text{ s}^{-1}}$

Example 15.9. Water issues into the air from a horizontal nozzle whose area of cross-section is 0.125×10^{-4} m^2. Its speed is such that 1.875 kg emerge in one minute. The water strikes a fixed wall which is at right angles to the nozzle and 0.5 m from it and then falls in a vertical plane. Calculate the vertical distance below the nozzle of the point where the jet strikes the wall and the force which the water exerts on the wall.

Solution. Volume of water flowing out per second $V = \frac{1.875}{60 \times 10^3} = 31.25 \times 10^{-6}$ m^3

Area of nozzle $a = 0.125 \times 10^{-4}$ m^2

If v is the velocity with which water issues, then $V = av$

$$v = \frac{V}{a} = \frac{31.25\times10^{-6}}{0.125\times10^{-4}} = 250 \times 10^{-2} \text{ ms}^{-1}$$

Distance of wall from the nozzle = 0.5 m

∴ Time taken by water to reach the wall = $\frac{0.5}{250\times10^{-2}} = 0.2$ sec.

Vertical distance through which water falls in 0.2 sec.

$$= \frac{1}{2}gt^2 = \frac{1}{2}\times9.8\times(0.02)^2 = 0.196$$

Hence, the water jet will strike the wall at a point 0.196 m below the nozzle.

Force exerted on wall = Momentum imparted by water in one second

$$= \frac{1.875}{60}\times250\times10^{-2} = 7.813 \times 10^{-2} \text{ N}$$

Example 15.10. A tank contains water to a height H. Calculate the range of flow of water from an orifice at depth $\frac{H}{4}, \frac{H}{2}$ and $\frac{3H}{4}$ from the surface of water. *(Indore U. 2001)*

Solution. (*i*) *When the orifice is at a depth* $\frac{H}{4}$

$$h = \frac{H}{4},\ h' = H - \frac{H}{4} = \frac{3H}{4}$$

Range $$S = 2\sqrt{hh'} = 2\sqrt{\frac{H}{4}\times\frac{3H}{4}} = \frac{\sqrt{3}}{2}H$$

(*ii*) *When the orifice is at a depth* $\frac{H}{2}$

$$h = \frac{H}{2},\ h' = H - \frac{H}{2} = \frac{H}{2}$$

Range $$S = 2\sqrt{hh'} = 2\sqrt{\frac{H}{2}\times\frac{H}{2}} = H$$

(*iii*) *When the orifice is at a depth* = $\frac{3H}{4}$

$$h = \frac{3H}{4},\ h' = H - \frac{3H}{4} = \frac{H}{4}$$

Range $$S = 2\sqrt{hh'} = 2\sqrt{\frac{3H}{4}\times\frac{H}{4}} = \frac{\sqrt{3}}{2}H.$$

Example 15.11. A flat plate of metal 100 sq. cm. in area rests on a layer of castor oil 2 mm thick whose co-efficient of viscosity is 15.5 poise. Calculate the horizontal force required to move the plate with a speed of 0.03 ms^{-1}.

Solution. Area A = 100 sq. cm. = 10^{-2} m^2, $v = 0.03$ ms^{-1}, r = 2 mm = 0.2×10^{-2} m,
η = 15.5 poise = 1.55 deca-poise

Horizontal viscous force $F = -\eta A\frac{v}{r} = \frac{-1.55\times10^{-2}\times0.03}{0.2\times10^{-2}} = -0.2325$ N

∴ External force required = 0.2325 N

II—Viscosity

Example 15.12. Calculate the mass of water flowing in 10 minutes through a tube 0.1 cm in diameter, 40 cm long if there is a constant pressure head of 20 cm of water. The coefficient of viscosity of water is 0.0089 C.G.S. Units. *(Agra)*

Solution. We have, from Poiseuille's equation, *volume rate of flow of water* given by

$$Q = \frac{\pi P r^4}{8\eta l} = \frac{\pi \times 20 \times 1 \times 981 \times (0.05)^4}{8 \times 0.0089 \times 40} = 0.1353 \text{ cc/sec.}$$

$\therefore$ volume of water flowing out in 10 minutes $= 0.1353 \times 10 \times 60$, say, $V = 81.18$ c.c.

Hence, *mass of water flowing out in 10 minutes* $= V \times \rho = 81.18 \times 1 = 81.18$ gm.

