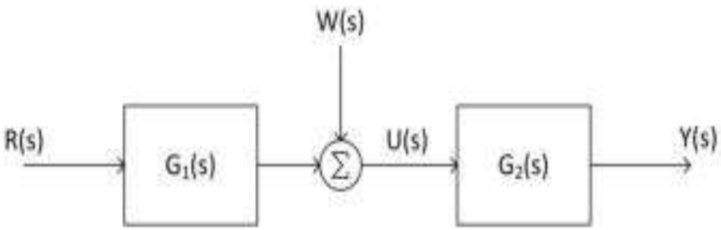
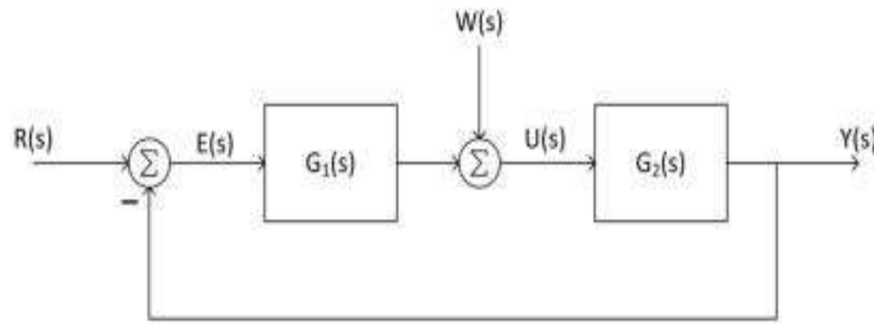
BLOCK DIAGRAMS

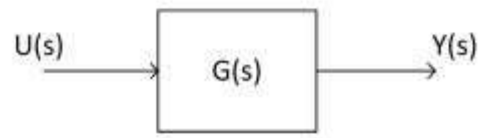
- In the introductory section we saw examples of ***block diagrams*** to represent systems, e.g.:





- Block diagrams consist of
  - ***Blocks*** – these represent subsystems – typically modeled by, and labeled with, a transfer function
  - ***Signals*** – inputs and outputs of blocks – signal direction indicated by arrows – could be voltage, velocity, force, etc.
  - ***Summing junctions*** – points were signals are algebraically summed – subtraction indicated by a negative sign near where the signal joins the summing junction

- The basic input/output relationship for a single block is:

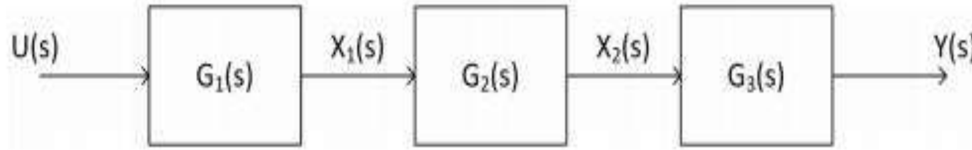


$$Y(s) = U(s) \cdot G(s)$$

- Block diagram blocks can be connected in three basic forms:
  - ***Cascade***
  - ***Parallel***
  - ***Feedback***

- We'll next look at each of these forms and derive a single-block equivalent for each

- Blocks connected in ***cascade***:



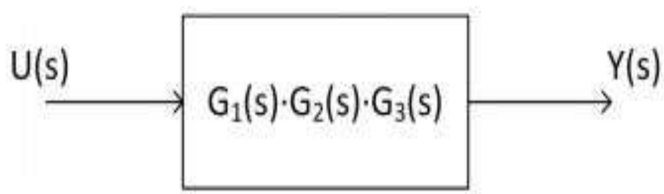
$$X_1(s) = U(s) \cdot G_1(s), \quad X_2(s) = X_1(s) \cdot G_2(s)$$

$$Y(s) = X_2(s) \cdot G_3(s) = X_1(s) \cdot G_2(s) \cdot G_3(s)$$

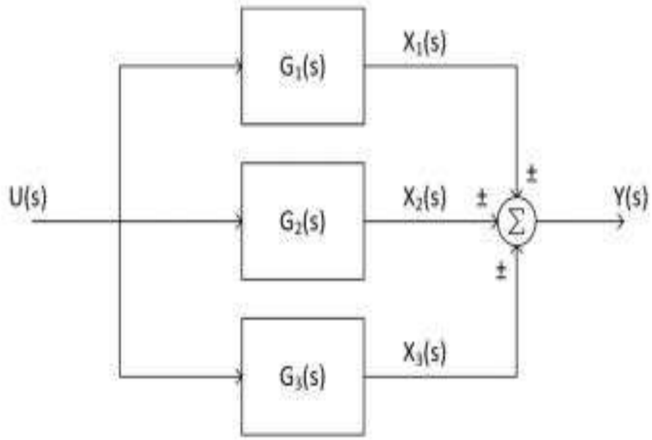
$$Y(s) = U(s) \cdot G_1(s) \cdot G_2(s) \cdot G_3(s) = U(s) \cdot G_{eq}(s)$$

$$G_{eq}(s) = G_1(s) \cdot G_2(s) \cdot G_3(s)$$

- The equivalent transfer function of cascaded blocks is the ***product*** of the individual transfer functions



