

Measurement of Resistance

Methods of measuring low, medium and high resistances

Classification of resistances:

Low resistance:

Resistance of the order of 1 ohm and under.

Medium resistance:

1 ohm to 100 Kilo ohm or 0.1Mohm.

High resistance:

100 k ohm and above.



Wheatstone Bridge

Definition: The device **uses** for the **measurement of minimum resistance** with the help of comparison method is **known** as the **Wheatstone bridge**. The value of **unknown resistance** is determined by **comparing** it with the **known resistance**. The Wheatstone bridge **works** on the **principle of null deflection**, i.e. the ratio of their **resistances** are equal, and no current flows through the **galvanometer**. The bridge is very **reliable** and gives an **accurate** result.

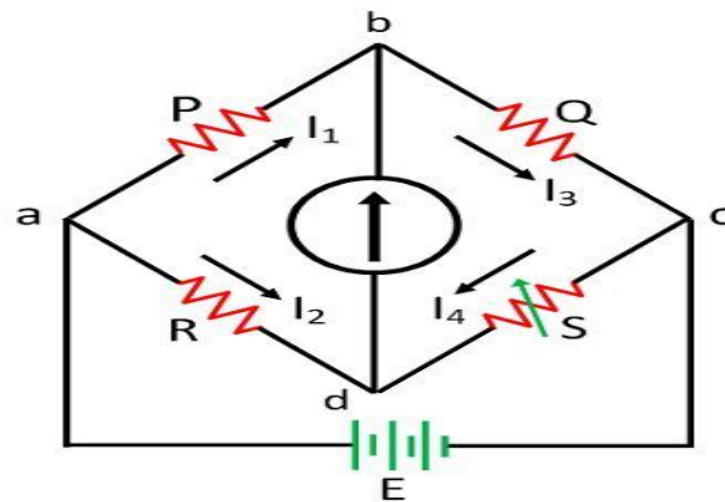
In normal condition, the bridge remains in the unbalanced condition, i.e. the current flow through the galvanometer. When zero current passes through the galvanometer, then the bridge is said to be in balanced condition. This can be done by adjusting the known resistance P, Q and the variable resistance S.

The working of the bridge is similar to the **potentiometer**. The Wheatstone bridge is only used for determining the medium resistance. For measuring the high resistance, the sensitive ammeter is used in the circuit.



Construction of Wheatstone Bridge

The basic circuit of the Wheatstone bridge is shown in the figure below. The bridge has four arms which consist two unknown resistance, one variable resistance and the one unknown resistance along with the emf source and galvanometer.



Wheatstone Bridge



Working of Galvanometer

The bridge is in balance condition when no current flows through the coil or the potential difference across the galvanometer is zero. This condition occurs when the potential difference across the a to b and a to d are equal, and the potential differences across the b to c and c to d remain same.

The current enters into the galvanometer divides into I_1 and I_2 , and their magnitude remains same. The following condition exists when the current through the galvanometer is zero.



$$I_1P = I_2R \dots \dots \dots equ(1)$$

The bridge in a balanced condition is expressed as

$$I_1 = I_3 = \frac{E}{P + Q}$$

$$I_2 = I_4 = \frac{E}{R + S}$$

Where E – emf of the battery.

By substituting the value of I_1 and I_2 in equation (1) we get.

$$\frac{PE}{P + Q} = \frac{RE}{R + S}$$

$$\frac{P}{P + Q} = \frac{R}{R + S}$$

$$P(R + S) = R(P + Q)$$



$$PR + PS = RP + RQ$$

$$PS = RQ \dots\dots\dots equ(2)$$

$$R = \frac{P}{Q} \times S \dots\dots\dots equ(3)$$

The equation (2) shows the balance condition of the Wheatstone bridge.

The value of unknown resistance is determined by the help of the equation (3). The R is the unknown resistance, and the S is the standard arm of the bridge and the P and Q are the ratio arm of the bridge.

Errors in Wheatstone Bridge

The following are the errors in the Wheatstone bridge.

1. The difference between the true and the mark value of the three resistances can cause the error in measurement.
2. The galvanometer is less sensitive. Thus, inaccuracy occurs in the balance point.
3. The resistance of the bridge changes because of the self-heating which generates an error.
4. The thermal emf cause serious trouble in the measurement of low-value resistance.
5. The personal error occurs in the galvanometer by taking the reading or by finding the null point.