Example 15.13. A cylindrical vessel of radius 7 cm is filled with water to a height of 50 cm. It has a capillary tube 10 cm long, 0.2 mm radius, protruding horizontally at its bottom. If he viscosity of water is 0.01 C.G.S. units and $g = 980$ cm/sec², find the time in which the level will fall to a height of 25 cm. ***(Punjab)***

Solution. Let h be the height of the water column in the vessel at any given instant and dh, the fall in its height in a small interval of time dt. Then, if A be the area of cross-section of the vessel, we have

rate of flow of water through the capillary tube, i.e., $Q = -A\, dh/dt$,

the –ve sign indicating that h decreases as t increases.

But, as we know, the rate of flow of water through a capillary tube is given by *Poiseuille's equation.* $Q = \pi P r^4/8\eta l$, where $P = h\rho g = hg$ (ρ being the density of water, equal to 1 gm/cc). We, therefore, have

$$-A\frac{dh}{dt} = \frac{\pi h g r^4}{8\eta l}. \quad \text{Or, } dt = -\frac{8\eta l A}{\pi g r^4}\cdot\frac{dh}{h}.$$

And $\therefore$
$$\int_0^t dt = \int_{h_1}^{h_2} -\frac{8\eta l A}{\pi g r^4}\cdot\frac{dh}{h}, \text{ whence, } t = \frac{8\eta l A}{\pi g r^4}\log_e\frac{h_1}{h_2}.$$

Or, putting the value of $A = \pi \times 7^2$, $r = 0.02$ cm, $h_1 = 50$ cm and $h_2 = 25$ cm, we have

$$t = \frac{8 \times 0.01 \times 10 \times \pi \times 7^2}{\pi \times 9.80 \times (0.02)^4} \times 2.3026 \log_{10}\frac{50}{25} = 1.734 \times 10^5 \text{ sec} = 48.16 \text{ hr.}$$

The water level in the vessel will thus fall to 25 cm in 48.16 hours.

Example 15.14. Write down Poiseuille's formula for the rate of flow of a liquid through a capillary tube. From this show that if two capillaries of radii a_1 and a_2, having lengths l_1 and l_2 respectively, are set in series, the rate of flow Q* is given by

$$Q = \frac{\pi P}{8\eta}\left(\frac{l_1}{a_1^4} + \frac{l_2}{a_2^4}\right)^{-1},$$

where P is the pressure across the arrangement and η, the coefficient of viscosity of the liquid. ***(Rajashtan)***

Solution. Let P_1 be the pressure across the first, and P_2 across the second, capillary. So that, $P = P_1 + P_2$ and, therefore, $P_2 = P - P_1$.

Obviously, in accordance with the equation of continuity, *the rate of flow through either capillary will be the same*, say Q and, therefore, from Poiseuille's equation, we have

$$Q = \pi P_1 a_1^4/8\eta l_1 = \pi P_2 a_2^4/8\eta l_2 = (P - P_1)a_2^4/8\eta l_2,$$

whence,
$$P_1 a_1^4/l_1 = P a_2^4/l_2 - P_1 a_2^4/l_2$$

Or,
$$P_1\left(\frac{a_1^4}{l_1} + \frac{a_2^4}{l_2}\right) = \frac{P a_2^4}{l_2}.$$

$\therefore$
$$P_1 = \frac{P a_2^4/l_2}{(a_1^4/l_1) + (a_2^4/l_2)} = \frac{P l_1}{a_1^4[(l_2/a_2^4) + (l_1/a_1^4)]}$$

* Here, symbol Q has been substituted for V in the original question.

$$= \frac{Pl_1}{a_1^4}\left(\frac{l_2}{a_2^4}+\frac{l_1}{a_1^4}\right)^{-1}.$$

Substituting this value of P_1 in the expression $Q = \pi P_1 a_1^4/8\eta l_1$, we have

$$Q = \frac{\pi P}{8\eta}\left(\frac{l_1}{a_1^4}+\frac{l_2}{a_2^4}\right)^{-1}.$$

Example 15.15. (*a*) Three capillaries of lengths $8L$, $0.2L$ and $2L$, with their radii r, $0.2r$ and $0.5\,r$ respectively, are connected in series. If the total pressure across the system in an experiment is p, deduce the pressure across the shortest capillary. **(*Agra*)**

(*b*) Fig. 15.20 below shows two wide tubes P and Q, connected by three capillaries A, B, C whose relative lengths and radii are indicated. If a pressure p is maintained across A, deduce (*i*) the ratio of liquid flowing through A and B, (*ii*) the pressure across B and across C.

