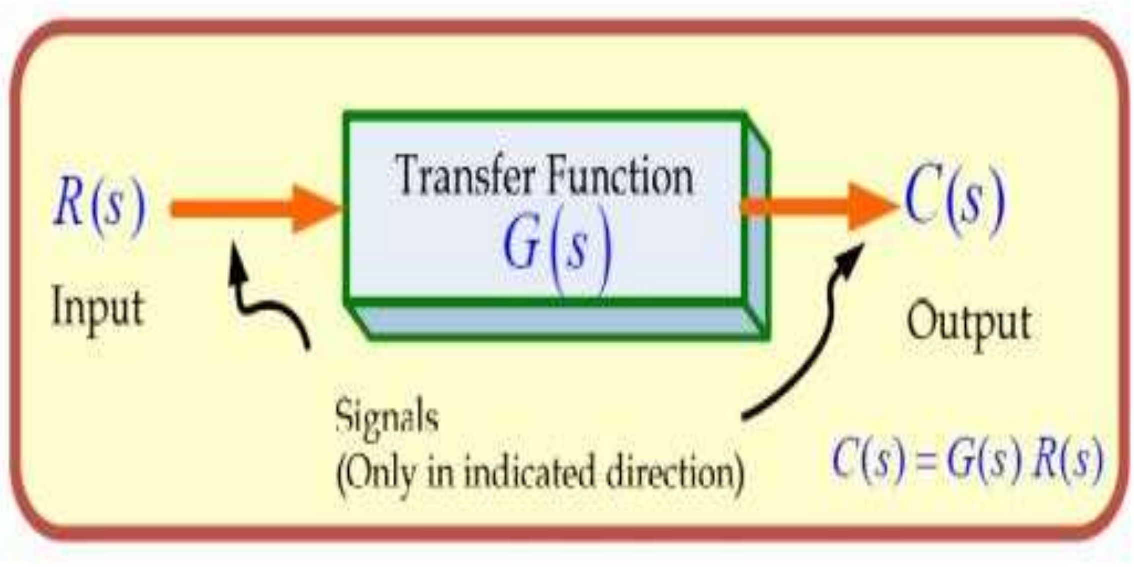
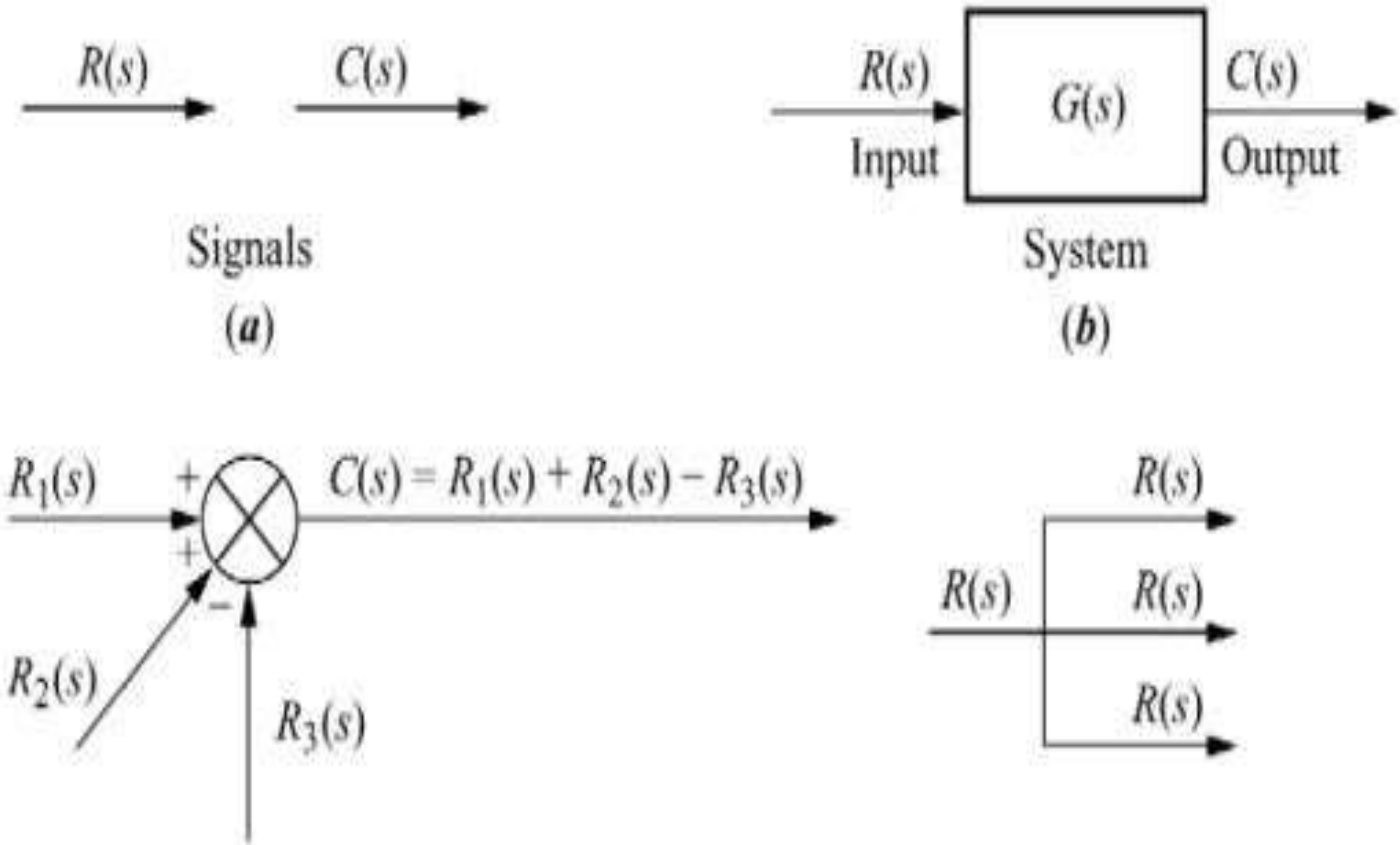
Derivation of transfer function using block diagrams reduction techniques

