Module: 02
FLUID MECHANICS

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Outlines:

- Torricelli’s Theorem
- Stokes's Law
- Terminal Velocity
- Motion of a rigid Body in Viscous medium

7(c).21. TORRICELLI'S THEOREM

It states that the velocity of efflux of a liquid, from an orifice, is equal to the velocity acquired by a body, falling freely, from the surface of liquid to the orifice.

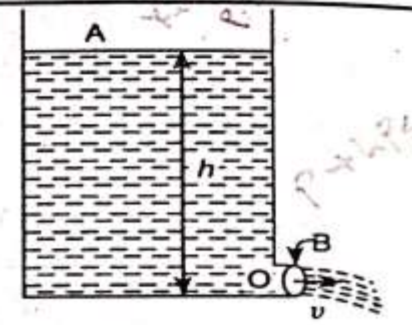


Fig. 7(c).28. Liquid coming out of an orifice

Consider a liquid coming out of an orifice at a depth 'h' below the liquid surface [Fig. 7(c).28]. Consider a unit mass (at a point A), of liquid, on the surface of liquid. Here the liquid is at rest (K.E. = 0), at a height h (potential energy = gh) and is under atmosphere pressure (pressure energy = p/ρ). Therefore, total energy/mass of liquid at A is

$$E_A = gh + \frac{p}{\rho}$$

The liquid at a point just inside the orifice is at rest under pressure ($p + h\rho g$) while that at a point just outside is at a pressure p only. This decrease in pressure in crossing the orifice results in decrease in pressure energy which appears in the form of kinetic energy. So, the liquid just outside the orifice possesses a velocity 'v' called ***"velocity of efflux"***. Here the potential energy is zero. Therefore, total energy per unit mass at a point outside is

$$E_B = \frac{1}{2}v^2 + \frac{p}{\rho}$$

According to the law of conservation of energy,

$$E_A = E_B$$

$$\frac{1}{2}v^2 + \frac{p}{\rho} = gh + \frac{p}{\rho} \qquad \therefore \quad \frac{1}{2}v^2 = gh$$

or

$$v = \sqrt{2gh}$$

Let 'V' be the velocity of a body after falling freely (under acceleration due to gravity) through a height 'h'. Using the relation $v^2 - u^2 = 2aS$, we get,

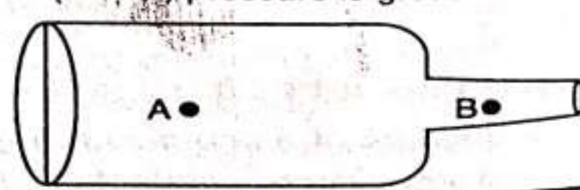
$$V^2 - 0 = 2gh \quad \text{or} \quad V = \sqrt{2gh}$$

Thus it can be seen that $V = v$

This is in accordance with the theorem.

CHECK LIST-2

1. Fire fighting persons have brass jets attached at the head of their water pipes. What is the purpose of this ? Is it done to have greater mass of water leaving the pipe or to have grater velocity of water ?
2. Liquid rushes out of a small orifice in one of the walls of the container. The height of liquid column, inside the vessel, above the orifice is h. How does the velocity of liquid vary with h ?
3. A strong wind flows over the roof of a hut. What is the direction of force experienced by the roof ?
4. A liquid flows in a streamlined flow through a pipe of variable diameter as shown in figure. At which place (A or B) pressure is greater ?



7(c).9. STOKE'S LAW

It deals with the force of viscosity acting on a spherical body as it moves through a fluid.

As spherical body of radius 'r' moves through a fluid, layers in contact with it stick to it and move along with it, with same velocity [Fig. 7(c).9]. Other layers which are situated at a distance from it move with gradually decreasing velocity, thus, bringing about a relative velocity between any of the two layers of fluid. Thus, a force of viscosity 'F', acting in opposite direction comes into play.

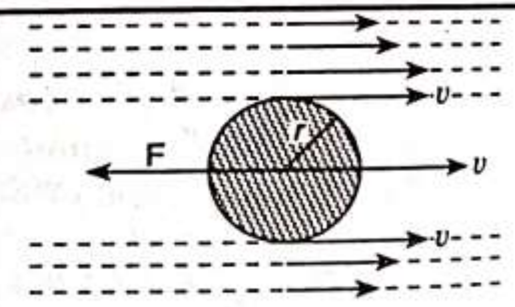


Fig. 7(*c*).9. A spherical body moving through a fluid

Force of viscosity 'F' is found to depend upon the following factors :

(i) co-efficient of viscosity of fluid, 'η'

(ii) radius of the moving body 'r'

(iii) velocity of body, 'v'.

An expression for 'F' can be derived by the method of dimensional analysis as follows :

Let 'F' vary as

$$F \propto \eta^a, \quad F \propto r^b, \; F \propto v^c,$$

or
$$F \propto \eta^a \,.\, r^b . \, v^c$$

or
$$F = k\eta^a \,.\, r^b \,.\, v^c \qquad ...(5)$$

where k is a dimensionless constant.