- Blocks connected in parallel:



$$X_1(s) = U(s) \cdot G_1(s)$$

$$X_2(s) = U(s) \cdot G_2(s)$$

$$X_3(s) = U(s) \cdot G_3(s)$$

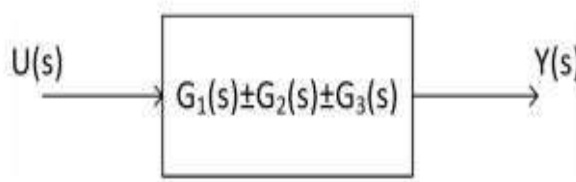
$$Y(s) = X_1(s) \pm X_2(s) \pm X_3(s)$$

$$Y(s) = U(s) \cdot G_1(s) \pm U(s) \cdot G_2(s) \pm U(s) \cdot G_3(s)$$

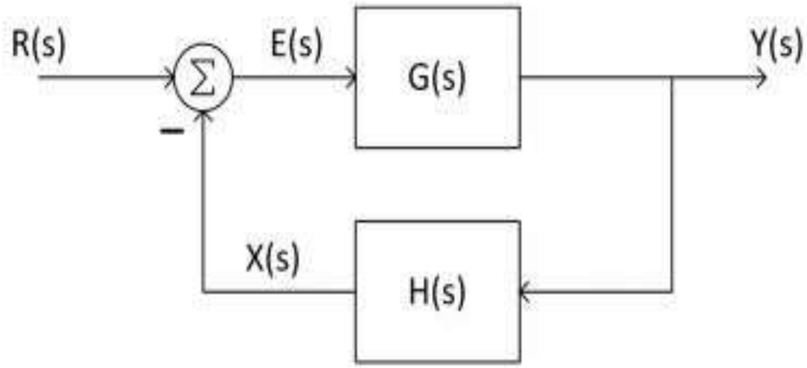
$$Y(s) = U(s)[G_1(s) \pm G_2(s) \pm G_3(s)] = U(s) \cdot G_{eq}(s)$$

$$G_{eq}(s) = G_1(s) \pm G_2(s) \pm G_3(s)$$

- The equivalent transfer function is the ***sum*** of the individual transfer functions:



- Of obvious interest to us, is the ***feedback form***:



$$Y(s) = E(s)G(s)$$

$$Y(s) = [R(s) - X(s)]G(s)$$

$$Y(s) = [R(s) - Y(s)H(s)]G(s)$$

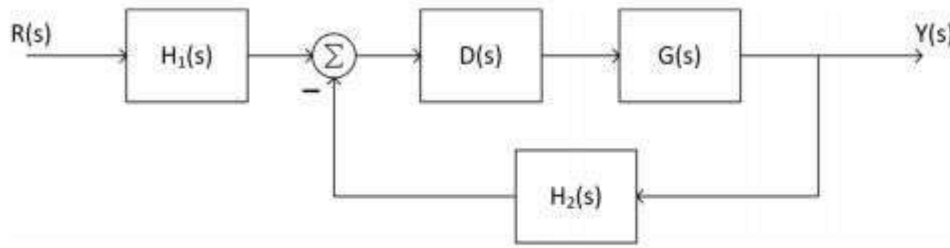
$$Y(s)[1 + G(s)H(s)] = R(s)G(s)$$

$$Y(s) = R(s) \cdot \frac{G(s)}{1 + G(s)H(s)}$$

- The ***closed-loop transfer function***, $T(s)$, is

$$T(s) = \frac{Y(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)}$$

- Calculate the closed-loop transfer function

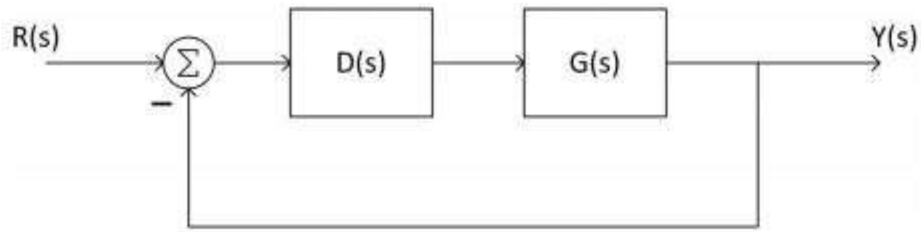


- $D(s)$ and $G(s)$ are in cascade
- $H_1(s)$ is in cascade with the feedback system consisting of $D(s)$, $G(s)$, and $H_2(s)$