The above mention error can be reduced by using the best qualities resistor and galvanometer. The error because of self-heating of resistance can minimise by measuring the resistance within the short time. The thermal effect can also be reduced by connecting the reversing switch between the battery and the bridge.

Limitation of Wheat Stone Bridge

The Wheatstone bridge gives inaccurate readings if it is unbalanced. The Wheatstone bridge measures resistance from few ohms to megohms.

The upper range of the bridge can be increased with the help of the applied emf, and the lower range is limited by connecting the lead at the binding post.



Sensitivity of Wheatstone Bridge

It is frequently desirable to know the galvanometer response to be expected in a bridge which is slightly unbalanced so that a current flows in the galvanometer branch of the bridge network. This may be used for

(i) selecting a galvanometer with which a given unbalance may be observed in a specified bridge arrangement,

(ii) determining the minimum unbalance which can be observed with a given galvanometer in the specified bridge arrangement, and

(iii) determining the deflection to be expected for a given unbalance.

The **sensitivity** to unbalance can be computed by solving the bridge circuit for a small unbalance. The solution is approached by converting the **Wheatstone bridge** of the figure above to its "Thevenin Equivalent" circuit.

Assume that the bridge is balanced when the branch **resistances** are P, Q, R, S so that $P / Q = R / S$. Suppose the resistance R is changed to $R + \Delta R$ creating an unbalance. This will cause an emf 'e' to appear across the galvanometer branch. With the galvanometer branch open, the **voltage** drop between points a and



$$E_{ab} = I_1 P = \frac{EP}{P+Q}$$

$$\text{Similarly, } E_{ad} = I_2(R + \Delta R) = \frac{E(R + \Delta R)}{R + \Delta R + S}$$

Therefore voltage difference between points d and b is :

$$e = E_{ad} - E_{ab} = E \left[\frac{R + \Delta R}{R + \Delta R + S} - \frac{P}{P + Q} \right]$$

and since $\frac{P}{P+Q} = \frac{R}{R+S}$

$$\begin{aligned} \therefore e &= E \left[\frac{R + \Delta R}{R + \Delta R + S} - \frac{P}{P + S} \right] \\ &= \frac{ES\Delta R}{(R + S)^2 + \Delta R(R + S)} \\ &\approx \frac{ES\Delta R}{(R + S)^2} \end{aligned}$$

as $\Delta R(R + S) \ll (R + S)^2$

Let S_v be the voltage sensitivity of galvanometer.

Therefore, deflection of galvanometer is

$$\theta = S_v e = S_v \frac{ES\Delta R}{(R+S)^2}$$

The bridge **sensitivity** is defined as the deflection of the galvanometer per unit fractional change in unknown **resistance**.

Sensitivity of Wheatstone bridge,

$$\begin{aligned} S_B &= \frac{\theta}{\Delta R / R} \\ &= \frac{S_v ESR}{(R+S)^2} \end{aligned}$$



Kelvin Bridge

Definition: The Kelvin bridge or Thompson bridge is used for measuring the unknown resistances having a value less than 1Ω . It is the modified form of the Wheatstone Bridge.

What is the need of Kelvin Bridge?

Wheatstone bridge use for measuring the resistance from a few ohms to several kilo-ohms. **But error occurs in the result when it is used for measuring the low resistance.** This is the reason because of which the [Wheatstone bridge](#) is modified, and the Kelvin bridge obtains. The Kelvin bridge is suitable for measuring the low [resistance](#).