Taking dimensions on both sides.

$$M^1 L^1 T^2 = [M^1 L^{-1} T^{-1}]^a . [L^1]^b . [L^1T^{-1}]^c$$
$$M^1 L^1 T^2 = M^a L^{-a+b+c} T^{-a-c}$$

According to the principle of homogeneity dimensions on the two sides of a correct relation must be same.

$$a = 1 \qquad ...(i)$$
$$-a + b + c = 1 \qquad ...(ii)$$
$$-a - c = -2 \qquad ...(iii)$$

Put $a = 1$ in Equation (*iii*), $\quad -1 - c = -2, \quad \therefore \quad c = 1$

Put $a = 1$, $c = 1$ in Equation (*ii*),

$$-1 + b + 1 = 1, \quad \text{or} \quad b = 1$$

Substituting for a, b and c in Equation (11),

$$F = k\eta r v$$

Stoke determined the value of 'k' experimentally and found that,

$$k = 6\pi$$

$$F = 6\pi\eta r v. \qquad \ldots(6)$$

Relation (6) is called **'Stoke's formula'**. Dependence of 'F' upon η, r and v, according to the relation (6), is termed as **Stoke's law.**

Stoke make the following assumptions while arriving at the law.

(*i*) The viscous fluid is indefinite in extent.

(*ii*) Spherical body is perfectly rigid and smooth.

(*iii*) There is no slip between the body and the fluid layer in contact with it.

(*iv*) The fluid is homogeneous.

(*v*) The velocity of the body is small, so that motion of fluid relative to body is streamlined.

7(*c*).10. APPLICATION OF STOKE'S LAW—"TERMINAL VELOCITY"

Almost all the applications of Stoke's law are based upon the concept of **terminal velocity.**

Consider a spherical body of radius r, density ρ (from rest) in a fluid of density σ and coefficient of viscosity η.

Case (*a*) If $\rho > \sigma$

Various forces acting on the body are shown in [Fig. 7(*c*).10].

(*i*) Weight W, acting downwards.

$$W = V \times \rho \times g = \frac{4}{3}\pi r^3 \rho g$$

where, ρ = density of body

(*ii*) Thrust U acting upwards.

$$U = V \times \sigma \times g = \frac{4}{3}\pi r^3 \sigma g$$

where, σ = density of fluid.

(*iii*) Force of viscosity, acting upwards.

$$F = 6\pi\eta r v$$

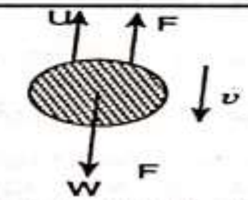


Fig. 7(*c*).10. A spherical body falling through a viscous medium.

Of all these three forces, only, F is velocity dependent. In the beginning ($v = 0$), it is zero. Since $\rho > \sigma$, downward weight W is greater than upthrust U. So, the body starts moving downward and gains velocity gradually. As a result of velocity, 'F' comes into existence (in upward direction) and keeps on increasing along with velocity.

$$F + U = W$$

At this stage, the resultant force on the body is zero. The body will not have its velocity increased any further. This uniform velocity of the body is called ***terminal velocity***. If 'v' is the terminal velocity.

$$6\pi\eta r v + \frac{4}{3}\pi r^3 \sigma g = \frac{4}{3}\pi r^3 . \rho g$$

$$\therefore \quad 6\pi\eta r v = \frac{4}{3}\pi r^3 (\rho - \sigma) g$$

or

$$v = \frac{2}{9}\left[\frac{r^2(\rho - \sigma)}{\eta}\right] g \qquad \ldots(13)$$

Surfing Aid

For virtual demonstration of terminal velocity reach following webaddress
<http://www.explorescience.com/activities/Activity_page.cfm?ActivityID=2>

Case (*b*) if $\rho < \sigma$

Again, F is zero initially. Since $\rho < \sigma$, downward weight is lesser than upthrust U. So, the body starts moving upwards. As it gains velocity, viscous drag F starts acting in downward direction [Fig. 7(*c*).11], and keeps on increasing with increase in velocity of body till the condition of equilibrium is achieved.

i.e., $$F + W = U \quad \text{or} \quad F = U - W$$

$$6\pi\eta r v = \frac{4}{3}\pi r^3 (\sigma - \rho) g$$

or $$v = \frac{2}{9}\left[\frac{r^2 (\sigma - \rho) g}{\eta}\right]$$

or $$v = -\frac{2}{9}\left[\frac{r^2 (\rho - \sigma) g}{\eta}\right] \quad \ldots(7)$$

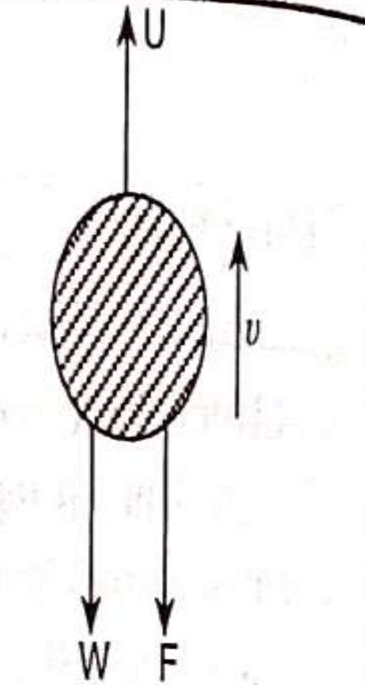


Fig. 7(*c*).11. A lighter body rising up in a denser fluid.

Since equation (6) represents a velocity in downward direction, equation (7) represents a velocity in upward direction. So, the body shall move upward till it acquires uniform velocity. As an example to this case we can consider an air bubble rising upward through water.

Thank You