$$T(s) = H_1(s) \cdot \frac{D(s)G(s)}{1 + D(s)G(s)H_2(s)}$$

$$T(s) = \frac{H_1(s)D(s)G(s)}{1 + D(s)G(s)H_2(s)}$$

- We're often interested in ***unity-feedback systems***

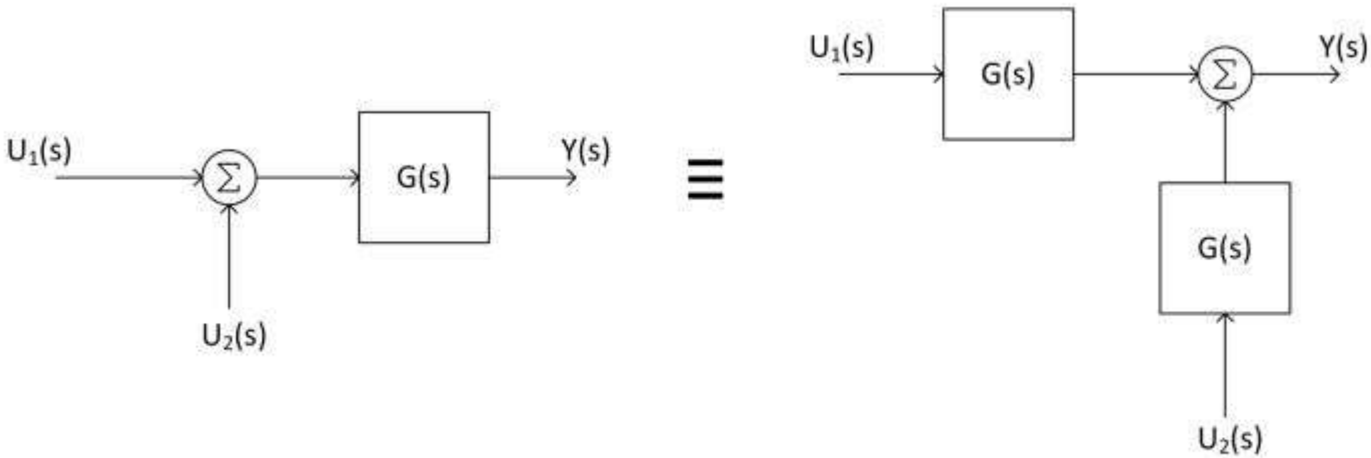


- Feedback path gain is unity
  - Can always reconfigure a system to unity-feedback form
- Closed-loop transfer function is:

$$T(s) = \frac{D(s)G(s)}{1 + D(s)G(s)}$$

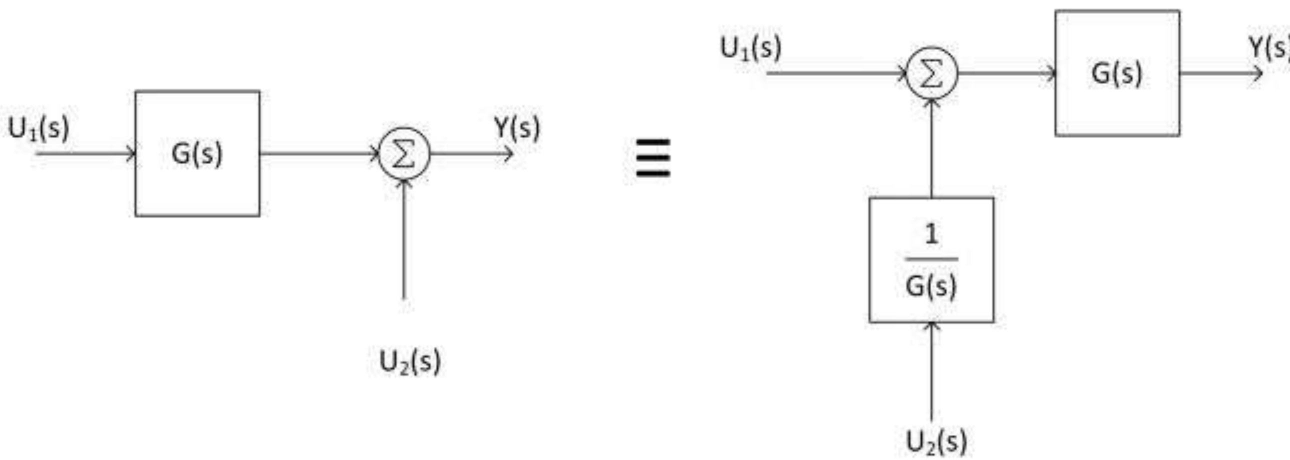
- Often want to simplify block diagrams into simpler, recognizable forms
  - To determine the equivalent transfer function
- Simplify to instances of the three standard forms, then simplify those forms
- ***Move blocks around relative to summing junctions and pickoff points*** – simplify to a standard form
  - Move blocks forward/backward past summing junctions
  - Move blocks forward/backward past pickoff points

- The following two block diagrams are equivalent:



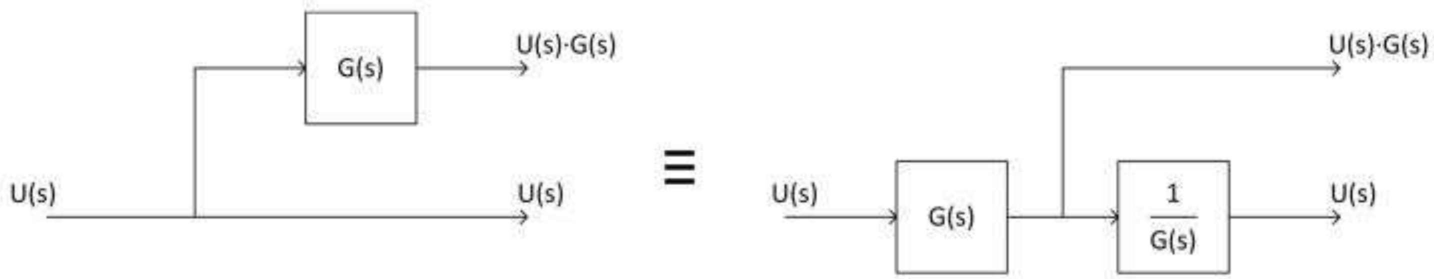
$$Y(s) = [U_1(s) + U_2(s)]G(s) = U_1(s)G(s) + U_2(s)G(s)$$