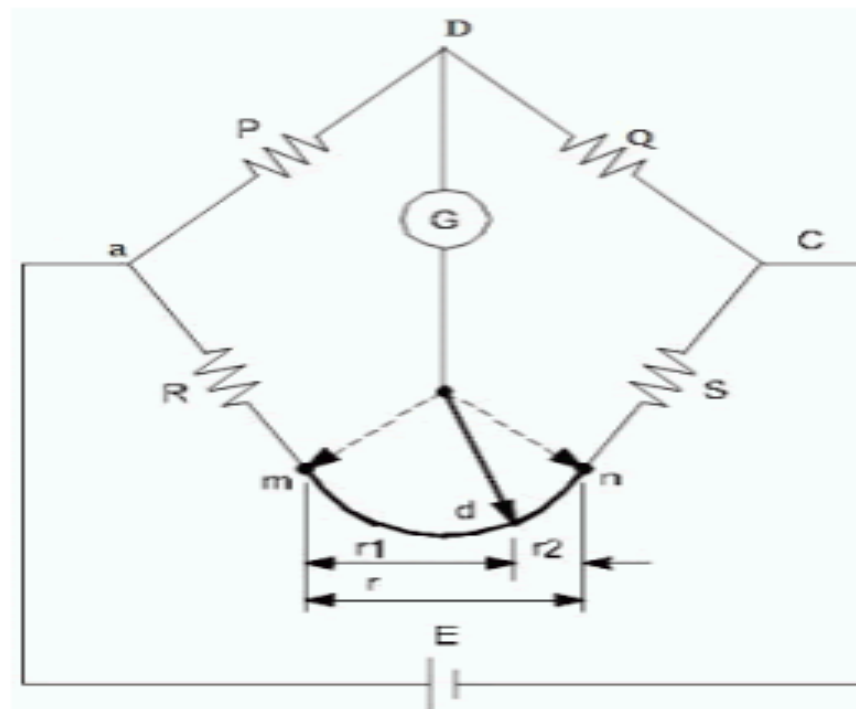
The **Kelvin Bridge circuit** is a modification of **Wheatstone Bridge**. An understanding of the kelvin Bridge arrangement may PV be obtained by a study of difficulties that arise in a **Wheatstone bridge** on account of the **resistance** of the leads and the contact resistances while measuring low valued resistors.

Consider the bridge circuit shown below where 'r' represents the resistance of the lead that connects the unknown resistance 'R' to standard resistance 'S'. Two galvanometer connections indicated by dotted lines are possible. The connection may be either to point 'm' or to point 'n'. When the galvanometer is connected to point 'm', the **resistance** 'r', of the connecting leads is added to the standard resistance 'S' resulting in indication of too low an induction for unknown resistance R. When the connection is made to point 'n', the resistance 'r', is added to the unknown resistance resulting in indication of too high a value for R.



Suppose that instead of using point 'm', which gives a low result or 'n', which makes the result high, we make the galvanometer connection to any intermediate point 'd' as shown by full line in below figure. If a point 'd' the **resistance** 'r' is divided into two parts r₁ and r₂, such that ,

$$\frac{r_1}{r_2} = \frac{P}{Q}$$



Then the presence of r_1 , the **resistance** of connecting leads, causes no error in the result. We have

$$R + r_1 = \frac{P}{Q} (s + r_2)$$

$$\text{but } \frac{r_1}{r_2} = \frac{P}{Q}$$

$$\text{or } \frac{r_1}{r_1 + r_2} = \frac{P}{P + Q}$$

$$\text{or } r_1 = \frac{P}{P + Q} r$$

$$\text{as } r_1 + r_2 = r \text{ and } r_2 = \frac{Q}{P + Q} r$$



we can write above eqn. as

$$\left(R + \frac{P}{P+Q} \right) = \frac{P}{Q} \left(S + \frac{Q}{P+Q} \right)$$

