For, Blasius flow, it is possible to identify $g(x)$ with the boundary layers thickness δ, we know

$$\varepsilon = \frac{\delta}{L} \sim \frac{1}{\sqrt{Re_L}}$$

in terms of x, we get

$$\frac{\delta}{x} \sim \frac{1}{\sqrt{\frac{U_\infty x}{\nu}}}$$

$$\delta \sim \sqrt{\frac{\nu x}{U_\infty}}$$

i.e

$$\frac{u}{U_\infty} = F\left(\frac{y}{\sqrt{\frac{\nu x}{U_\infty}}}\right) = F(\eta) \longrightarrow (4)$$

where, $\eta \sim \frac{y}{\delta}$ and $\delta \sim \sqrt{\frac{\nu x}{U_\infty}}$

or, more precisely,

$$\eta = \frac{y}{\sqrt{\frac{\nu x}{U_\infty}}} \longrightarrow (5)$$

$$y = \eta \sqrt{\frac{\nu x}{U_\infty}}$$

$$dy = \sqrt{\frac{\nu x}{U_\infty}}\, d\eta$$

The stream function can now be obtained in terms of the velocity components as

$$\psi = \int u\, dy = \int U_\infty F(\eta) \sqrt{\frac{\nu x}{U_\infty}}\, d\eta = \sqrt{U_\infty \nu x} \int F(\eta)\, d\eta$$

$$\Rightarrow \psi = \sqrt{U_\infty \nu x}\, f(\eta) + D \longrightarrow (6)$$

where, D is a constant also $\int F(\eta)\, d\eta = f(\eta)$ and the constant of integration is zero if the stream function at solid surface is set equal to zero.

Now, the velocity components and their derivatives are

$$u = \frac{\partial \psi}{\partial y} = \frac{\partial \psi}{\partial \eta}\frac{\partial \eta}{\partial y} = U_\infty f'(\eta) \longrightarrow 7(a)$$

$$v = -\frac{\partial \psi}{\partial x} = -\sqrt{U_\infty \nu}\left[\frac{1}{2}\frac{1}{\sqrt{x}}f(\eta) + \sqrt{x}f'(\eta)\left\{-\frac{1}{2}\frac{y}{\sqrt{\nu x/U_\infty}}\frac{1}{x}\right\}\right]$$

$$\Rightarrow v = \frac{1}{2}\sqrt{\frac{\nu U_\infty}{x}}\left[\eta f'(\eta) - f(\eta)\right] \longrightarrow 7.(b)$$

Now,

$$\frac{\partial u}{\partial x} = U_\infty f''(\eta)\frac{\partial \eta}{\partial x}$$

$$= U_\infty f''(\eta)\left[-\frac{1}{2}\frac{y}{\sqrt{\frac{\nu x}{U_\infty}}}\frac{1}{x}\right]$$

$$= -\frac{U_\infty}{2}\frac{\eta}{x}f''(\eta) \longrightarrow 7.(c)$$

and

$$\frac{\partial u}{\partial y} = U_\infty f''(\eta)\frac{\partial \eta}{\partial y}$$

$$= U_\infty f''(\eta)\left[\frac{1}{\sqrt{\nu x/U_\infty}}\right]$$

$$= U_\infty\sqrt{\frac{U_\infty}{\nu x}}f''(\eta) \longrightarrow 7.(d)$$

$$\frac{\partial^2 u}{\partial y^2} = U_\infty\sqrt{\frac{U_\infty}{\nu x}}f''(\eta)\left\{\frac{1}{\sqrt{\nu x/U_\infty}}\right\}$$

$$= \frac{U_\infty^2}{\nu x}f'''(\eta) \longrightarrow 7(e)$$

Substituting these values in equations (1)

$$-\frac{U_\infty^2}{2}\frac{\eta}{x}f'(\eta)f''(\eta) + \frac{U_\infty^2}{2x}\left[\eta f'(\eta) - f(\eta)\right]f''(\eta) = \frac{U_\infty^2}{x}f'''(\eta)$$

$$\Rightarrow \boxed{-\frac{1}{2}\frac{U_\infty^2}{x}f(\eta)f''(\eta) = \frac{U_\infty^2}{x}f'''(\eta)}$$

$$\Rightarrow \boxed{2f'''(\eta) + f(\eta)\cdot f''(\eta) = 0}$$

where

$$f(\eta) = \int F(\eta)\,d\eta + C = \int \frac{u}{U_\infty}\,d\eta + C$$

and

$$\eta = \frac{y}{\sqrt{\frac{\nu x}{U_\infty}}}$$

This is known a Blasius Equation