- The following two block diagrams are equivalent:

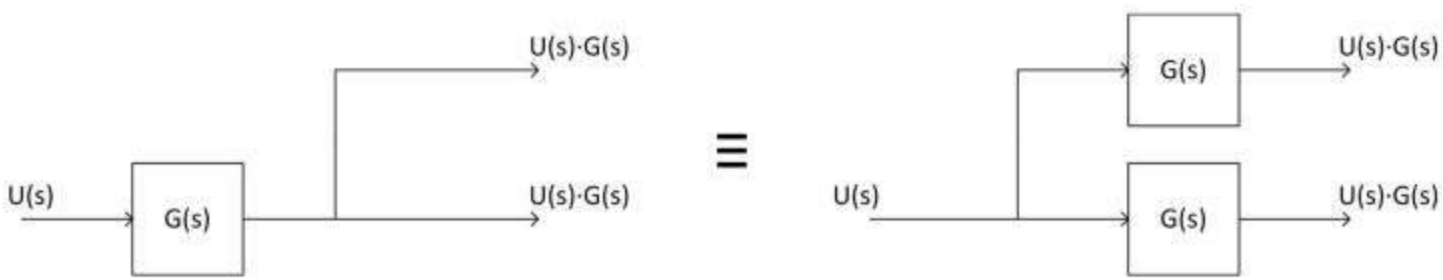


$$Y(s) = U_1(s)G(s) + U_2(s) = \left[U_1(s) + U_2(s)\frac{1}{G(s)}\right]G(s)$$

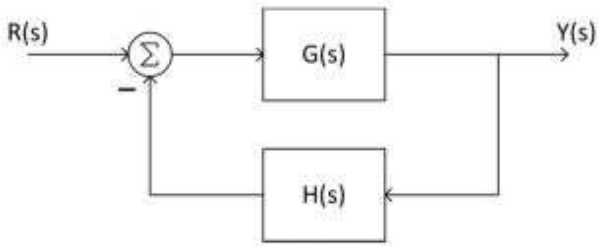
- We can move blocks backward past pickoff points:



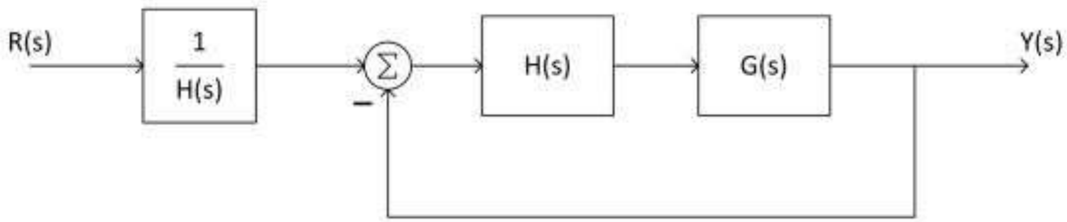
- And, we can move them forward past pickoff points:



- Rearrange the following into a unity-feedback system



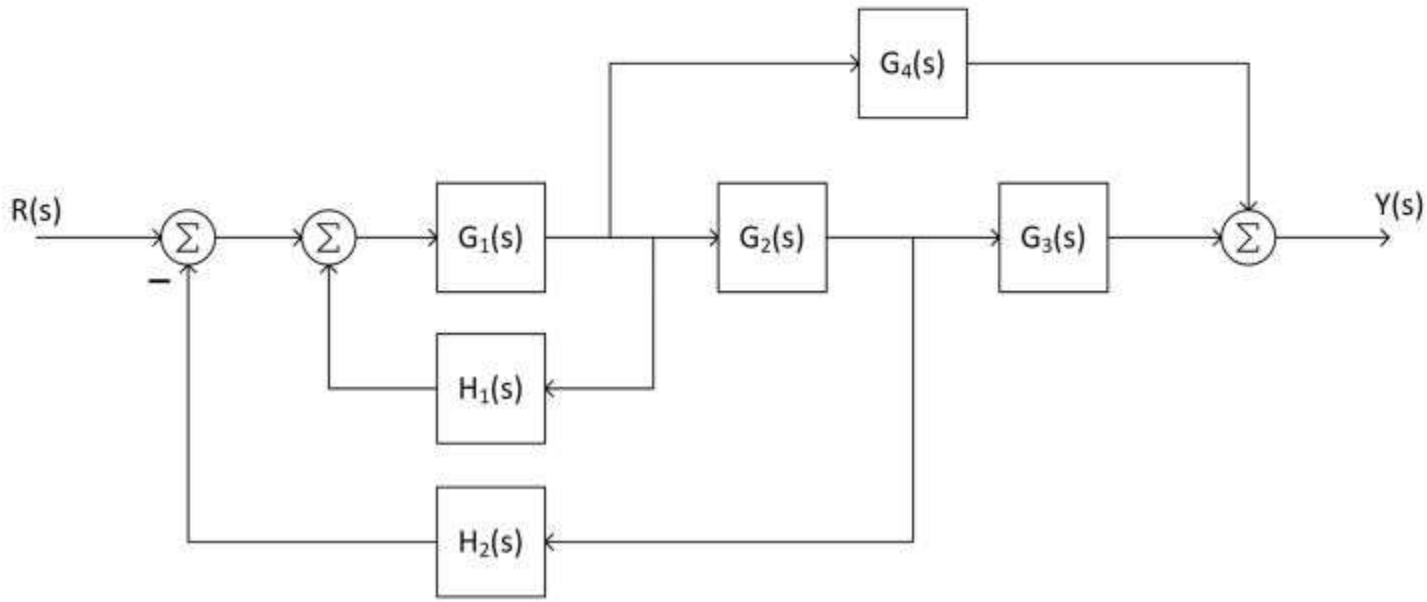
- Move the feedback block, $H(s)$, forward, past the summing junction
- Add an inverse block on $R(s)$ to compensate for the move