or

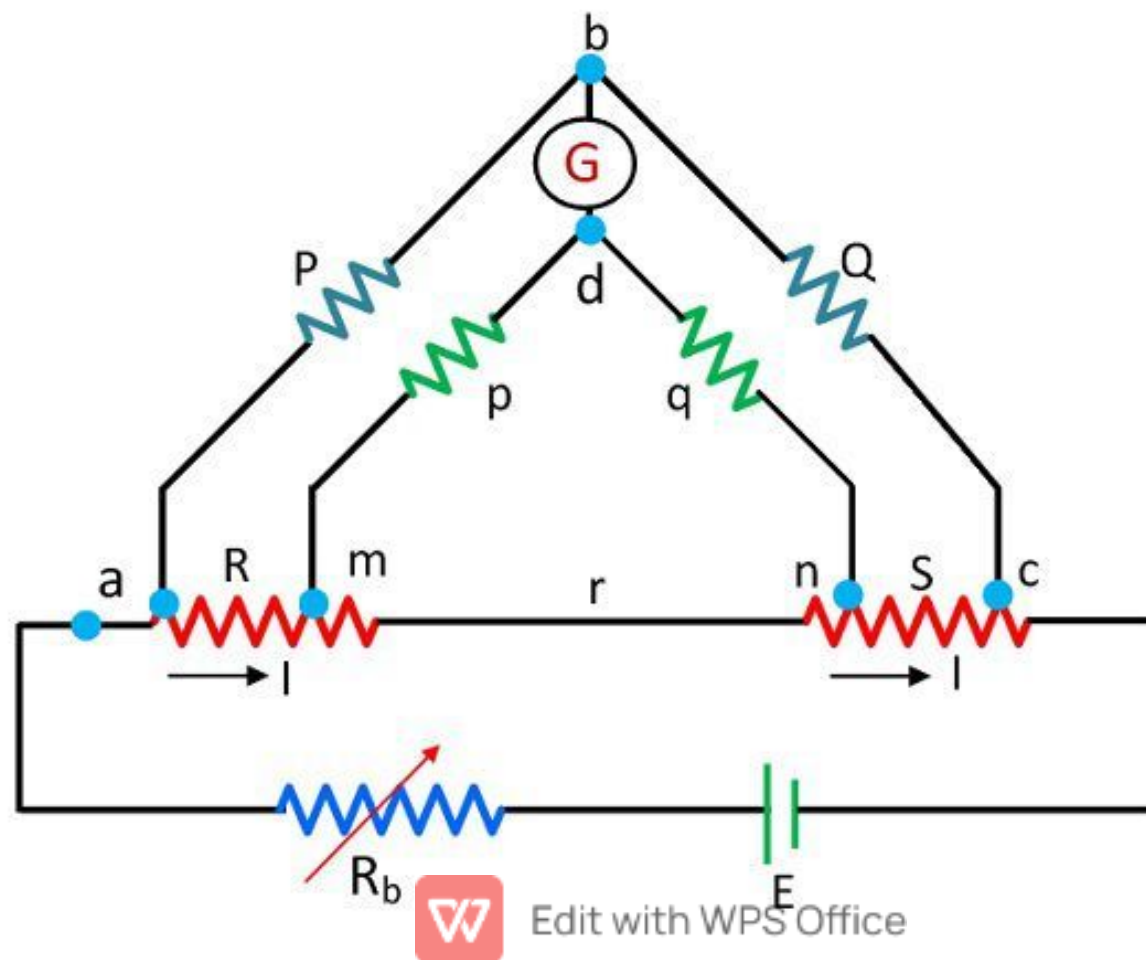
$$R = \frac{P}{Q} S$$

Therefore we conclude that making the galvanometer connection as at 'c', the **resistance** of leads does not affect the result.

The **Kelvin double bridge** method described above is obviously not a practical way of achieving the desired, result, as there would certainly be a trouble in determining the correct point for galvanometer connections. So the simple modification is that two actual resistance units of character ratio be connected between points m and n, the galvanometer be connected to the junction of the resistors. This is the actual **kelvin bridge arrangement** which is shown below.



The **Kelvin double bridge** incorporates the idea of a second set of ratio arms - hence the name double bridge has come and the use of four terminal resistors for low **resistance** arms. Below figure shows the schematic diagram of **kelvin double bridge**. The second set of ratio arms p and q is used to connect the galvanometer to a point d at the appropriate potential between points m and n to eliminate the effect of connecting lead of resistance r between the known resistance R and the standard resistance S .



The ratio p/q is made equal to P/Q . Under balanced conditions, there is no current through the galvanometer, which means that the **voltage** drop between a and b, E_{ab} is equal to the voltage drop E_{amd} between a and c.

Now,

$$E_{ab} = \frac{P}{P+Q} E_{ac}$$

And,

$$E_{ac} = I \left[R + S + \frac{(p+q)r}{p+q+r} \right]$$

$$\begin{aligned} E_{amd} &= I \left\{ R + \frac{p}{p+q} \frac{(p+q)r}{p+q+r} \right\} \\ &= I \left[R + \frac{pr}{p+q+r} \right] \end{aligned}$$



For zero galvanometer deflection,

$$E_{ab} = E_{amd}$$

$$\frac{P}{P+Q} I \left[R + S + \frac{(p+q)r}{p+q+r} \right] = I \left[R + \frac{pr}{p+q+r} \right]$$

$$R = \frac{P}{Q} S + \frac{qr}{p+q+r} \left[\frac{P}{Q} - \frac{p}{q} \right] \quad \text{eqn.1}$$

Now if $P/Q = p/q$, equation above becomes

$$R = \frac{P}{Q} S \quad \text{eqn.2}$$

Equation 2 to is the usual working equation for **Kelvin Bridge circuit**. It indicates that the **resistance** of connecting lead 'r' has no effect on the measurement, provided that the two sets of ratio arms have equal ratios. Equation 1, however, as it shows the error that is introduced in case the ratios are not exactly Ω equal. It indicates that it is desirable to keep r as small as possible in order to minimise the errors in case there is a difference between ratios P/Q and p/q .

