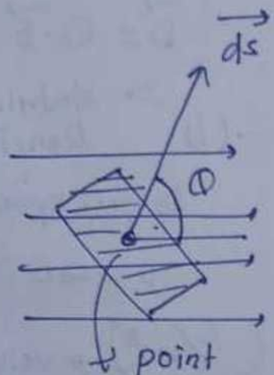
## * maxwell's equation :

- its derived from Gauss law.

flux → It is a measure of how much field lines passes through any surface.

→ surface direction is always normal to the surface i.e perpendicular to the surface.

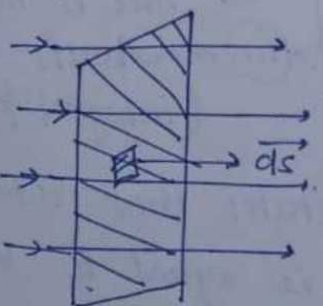



$d\psi = \vec{E} \cdot \vec{ds}$ → flux for one point

$\psi = \int_s \vec{E} \cdot \vec{ds}$ → flux through ~~whole~~ all the surface

$E \cdot ds = E\, ds \cos\theta$ $\quad\quad \theta = 0^\circ$ → maximum value.

⇒ When both the direction $\vec{ds}$ and $\vec{E}$ are same, then.




$\vec{E}$

$\theta = 0$

$\cos 0^\circ = 1$

so, maximum flux will generate.

maximum field passing = maximum ~~field~~ flux will generate

∴ $\psi = \int_s E \cdot ds \cos 0^\circ$

$= \int_s E \cdot ds$

$$\boxed{\psi = E \times \text{Area}}$$

(ii)

Gauss law :- it states that total electric flux passing through any close surface is equal to charge enclosed by closed surface. divided by $\epsilon_0$.

$$\oint_s \vec{E}\cdot\vec{ds} = \frac{Q_{enclosed}}{\epsilon_0} \qquad \because \psi = \text{flux} \int \vec{E}\cdot\vec{ds}$$

$$\oint_s \epsilon_0 \vec{E}\cdot\vec{ds} = Q_{enc} \qquad \therefore \vec{D} = \epsilon_0 \cdot \vec{E}$$

→ electric flux Density.

$$\oint_s \vec{D}\cdot\vec{ds} = Q_{enc} \longrightarrow (i)$$

unit → $C/m^2$

In free space

$D = \epsilon_0 E$

let $\rho_v$ be the volume charge density ($C/m^3$) → volume

→ To find out the charge from the volume i.e.

$$Q = \int_v \rho_v \, dv \longrightarrow (ii)$$

From equation (i) and (ii)

$$\boxed{\oint_s \vec{D}\cdot\vec{ds} = \int_v \rho_v \, dv} \rightarrow (iii)$$

→ This is nothing but the maxwell's 1st equation (integral form)

⇒ Divergence theorem :- it states that total flux passing through any closed surface is equal to volume integral of divergence of the vector

$$\oint_s \vec{D}\cdot\vec{ds} = \int_v (\nabla\cdot\vec{D}) \, dv \longrightarrow (iv)$$

From equation (iii) & (iv)

$$\int_v (\nabla\cdot D) \, dv = \int_v \rho_v \, dv$$

$$\boxed{\nabla\cdot\vec{D} = \rho_v} \rightarrow (v)$$

maxwell's 1st equation in (Differential form or point form)

(iii)

⇒ For Source free Region. (charge is Zero)

$\rho_v = 0$

$\boxed{\nabla \cdot \vec{D} = 0}$ { $\vec{D}$ is a divergenceless if $\nabla \cdot \vec{D} = 0$ } or Solenoid.

2nd maxwell's equation

conservative nature of electric field (for static)
(static means → field does not change w.r.t time)

Total work done to move a point charge around the closed loop in presence of electric field is always equal to Zero.

$$\boxed{\oint_l \vec{E} \cdot \vec{dl} = 0}$$

maxwell's second (2nd) equation for static field.
(integral form) for static field

applying stoke's theorem.

Stokes theorem: A circulation of Vector $\vec{A}$ ~~in a~~ around the closed loop is equal to the Surface integral of curl of that Vector over the circle bounded by closed loop.

$$\oint_l \vec{A} \cdot \vec{dl} = \int_s (\nabla \times \vec{A}) \cdot \vec{ds}$$

$$\int_s (\nabla \times \vec{E}) d\vec{s} = 0$$

$$\boxed{\nabla \times \vec{E} = 0}$$ Maxwell's 2nd equation, (in Differential form or point form)

Note: if the curl is Zero, then its nothing but conservative field.

→ conservative in nature

Now, for Time Varying field :

2) – it comes from faraday's law.

Faraday's law:

it states that induced emf in a closed circuit is equal to time rate of change in flux linkage in the circuit.

$$V_{emf} = -\frac{d\lambda}{dt}$$ , where

$\lambda$ = flux linkage (flux linking with no. of turns)

$= N\psi$

$$V_{emf} = -\frac{N\,d\psi}{dt}$$

$\psi$ = flux.

$$\psi = \int_s \vec{B}\cdot d\vec{s}$$

D = electric ~~field~~ flux Density

B = magnetic ~~field~~ flux Density.

$\vec{B}$ = magnetic flux density.

unit = $wb/m^2$ or Tesla.

$\psi_{gai}$

$$V_{emf} = -N\frac{d}{dt}\int_s \vec{B}\cdot d\vec{s} \quad \text{—— (1)}$$

Potential,

$$V = \oint_l \vec{E}\cdot d\vec{l} \quad \text{—— (2)}$$

both this two equations are difining emf. so,

Therefore from eqn (1) & (2) we can write.

$$\boxed{\oint \vec{E}\cdot d\vec{l} = -N\frac{d}{dt}\int_s \vec{B}\cdot d\vec{s}} \quad \text{— (3)}$$

↳ Maxwell's $2^{nd}$ equation (integral form)
For time varying.

applying stokes theorem, $\oint \vec{A}\cdot d\vec{l} = \int_s (\nabla\times\vec{A})\cdot d\vec{s}$

From (3) eqn $\quad \int_s (\nabla\times\vec{E})\cdot d\vec{s} = -N\int_s \frac{d\vec{B}}{dt}\cdot d\vec{s}$

$$\nabla \times \vec{E} = -N \frac{d\vec{B}}{dt}$$

if $N=1$

$$\boxed{\nabla \times \vec{E} = -\frac{\partial \vec{B}}{dt}}$$

→ Maxwell's $2^{nd}$ eqⁿ for ~~static field~~ time Varying field (differential form or point form)