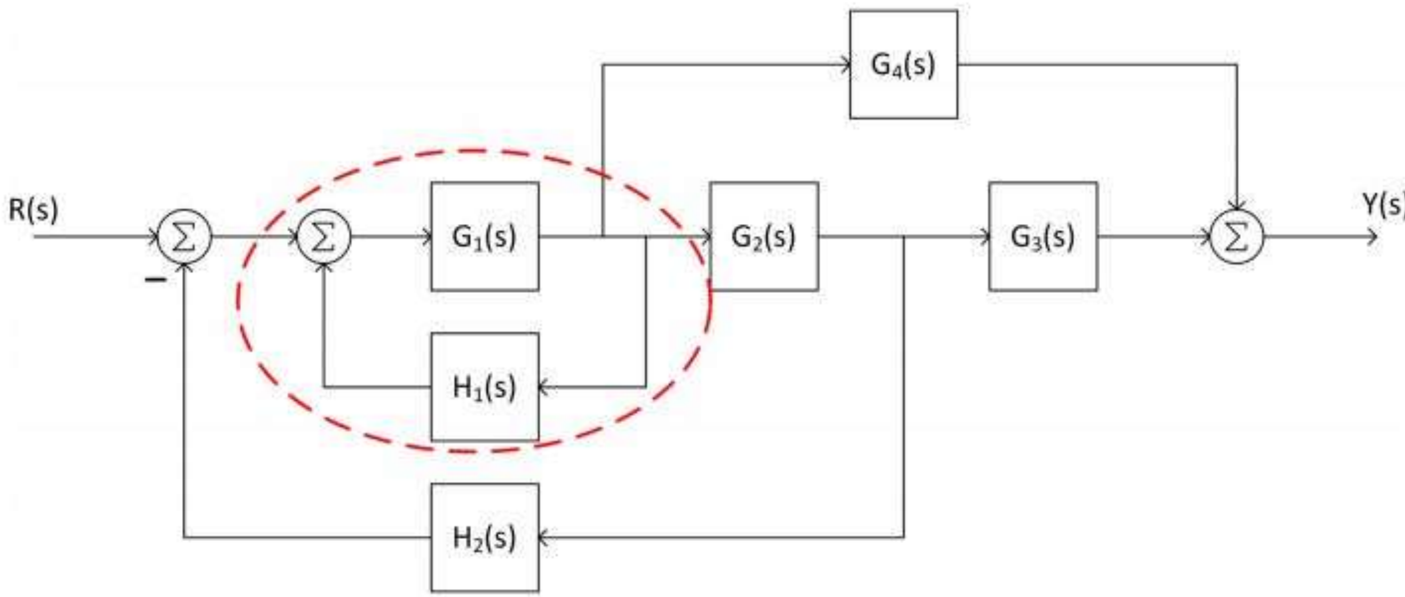
- Closed-loop transfer function:

$$T(s) = \frac{\frac{1}{H(s)} H(s)G(s)}{1 + G(s)H(s)} = \frac{G(s)}{1 + G(s)H(s)}$$

- Find the closed-loop transfer function of the following system through block-diagram simplification

