Module: 02
FLUID MECHANICS

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Outlines:

- Rotational Viscometer

15.19 ROTATION VISCOMETER

The principle and working of a *rotation viscometer* will be understood from the following:

Let A be a cylinder, of radius a, suspended coaxially, by means of a long and thin suspension wire, inside a cylinder B, of radius b, clamped on to a table which can be rotated by means of a small electric motor, and let the space between the two be filled with a fluid, say, a liquid, up to a height l, say, Fig. [15.18 (*a*)].

As the outer cylinder B is rotated with a suitable angular velocity Ω (so as not to cause turbulence in the fluid), its rotation communicates a torque to the stationary cylinder A, the magnitude of the torque depending upon the coefficient of viscosity of the fluid.

Representing the two cylinders by the full line circles A and B, of radii a and b respectively. [Fig. 15.18 (*b*)], with their common axis perpendicular to the plane of the paper and passing through O, let us consider a coaxial cylindrical layer of the fluid of thickness dr at a distance r from O (shown dotted). If ω be its angular velocity and hence its linear speed, $v = r\omega$, the *viscous drag* on it is, in accordance with Newton's formula, given by F = its area × its coefficient of viscosity × velocity gradient $= 2\pi r l \eta\, dv/dr$.

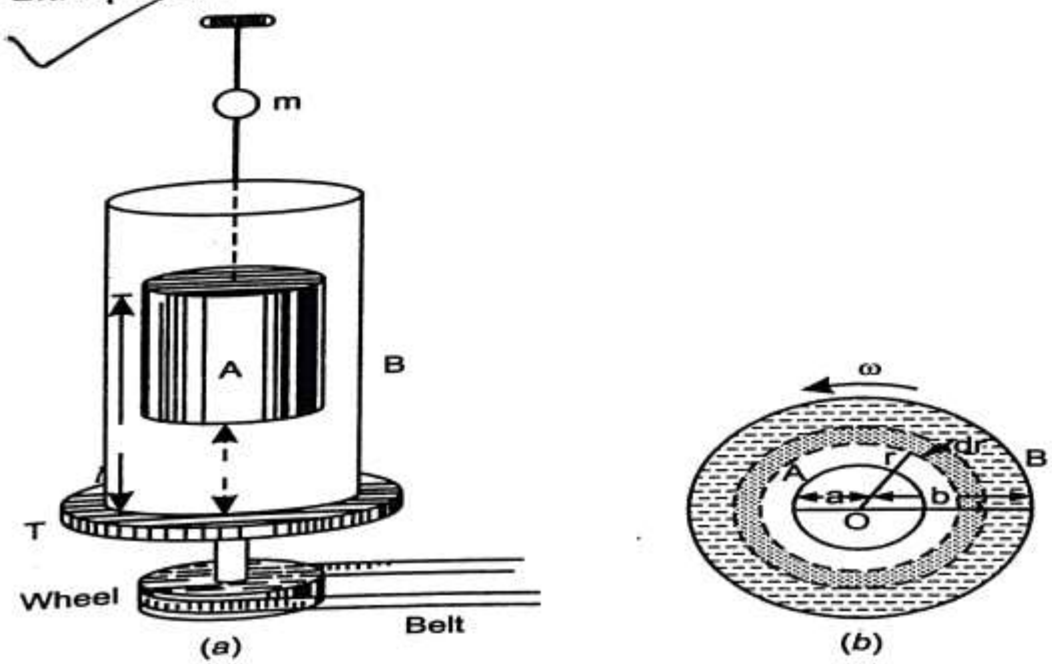

Fig. 15.18

Now, $dv/dr = d(r\omega)/dr = \omega + r\, d\omega/dr$, where ω being a constant quantity, only the second term is operative and the *effective velocity gradient* is thus $r d\omega/dr$. So that, $F = 2\pi r l \eta r\, d\omega/dr = 2\pi r^2 l \eta\, d\omega/dr$.

Hence, *moment of this force or the torque acting on this layer or on the inner cylinder A* (the fluid in between the two being in a steady sate) is given by

$$\tau_1 = 2\pi r^2 l\eta\, d\omega/dr \times r = 2\pi r^3 l\eta\, d\omega/dr.$$

Putting the relation in the form $2\pi l\eta\, d\omega = \tau_1 dr/r^3$ and integrating for the limits $\omega = 0$ and $\omega = \Omega$ and $r = a$ and $r = b$, we have

$$\int_0^{\Omega} 2\pi l\eta d\omega = \tau_1 \int_b^a \frac{dr}{r^3}, \text{ which gives } \tau_1 = \frac{4\pi l\eta\Omega a^2 b^2}{b^2 - a^2}.$$

Here, τ_1 is clearly the torque on the sides of the cylinder A. There is also a torque on its bottom, depending on the radii of the two cylinders and the distance between their bottoms. Let this be τ_2. Then, *total torque on cylinder A* is $\tau = \tau_1 + \tau_2$.

This torque (τ) tends to accelerate the motion of the fluid between the layer and the inner cylinder A and hence also the inner cylinder A (the fluid in between being in a steady state), tending to rotate it through an angle θ, say, until the *restoring torsional couple* $C\theta$, set up in the suspension wire, just balances it (C being the torsional couple per unit twist of the wire). We, therefore, have

$$\textit{total torque } \tau = C\theta = \frac{4\pi l\eta\Omega a^2 b^2}{b^2 - a^2} + \tau_2. \quad \ldots(i)$$

To eliminate τ_2, the experiment is repeated with a different height l' of the fluid in between the two cylinders when, since the radii a and b of the cylinders and the distance between their bottoms remain the same as before, τ_2 remains unchanged. If the torque on the sides of the cylinder be now τ_1 and cylinder A rotates through an angle θ', we have

$$\textit{total torque } \tau' = C\theta' = \frac{4\pi l'\eta\Omega a^2 b^2}{b^2 - a^2} + \tau_2. \quad \ldots(ii)$$

From relations (i) and (ii), therefore, we have

$$\tau - \tau' = C(\theta - \theta') = \frac{4\pi\eta\Omega a^2 b^2}{b^2 - a^2}(l - l'), \text{ whence, } \eta = \frac{C(b^2 - a^2)(\theta - \theta')}{4\pi\Omega a^2 b^2 (l - l')}.$$

The angle θ and θ' can be easily read by the scale and telescope method, for which purpose a small mirror m is fixed on to the suspension wire as shown, and the value of C obtained by noting the time-periods T_1 and T_2 of the torsional vibration of cylinder A alone and in combination with a hollow metallic disc, of a known moment of inertia, respectively. If I be the moment of inertia of cylinder A alone about the suspension wire and I' that of the combination of A and the disc, we have

$$T_1 = 2\pi\sqrt{I/C} \text{ and } T_2 = 2\pi\sqrt{(I + I')/C}, \text{ whence, } C = 4\pi^2 I/(T_2^2 - T_1^2).$$

This value of C is then substituted in the expression for η above.

In case the fluid be a *gas*, we repeat the experiment with two different inner cylinders of *different lengths* but the *same radius*, keeping the distance of the bottom of each from that of the outer cylinder the same (instead of taking two observations with different heights of the liquid in between the same two cylinders). Thus, in the expression for η above, l and l' will now stand for the lengths of the two inner cylinders used, with a as the radius of each inner cylinder and b, that of the outer cylinder.

Thank You