(*Agra (Supp.)*)

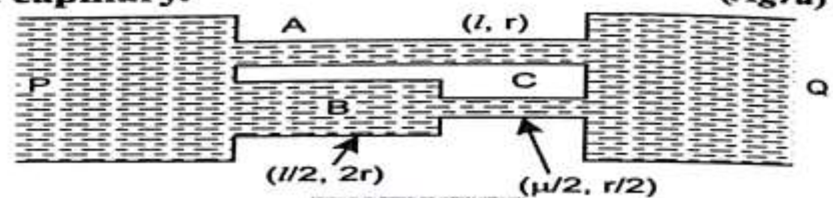

Fig. 15.20

Solution. (*a*) Obviously, the rate of flow of liquid across each capillary is the same. So that, if p_1, p_2 and p_3 be the pressures across the three capillaries respectively, we have, in accordance with Poiseuille's equation,

$$Q = \frac{\pi p_1 r^4}{8\eta(8L)} = \frac{\pi p_2 (0.2r)^4}{8\eta(0.2L)} = \frac{\pi p_3 (0.5r)^4}{8\eta(2L)},$$

whence,
$$\frac{p_1}{64} = \frac{p_2}{1000} = \frac{p_3}{256},$$

which gives
$$p_1 = \frac{p_2}{1000}\times 64 = \frac{8}{125}p_2 \text{ and } p_3 = \frac{p_2}{1000}\times 256 = \frac{32}{125}p_2.$$

Now,
$$p = p_1 + p_2 + p_2 = \frac{8}{125}p_2 + p_2 + \frac{32}{125}p_2 = \frac{165}{125}p_2.$$

And, therefore, *pressure across the shortest capillary, i.e.*,

$$p_2 = \frac{125}{165}p = 0.7575\,p.$$

(*b*) (*i*) Clearly, *rate of liquid-flow through capillary A* is $Q = \pi p r^4/8\eta l$, and

rate of liquid-flow through B and C is, say, $Q' = \frac{\pi p_1 (2r)^4}{8\eta(l/2)} = \frac{\pi p_2 (r/2)^4}{8\eta(l/2)}$,

whence, $(2r)^4 p_1 = (r/2)^4 p_2$. Or, $p_2 = 256\,p_1$.

Or, since $p = p_1 + p_2$, we have $p = p_1 + 256\,p_1 = 257\,p_1$.

$\therefore$ *ratio of liquid flowing through A and B* $= \frac{Q}{Q'} = \frac{\pi p r^4/8\eta l}{\pi p_1 (2r)^4/8\eta(l/2)} = \frac{257\pi p_1 r^4/8\eta l}{32\pi p_1 r^4/8\eta l} = 257 : 32.$

(*ii*) since $p = 257\,p_1$, we have

Pressure across capillary B, i.e., $p_1 = p/257$

and *pressure across capillary C, i.e.*, $p_2 = p - p_1 = p - p/257 = \frac{256}{257}p.$

Example 15.16. A gas bubble of diameter 2 cm rises steadily through a solution of density 1.75 gm/c.c. at the rate of 0.35 cm/sec. Calculate the coefficient of viscosity of the solution. (Neglect density of the gas).

Solution. As we know, the coefficient of viscosity of the solution is given by the relation

$$\eta = \frac{2}{9}\left(\frac{\rho-\sigma}{v}\right)r^2 g.$$

Here, *radius of the bubble*, $r = 2/2 = 1$ cm, *density of the solution*, $\sigma = 1.75$ gm/c.c. *density of the bubble*, $\rho = 0$ (negligible) and *velocity of the bubble*, $v = -0.35$ cm/sec, (because it is directed upwards). We, therefore, have

coefficient of viscosity of the solution,

$$\eta = \frac{2}{9}\frac{\sigma r^2 g}{v} = \frac{2}{9}\times\frac{1.75\times(1)^2\times 981}{0.35} = 1.09\times 10^3 \text{ poise.}$$

Example 15.17. A glass bulb of volume 500 c.c. has a capillary tube of length 40 cm and radius 0.020 cm leading from it. The bulb is filled with hydrogen at an initial pressure of 86 cm of mercury, density 13.6 gm/c.c., and it is found that if the volume of the gas remaining in the vessel is kept constant, the pressure falls to 80 cm of mercury in 25.4 sec. If the height of the barometer is 76 cm and g = 981 cm/sec², find the viscosity of hydrogen.