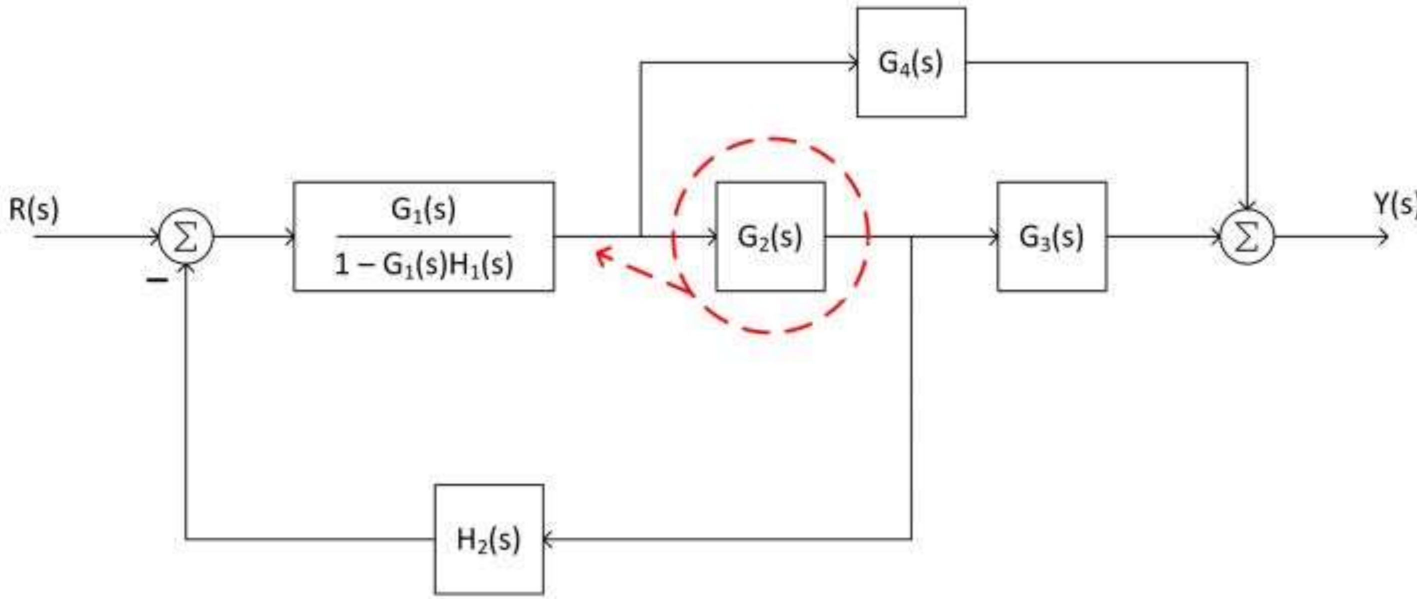


- $G_1(s)$ and $H_1(s)$ are in feedback form



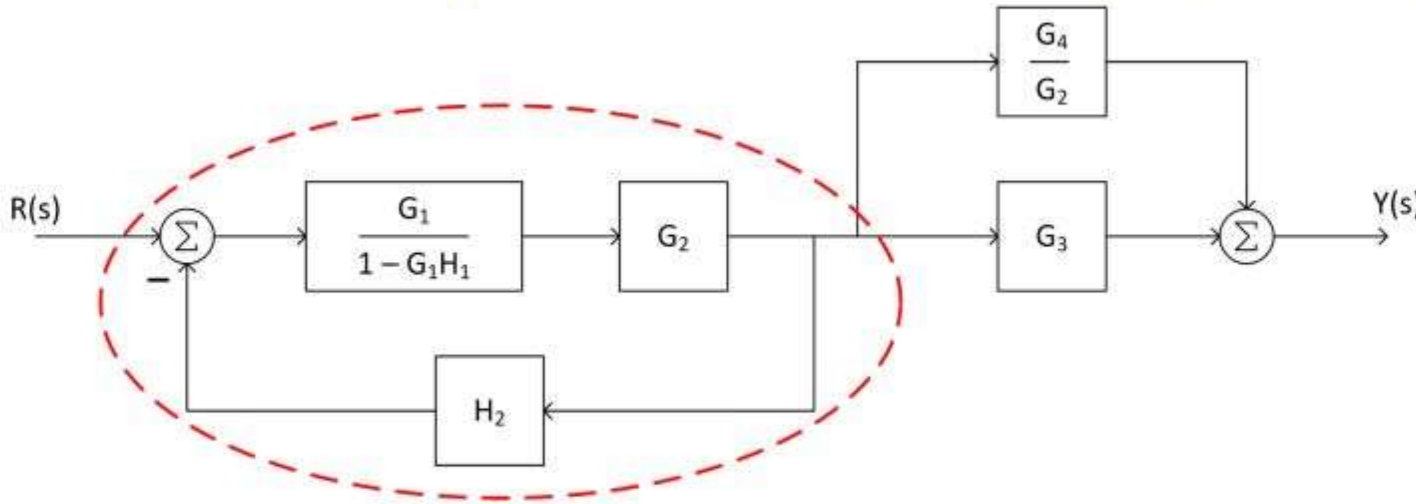
$$G_{eq}(s) = \frac{G_1(s)}{1 - G_1(s)H_1(s)}$$

