Module: 02
FLUID MECHANICS

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Outlines:

- Ideal Fluid
- Streamline & Laminar Flow
- Critical Velocity
- Reynolds's Number
- Viscosity & Co-efficient of Viscosity

7(c).1. IDEAL FLUID

Motion of a fluid may be constrained to many restrictions. It may undergo a change in density due to sudden change in diameter of pipe. There may be frictional interaction with the side of pipe and may be inbetween its different layers. Sometimes rotational motion may appear alongwith translatory motion. All these constraints can make the study of motion of fluid quite complicated. So, we evolve the concept of an *ideal fluid* which is supposed to be free from these complications.

Characteristics of ideal fluid : An ideal fluid is supposed to possess following characteristics :

(*i*) ***It is incompressible.*** Its volume can not be changed by the application of pressure. Since mass of fluid is a constant quantity, its density shall also taken to be constant.

(*ii*) ***It is nonviscous.*** As ideal fluid moves, there is no frictional drag between its layers. Therefore, there will be no dissipation of energy during the flow.

(*iii*) ***Flow of ideal fluid is irrotational.*** Due to flow of fluid, no internal torques, causing rotational motion, can come into play.

(*iv*) ***It is capable of exhibiting steady flow.*** Velocity of flow and pressure, at any point, do not change with time.

Concept of ideal fluid is only theoretical. In actual practice we do not find any fluid which satisfies all these characteristics. All the formulation to be discussed here pertain to ideal fluid. So, practical results are likely to show some deviation due to the deviation of behaviour of fluid from that of an ideal one.

7(c).2. FLOW OF FLUID

A flowing liquid may be regarded as consisting of a number of layers one above the other. Flow of liquid can be put into two categories.

(*i*) **Streamline flow.** *Flow of a liquid fluid is said to be streamlined if the velocity of a molecule, at any point, coincides with that of the preceding one.*

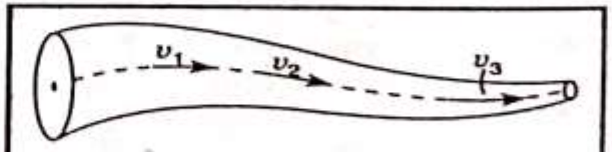

Fig. 7(c).1. Flow of liquid through a non-uniform tube.

Consider a liquid flowing through a tube of non-uniform area of cross-section as shown in Fig. 7(*c*).1. If all the molecules while at A, B, and C possess velocities v_1, v_2 and v_3 respectively, the motion is said to be **streamlined** or **steady** or **orderly.**

This type of flow is found to have following characteristics :

1. The velocity of any molecule, at a point, is independent of time.
2. The layer of liquid in contact with the solid surface is at rest.
3. The liquid in streamlined flow can be supposed to be in the form of parallel layers one above the other. This is why the flow is also called ***laminar flow.***

4. Motion is governed by Newton's law of viscosity.

5. In this flow, the loss of energy varies as the first power of velocity.

The path of molecule, in such a flow, is called ***a streamline.*** A streamline may be strength or curved. Direction of motion of any molecule, at any point, on a streamline is given by the direction of tangent drawn at that point. For this reason no two streamlines can ever intersect each other.

A bundle of streamlines having same velocity of fluid elements, over any cross-section perpendicular to the direction of flow, is called a ***tube of flow.***

Laminer flow : It is a special case of streamline flow in which velocities of all the molecules on one streamline is same throughout its motion. The velocities of molecules lying on different stream lines may be different.

Consider the motion of water in a canal [Fig. 7(*c*).2]. Velocity of molecules in contact with the bed of the canal is zero. The velocities of molecules in different layers, as we move away from bed, increases. Velocities of all the molecules in one layer is same and remains the same throughout the motion. The gradation in the velocities of different layers is due to the property ***viscosity*** of the liquid.

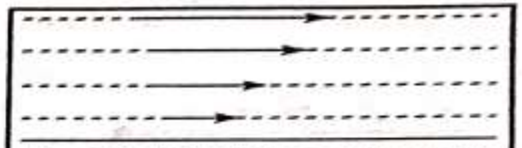

Fig. 7(*c*).2. Motion of water in a canal

(*ii*) **Turbulent flow.** Whenever the velocity of a fluid is very high or it rushes past an obstacle so that there is a sudden change in its direction of motion, the motion of fluid becomes irregular, forming eddies or whirlpools. This type of motion of fluids is called ***turbulent flow.*** A fluid moving in turbulent flow exerts greater thrust on an obstacle in its path. Of course, it depends on how suddenly the direction of motion of fluid, is changed by the obstacle.