Solution. This is obviously a straight applicationof Searle's method for determining the coefficient of viscosity of a gas and we, therefore, have

$$\eta = \frac{\pi r^4 H\rho g t}{8lV\times 2.3026\log_{10}\left(\frac{h_1}{h_2}\cdot\frac{2H+h_2}{2H+h_1}\right)}$$

Substituting the given values, *viz.*, $H = 76$ cm, $\rho = 13.6$ gm/c.c., $g = 981$ cm/sec², $t = 25.4$ sec, $l = 40$ cm, $V = 500$ c.c., $h_1 = 86 - 76 = 10$ cm and $h_2 = 80 - 76 = 4$ cm, we have

coefficient of viscosity of hydrogen,

$$\eta = \frac{\pi(0.02)^4\times 76\times 13.6\times 981\times 25.4}{8\times 40\times 500\times 2.3026\log_{10}\left(\frac{10}{4}\cdot\frac{152+4}{152+10}\right)}$$

$$= \frac{\pi(0.02)^4\times 76\times 13.6\times 981\times 25.4}{8\times 40\times 500\times 2.3026\times 0.3815} = 9.204\times 10^{-5} \textit{ poise.}$$

Example 15.18. Two horizontal capillary tubes *A* and *B* are connected together in series so that a steady stream of fluid flows through them. *A* is 0.4 mm in internal radius and 256 cm long. *B* is 0.3 mm in internal radius and 40.5 cm long. The pressure of the fluid at the entrance is 3 inches of mercury above the atmosphere. At the exit end of *B*, it is atmospheric (30 inches of mercury). What is the pressure at the junction of *A* and *B* if the fluid is (*i*) a liquid (*ii*) a gas?

Solution. Case (*i*) When the fluid is liquid. Let the pressure at the junction of capillary tubes *A* and *B* be *h* inches of mercury, so that

pressure across capillary A is, say, $P_1 = (33 - h)$ *inches of mercury*, and

pressure across capillary B is, say, $P_2 = (h - 30)$ *inches of mercury*.

The two tubes being connected *in series, the volume rate of flow of the liquid is the same through both* and we, therefore, have

$$Q = \frac{\pi P_1 r_1^4}{8\eta l_1} = \frac{\pi P_2 r_2^4}{8\eta l_2}, \text{ whence, } \frac{P_1}{P_2} = \frac{l_1}{l_2}\left(\frac{r_2}{r_1}\right)^4,$$

where l_1, l_2 and r_1, r_2 are the lengths and the radii of the two tubes respectively.

Substituting the values of P_1 and P_2 from above, we have

$$\frac{33-h}{h-30} = \frac{256}{40.5}\left(\frac{0.3}{0.4}\right)^4 = \frac{256}{40.5}\times\frac{81}{256} = 2.$$

Or, $33 - h = 2(h - 30) = 2h - 60$. Or, $3h = 93$, whence, $h = 31$ inches,
i.e., the pressure at the junction of the two capillary tubes is 31 inches *of mercury.*

Case (*ii*) When the fluid is a gas. Here, *the mass rate of flow of the gas through either capillary tube is the same* and we thus have

$$\frac{\pi r_1^4(33^2 - h^2)}{16\eta l_1} = \frac{\pi r_2^4(h^2 - 30)}{16\eta l_2}, \qquad \text{[See § 15.20]}$$

where h, as before, is the pressure in inches at the junction of the two tubes.

We, therefore, have $\dfrac{33^2 - h^2}{h^2 - 30^2} = \left(\dfrac{r_2}{r_1}\right)^4 = 2.$ [From above.]

Or, $33^2 - h^2 = 2h^2 - 2 \times 30^2$. Or, $h^2 = (2 \times 30^2 + 33^2)/3 = 2889/3 = 963$,

whence, $h = \sqrt{963} = 31.03$ inches.

Thus, *the pressure at the junction of the two capillary tubes, in this case, equal to* 31.03 inches *of mercury.*

Example 15.19. A horizontal tube of 1 mm bore is joined to another horizontal tube of 0.5 mm bore. Water enters at the free end of the first tube at a pressure equal to 0.5 m of water above the atmospheric pressure and leaves at the free end of the second tube at the atmospheric pressure. Calculate the pressure at the junction of the tubes if the lengths of the tubes are equal.