- Move $G_2(s)$ backward past the pickoff point



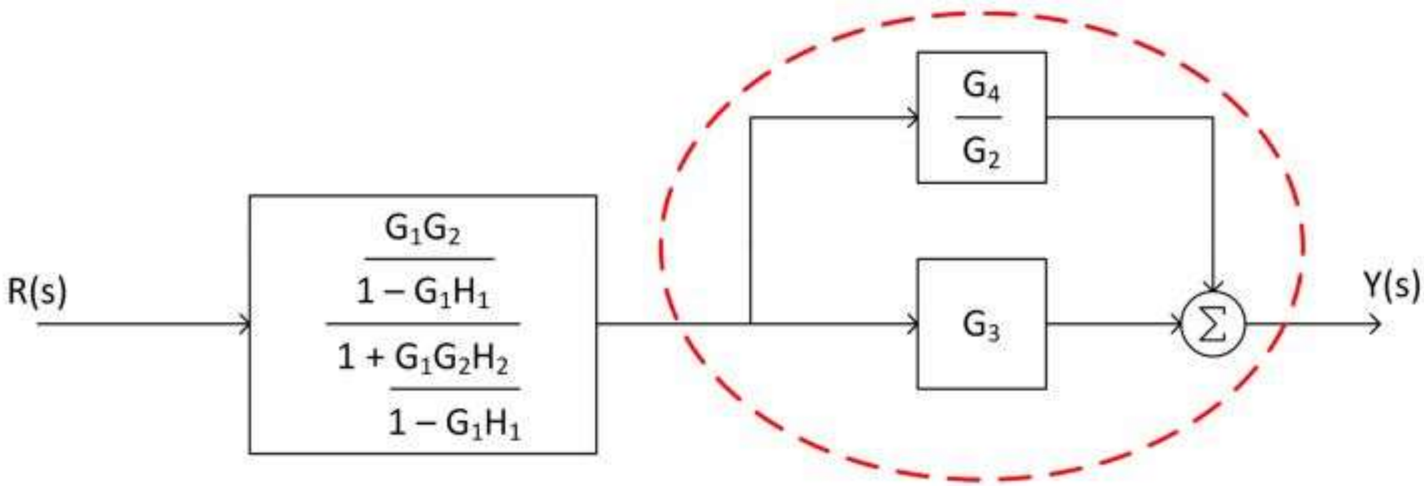
- Block from previous step, $G_2(s)$, and $H_2(s)$ become a feedback system that can be simplified

- Simplify the feedback subsystem
- Note that we've dropped the function of s notation, (s), for clarity



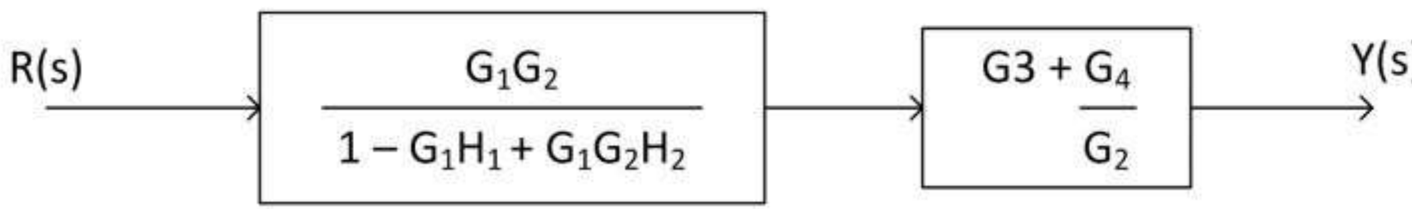
$$G_{eq}(s) = \frac{\dfrac{G_1 G_2}{1 - G_1 H_1}}{1 + \dfrac{G_1 G_2 H_2}{1 - G_1 H_1}} = \frac{G_1 G_2}{1 - G_1 H_1 + G_1 G_2 H_2}$$