Turbulance may happen due to two reasons :

(i) Speed of fluid : Consider flow of fluid through a pipe ABC, having a constriction at B [Fig. 7(*c*).3] As the fluid passes through B, its velocity increases. It will be observed that while motion of fluid in A and C is steady, it is turbulent at B.

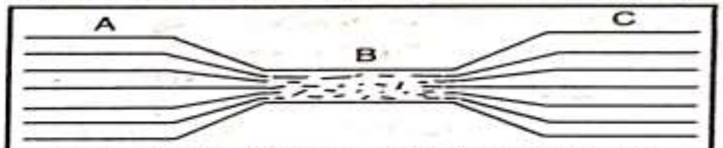

Fig. 7(*c*).3. Motion of liquid through a constriction

(ii) Shape of obstacle : Presence of an obstacle in the path of fluid imparts a sudden change in direction of its motion. Fig. 7(*c*).4(*a*) shows a pinpointed shape of obstacle in which there is no sudden change in direction of motion. So this offers minimum resistance to motion. Fig. 7(*c*).4(*b*) shows a spherical ball as obstacle. In this the resistance is comparatively greater while it is maximum if the obstacle is a flat one as shown in Fig. 7(*c*).4(*c*). So, the turbulence is maximum in case of flat obstacle.

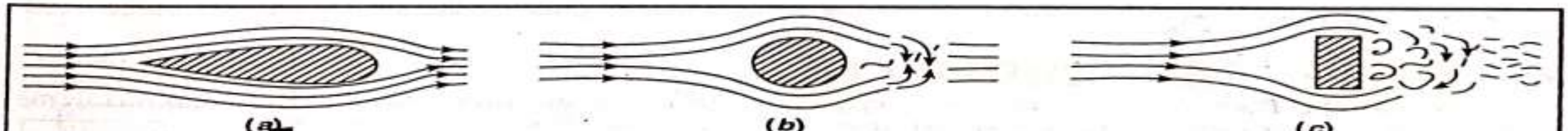

Fig. 7(*c*).4. Effect of obstacle on flow of fluid.

7(c).3. CRITICAL VELOCITY — "REYNOLD'S NUMBER"

If the velocity of a flow of liquid is increased gradually, the flow remains streamlined up to a certain value of velocity of liquid, beyond which the flow becomes turbulent.

Critical velocity is the maximum velocity of the flow of liquid flowing in a streamlined flow.

Critical velocity 'v_c' of a liquid flowing through a tube depends upon the co-efficient of viscosity (η) of the liquid, density (ρ) of the liquid and the diameter (D) of the tube. An expression for 'v_c' can be obtained by the method of dimensions.

Let $\quad v_c \propto \eta^x, v_c \propto \rho^y$

and $\quad v_c \propto D^z \quad \therefore \quad v_c = R\eta^x \rho^y D^z$ ...(1)

where 'R' is dimensionless constant.

Writing the dimensional formulae of the quantities on both sides,

$$M^0L^1T^{-1} = R[ML^{-1}T^{-1}]^x [ML^{-3}]^y [L]^z$$

or

$$M^0 L^1T^{-1} = RM^{x+y} L^{-x-3y+z} T^{-x}$$

Using principle of homogeneity,

$$x + y = 0, \ -x - 3y + z = 1$$

$$-x = -1.$$

Solving these equations simultaneously, we obtain $x = 1$, $y = -1$ and $z = -1$

Substituting for x, y and z in Equation (1),

$$v_c = R\frac{\eta}{\rho D}$$

'R' is called Reynold's constant or **Reynold's number.**

Reynold's number is pure number and, therefore, its numerical value is same in every set of units. The flow of viscous liquid is said to be steady when R lies between 0 and 2000. For values of R about 3000 flow is turbulent. For 'R' lying in between 2000 and 3000, the flow is unstable and may switch over from one type to another.

7(c).4. VISCOSITY

Put a glass rod in a beaker containing liquid and start rotating the rod. The liquid, also, starts rotating and acquires the shape of a cone as shown in Fig. 7(c).5. Lowest layer of the liquid is at rest due to the force of adhesion while other layers are in motion. Velocity of top layer is maximum, say "v_{max}" since it is removed maximum distance away from the axis of rotation. Velocities of layers in between lie between 0 and v_{max}. Thus, *there exists a relative velocity between every two layers.*

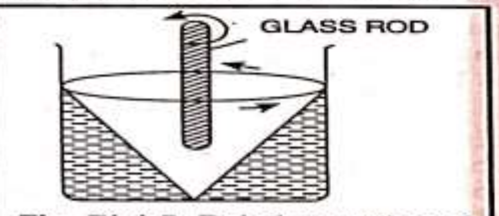

Fig. 7(c).5. Relative motion of different layers of liquid.