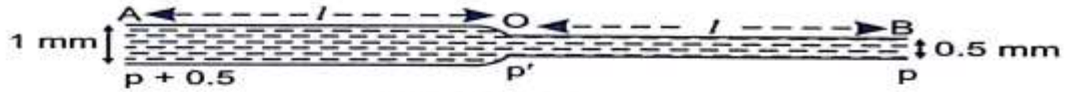

Fig. 15.21

Solution. According to Poiseuille's equation $V = \dfrac{\pi P a^4}{8\eta l}$

If p' is the pressure at the junction O of the two tubes each of length l, then difference of pressure between A and $O = p + 0.5 - p'$

∴ Volume of water flowing through AO per second $= \dfrac{\pi(p + 0.5 - p')(0.5 \times 10^{-3})^4}{8\eta l}$

Difference of pressure between O and $B = p' - p$

∴ Volume of water flowing per second through $OB = \dfrac{\pi(p' - p)(0.25 \times 10^{-3})^4}{8\eta l}$

As the two tubes are joined end to end, the volume of water flowing per second through them is the same.

$$\therefore \quad \frac{\pi(p + 0.5 - p')(0.5 \times 10^{-3})^4}{8\eta l} = \frac{\pi(p' - p)(0.25 \times 10^{-3})^4}{8\eta l}$$

or

$$p + 0.5 - p' = \frac{(p' - p)}{16}$$

$$17(p' - p) = 8$$

$$\therefore \quad p' - p = \frac{8}{17} = \mathbf{0.47\ m\ of\ water\ column.}$$

Hence, pressure at O is 0.47 m of water column.

Example 15.20. A capillary tube of radius *a* and length *l* is fitted horizontally at the bottom of a cylindrical flask of cross-section area *A*. Initially there is water in the flask up to a height *h*. What time would be required for half the liquid to flow out, if the coefficient of viscosity of the liquid is η?

Solution. According to Poiseuille's equation for volume V of the water flowing through a tube of length l and radius a is given by

$$V = \frac{\pi P a^4}{8\eta l}$$

As A is the area of cross-section of the vessel and h the height of water above the capillary tube.

$\therefore$ Pressure head $P = hg$ [$\because \rho = 1$ for water in C.G.S. system]

Suppose in a small time dt the level of water in the vessel falls through a height dh, then

Volume of water flowing in time $dt = A.dh$

$\therefore$ Rate of flow $V = -A\frac{dh}{dt}$

The negative sign shows that the height decreases with time.

Substituting the value of P and V in (*i*), we have

$$-A\frac{dh}{dt} = \frac{\pi hga^4}{8\eta l} \text{ or } dt = -\frac{8\eta lA}{\pi ga^4}\cdot\frac{dh}{h} \quad \ldots(ii)$$

Let t be the time in which the initial height h is reduced to $h/2$, then

$$\int_0^t dt = \frac{8\eta lA}{\pi ga^4}\int_h^{h/2}\frac{dh}{h}$$

$$\therefore \quad t = -\frac{8\eta lA}{\pi ga^4}[\log_e h]_h^{h/2} = \frac{8\eta lA}{\pi ga^4}\left[\log_e h - \log_e \frac{h}{2}\right]$$

$$= \frac{8\eta lA}{\pi ga^4}\log_e 2 = \mathbf{2.3026 \times A\frac{8\eta l}{\pi ga^4}\log_{10} 2}$$

Example 15.21. In the Poiseuille experiment the following observations were made. Volume of water collected in 5 minutes = 40 c.c.; Head of water 0.4 m; length of capillary tube = 0.602 m and radius of capillary tube = 0.52×10^{-3} m. Calculate the coefficient of viscosity of water.

Solution. Volume of water collected per second

$$V = \frac{40}{5\times 60}\text{ cm}^3 = \frac{40}{5\times 60}\times 10^{-6}\text{ m}^3 = \frac{2}{15}\times 10^{-6}\text{ m}^3$$

Head of water $h = 0.4$ m

$\therefore$ Difference of pressure $P = h\rho g = 0.4 \times 10^3 \times 9.8 \text{ Nm}^{-2} = 3.92 \times 10^3 \text{ Nm}^{-2}$

Length of capillary tube $l = 0.602$ m

Radius of the capillary tube $a = 0.52 \times 10^{-3}$ m

Now, coefficient of viscosity $\eta = \frac{\pi P a^4}{8lV} = \frac{3.142\times 3.92\times 10^3\times(0.52)^4\times 10^{-12}\times 15}{8\times 0.602\times 2\times 10^{-6}}$

$$= \mathbf{1.4 \times 10^{-3}\ Nm^{-2}\ (or\ deca\text{-}poise)}$$

Example 15.22. If two capillaries of radii r_1 and r_2 and length l_1 and l_2 are joined in series, derive an expression for the rate of flow of the liquid through the arrangement using Poiseuille's formula.