- Simplify the two parallel subsystems

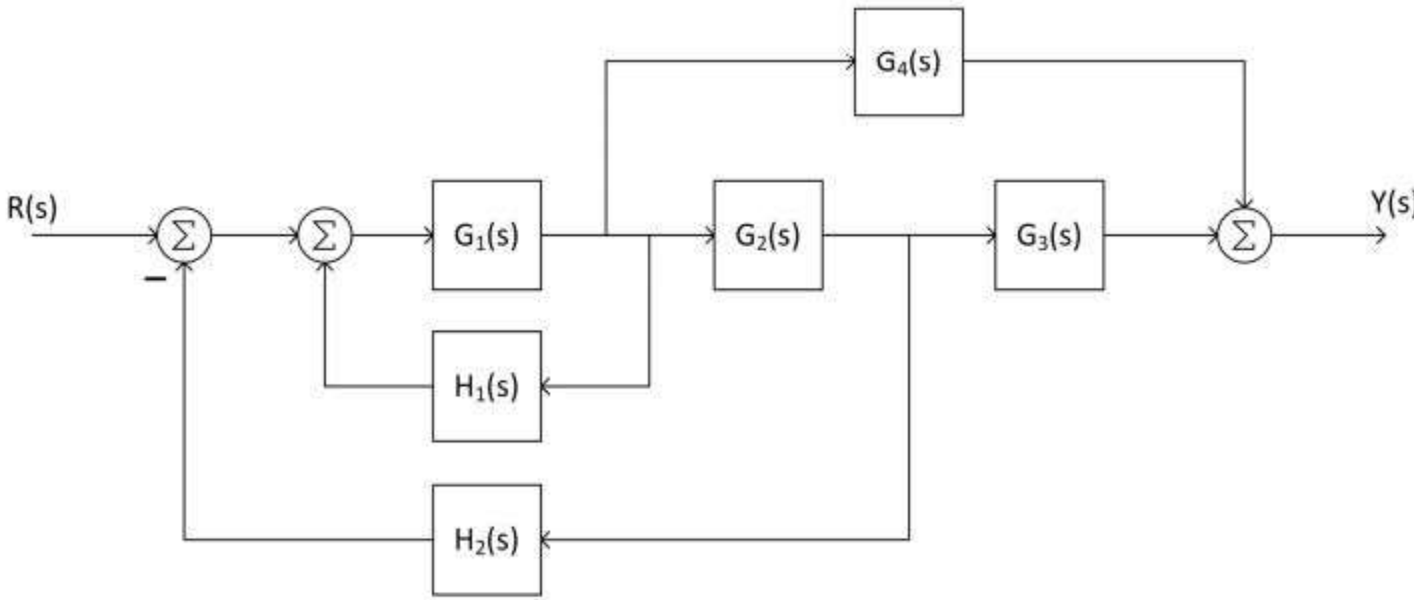


$$G_{eq}(s) = G_3 + \frac{G_4}{G_2}$$

- Now left with two cascaded subsystems
  - Transfer functions multiply



$$G_{eq}(s) = \frac{G_1G_2G_3 + G_1G_4}{1 - G_1H_1 + G_1G_2H_2}$$

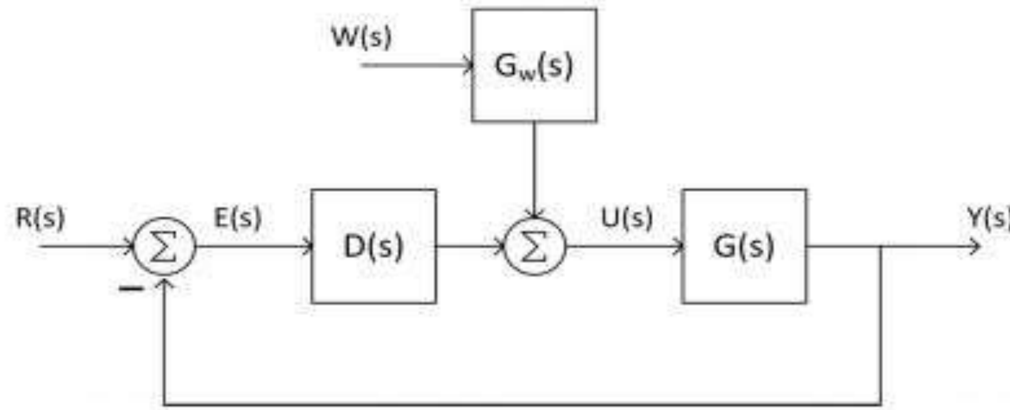


- The equivalent, close-loop transfer function is

$$T(s) = \frac{G_1G_2G_3 + G_1G_4}{1 - G_1H_1 + G_1G_2H_2}$$

Multiple-Input Systems

- Systems often have more than one input
  - E.g., reference, $R(s)$, and disturbance, $W(s)$



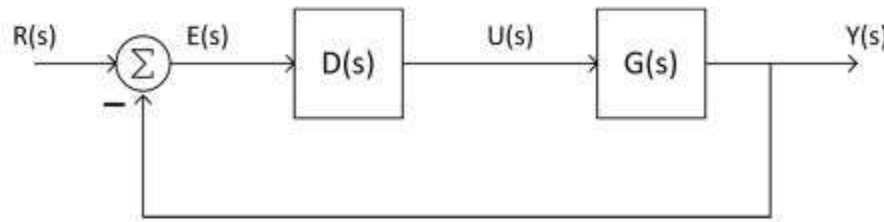
- Two transfer functions:
  - From reference to output

$$T(s) = Y(s)/R(s)$$

  - From disturbance to output

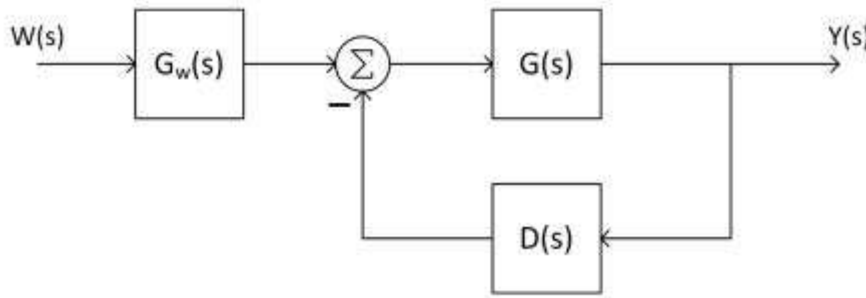
$$T_w(s) = Y(s)/W(s)$$

- Find transfer function from $R(s)$ to $Y(s)$
  - A linear system – superposition applies
  - Set $W(s) = 0$



$$T(s) = \frac{Y(s)}{R(s)} = \frac{D(s)G(s)}{1 + D(s)G(s)}$$

- Next, find transfer function from $W(s)$ to $Y(s)$
  - Set $R(s) = 0$
  - System now becomes:



$$T_w(s) = \frac{Y(s)}{W(s)} = \frac{G_w(s)G(s)}{1 + D(s)G(s)}$$