Now take the rod out, it will be observed that all the layers come to rest after some time, meaning that their relative velocity has been destroyed. According to Newton's first law, a motion cannot be destroyed without the help of an external force. Therefore, within the liquid, there must be a force which opposes the relative motion between its layers. This property of liquids or, in general, of fluids is called ***viscosity.***

Viscosity is the property of fluids by virtue of which they tend to destroy any relative motion between their layers.

7(c).5. COEFFICIENT OF VISCOSITY (η)

Consider a section of a canal moving over the earth's surface. Water flows through the canal due to the slope of the bed. A component ($mg \sin \theta$) of the weight of water along the bed of canal is responsible for its motion. Lowest layer is at rest due to the force of adhesion between the layer and the bed of canal while top layer is moving with maximum velocity [Fig. 7(c).6]. Thus, there is some relative velocity between any of its two layers.

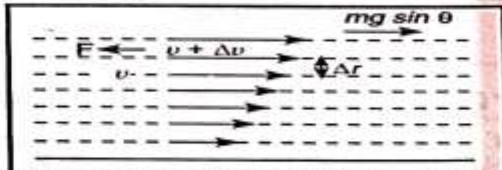

Fig. 7(c).6. Velocity gradation of different layers of liquid.

Consider two layers of liquid separated a small distance Δr apart having velocities v and $v + \Delta v$. The relative velocity between them is Δv.

$\Delta v/\Delta r$ is known as the ***average velocity gradient.***

Average velocity gradient is the difference between velocities of two layers separated a unit distance apart.

7(c).8. SOME TERMS CONNECTED WITH VISCOSITY

1. Coefficient of viscosity (Absolute viscosity or Dynamic viscosity)

According to Equation (2),

$$F = \eta \frac{A\,dv}{dr} \qquad \text{If } A = 1,\ \frac{dv}{dr} = 1$$

$$F = \eta$$

Coefficient of viscosity of a fluid is defined as the tangential force per unit area which is required to maintain a unit velocity gradient between its layers.

Or

Coefficient of viscosity of a fluid is defined as the tangential force per unit area which is required to maintain (or resist) a unit relative velocity between two layers a unit distance apart.

Units of η (*i*) In C.G.S. system. Coefficient of viscosity η is measured in '*poise*'.

If $A = 1\ cm^2,\ v = 1\ cm\ s^{-1}$

$r = 1\ cm$ and $F = 1\ dyn$

From equation (2), $1 = \eta \frac{1 \times 1}{1}$ or $\eta = 1$ *poise*.

Poise. *Coefficient of viscosity of a fluid is said to be one 'poise' if a tangential force of 1 dyn per square cm is required to maintain a velocity gradient of 1 s^{-1} between its layers.*

(*ii*) **In S.I.** Coefficient of viscosity in S.I. is '*deca poise*'.

If $A = 1\ m^2,\ \frac{dv}{dr} = 1\ s^{-1}\ m$ and $F = 1\ N$

From equation (1), η = 1 *deca poise* or 1 *pascal second* (*Pa s*)

Decapoise (or *Pa s*). *Coefficient of viscosity of a liquid is said to be one decapoise if a tangential force of 1 N per metre square is required to maintain a velocity gradient of 1 s^{-1} between its layers.*

1 *decapoise* (or *Pa s*) = $1\ N\ sm^{-2} = 1\ kg\ m^{-1}\ s^{-1}$

Dimensional formula of η

$$\eta = \frac{F\,dr}{A\,dv} = \frac{[M^1L^1T^{-2}][L]}{[L^2][LT^{-1}]} \quad \text{or} \quad \eta = [M^1L^{-1}T^{-1}]$$

2. Fluidity. *Reciprocal of coefficient of viscosity of a fluid is called its* ***fluidity.***

$$\text{Fluidity} = \frac{1}{\eta}$$

Fluidity of a fluid is a measure of its ability to flow with ease. Greater the fluidity more easily the fluid can flow.

Unit of fluidity : *poise*$^{-1}$

Dimensional formula of fluidity = $[M^{-1}L^1T^1]$

Thank You