Solution. According to Poiseuille's formula, the rate of flow V of a liquid through a capillary tube of length l and radius r is given by

$$V = \frac{\pi p r^4}{8\eta l}$$

where p is the pressure difference across the ends of the tube and η the co-efficient of viscosity.

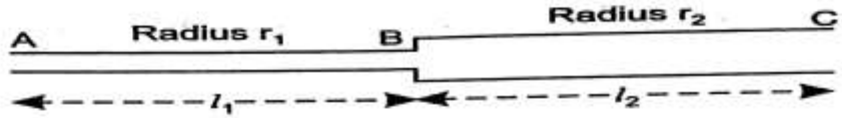

Fig. 15.22

Consider two capillaries of lengths l_1 and l_2 having radii r_1 and r_2 respectively connected in series. If p_1 is the pressure difference between the ends of capillary AB and p_2 that between the ends of the capillary BC, then as the same volume of liquid is flowing through each of the capillaries

$$V = \frac{\pi p_1 r_1^4}{8\eta l_1} = \frac{\pi p_2 r_2^4}{8\eta l_2}$$

So that $$p_1 = \frac{8\eta l_1}{\pi r_1^4}V \text{ and } p_2 = \frac{8\eta l_2}{\pi r_2^4}V$$

If p is the effective pressure across the ends A and C, then

$$p = p_1 + p_2 = \left(\frac{8\eta l_1}{\pi r_1^4} + \frac{8\eta l_2}{\pi r_2^4}\right)V$$

∴ Rate of flow $$V = \frac{\pi p}{8\eta\left(\frac{l_1}{r_1^4} + \frac{l_2}{r_2^4}\right)}$$

Example 15.23. Two drops of water of the same size are falling through air with terminal velocity 1 ms^{-1}. If the two drops combine to form a single drop, calculate the terminal velocity.

Solution. Let r be the radius of each drop and r_1 that of the combined drop, then

$$4/3\pi r_1^3 = 2 \times 4/3\pi r^3$$

∴ $$r_1 = 2^{1/3} r$$

If v is the terminal velocity of each drop and v_1 that of the combined drop then according to Stoke's law

$$v = \frac{2r^2(\rho - d)g}{9\eta}$$

and $$v_1 = \frac{2r_1^2(\rho - d)g}{9\eta}$$

∴ $$\frac{v_1}{v} = \frac{r_1^2}{r^2}$$

$$= \frac{2^{2/3}r^2}{r^2} = 2^{2/3} = 1.588 \text{ ms}^{-1} \qquad [\because V = 1 \text{ ms}^{-1}]$$

Example 15.24. Eight drops of water of the same size are falling through air with terminal velocity of 10 m/sec. If the eight drops combine to form a single drop what will the new terminal velocity?

Solution. $\frac{4}{3}\pi r_1^3 = 8 \times \frac{4}{3}\pi r^3$ or $r_1 = 2r$

$$\frac{v_1}{v} = \frac{r_1^2}{r^2} = \frac{2^2 r^2}{r^2} = 4 \text{ or } v_1 = 4 \times 10 = 40 \text{ m/s}$$

Example 15.25. A steel ball of radius 2×10^{-3} m falls in a vertical column of castor oil. The co-efficient of viscosity of castor oil is 0.7 Nm^{-2} and its density 0.98×10^3 kg m^{-3}. The density of steel is 7.8×10^3 kg m^{-3} and g = 9.8 ms^{-2}. Find its terminal velocity.

Solution. According to Stokes' formula, the terminal velocity is given by

$$v = \frac{2r^2(\rho - d)g}{9\eta}$$

Now, radius of the ball $r = 2 \times 10^{-3}$ m

Density of steel ball $\rho = 7.8 \times 10^3$ kg m^{-3}

Density of castor oil $d = 0.98 \times 10^3$ kg m^{-3}; $g = 9.8$ ms^{-2}

Viscosity of castor oil $\eta = 0.7$ Nm^{-2}

$$v = \frac{2 \times (2 \times 10^{-3})^2 (7.8 \times 10^3 - 0.98 \times 10^3) 9.8}{9 \times 0.7}$$

$$= \frac{2 \times 4 \times 10^{-3} \times 6.82 \times 9.8}{9 \times 0.7}$$

$$= 84.87 \times 10^{-3} \text{ ms}^{-1}$$

Thank You