# WALL SHEAR & BOUNDARY LAYER THICKNESS

Wall shear can be evaluated as,

$$\tau_w = \mu \frac{\partial u}{\partial y}\Big|_{y=0} \quad [\text{here } y \to 0]$$

$$\Rightarrow \tau_w = \mu U_\infty \frac{\partial}{\partial \eta} f'(\eta) \cdot \frac{\partial \eta}{\partial y}\Big|_{\eta=0}$$

$$= \mu U_\infty \times 0.33206 \times \frac{1}{\sqrt{\frac{\nu x}{U_\infty}}}$$

[we know from table $f''(0) = 0.33206$]

$$= \frac{0.332 \rho U_\infty^2}{\sqrt{Re_x}} \longrightarrow (A)$$

and the local skin friction coefficient is

$$C_{fx} = \frac{\tau_w}{\frac{1}{2}\rho U_\infty^2}$$

Now, substituting (A)

$$C_{fx} = \frac{0.332 \rho U_\infty^2}{\sqrt{Re_x}} \times \frac{2}{\rho U_\infty^2}$$

$$= \frac{0.664}{\sqrt{Re_x}} \longrightarrow (B)$$

Total frictional force per width for the plate of length L

$$F = \int_0^L \tau_w dx$$

$$= \int_0^L \frac{0.332 \rho U_\infty^2}{\sqrt{\frac{U_\infty}{\nu}}} \frac{dx}{\sqrt{x}}$$

$$= \left[\frac{0.332 \rho U_\infty^2}{\sqrt{\frac{U_\infty}{\nu}}} \times \frac{x^{1/2}}{\left(\frac{1}{2}\right)}\right]_0^L$$

$$= 0.664 \times \rho U_\infty^2 \sqrt{\frac{\nu L}{(1/2)}} \longrightarrow (C)$$

and, the average skin friction coefficient is

$$\overline{C_f} = \frac{F}{\frac{1}{2}(\rho U_\infty^2 L)} = \frac{1.328}{\sqrt{Re_L}} \rightarrow (D) \quad \text{[from table]}$$

where, $Re_L = \frac{U_\infty L}{\nu}$

~~From~~ Since, $\frac{u}{U_\infty}$ approaches 1.0 as $y \rightarrow \infty$, it is customary to select the boundary layer thickness $\delta$ as that point where $\frac{u}{U_\infty}$ approaches 0.99. From table, $\frac{u}{U_\infty}$ reaches 0.99 at $\eta = 5.0$ and we can write

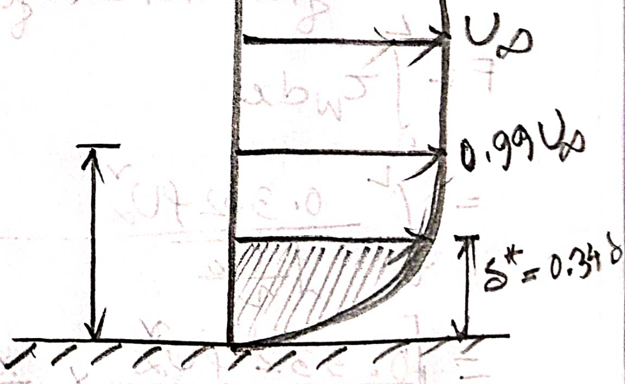
$$\frac{\delta}{\sqrt{\frac{\nu x}{U_\infty}}} = 5.0$$

$$\Rightarrow \delta = 5.0\sqrt{\frac{\nu x}{U_\infty}} = \frac{5.0x}{\sqrt{Re_x}} \rightarrow (E)$$

* From the defination of boundary layer thickness is ~~somewhat~~ arbitary, a physically more meaningful measure of boundary layer estimation is expressed through displacement thickness.

- Displacement Thickness ($\delta^*$):

is defined as the distance by which the external potential flow is displaced outwards due to the decrease in velocity in the boundary layer.




$$U_\infty \delta^* = \int_0^\infty (U_\infty - u)\,dy$$

$$\delta^* = \int_0^\infty \left(1 - \frac{u}{U_\infty}\right) dy \rightarrow (i)$$