Transfer function of a system



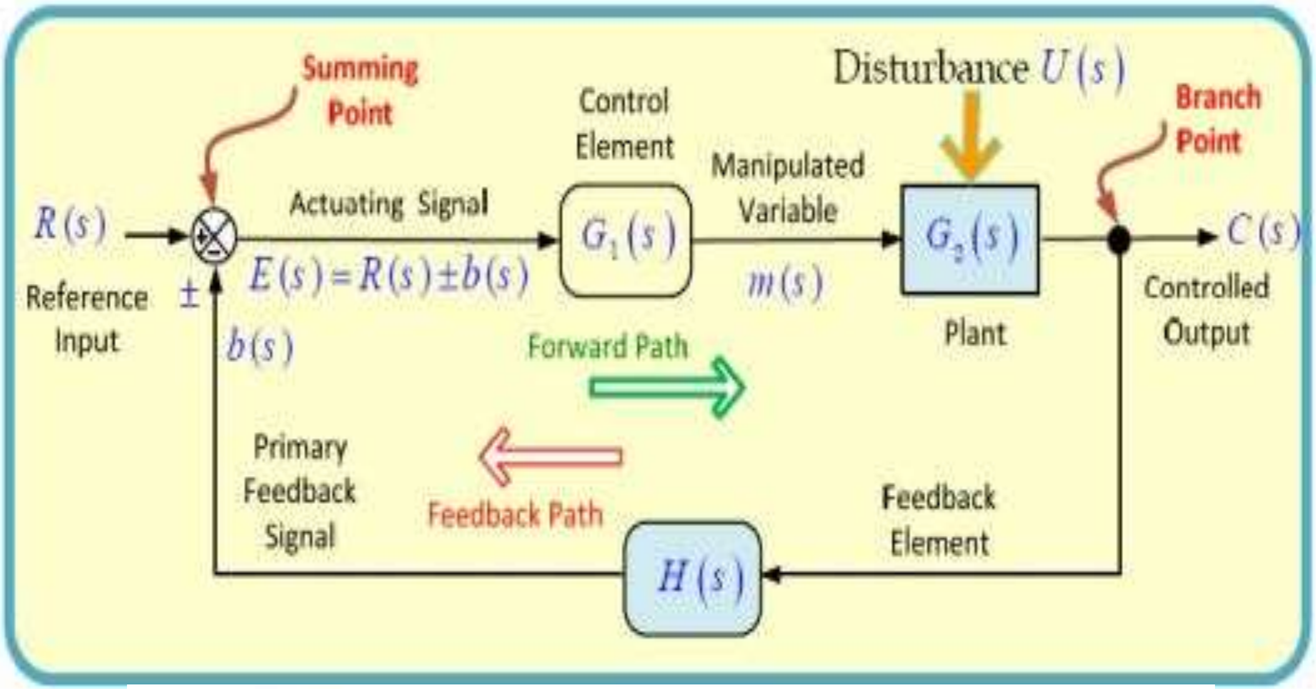


Signals
(*a*)

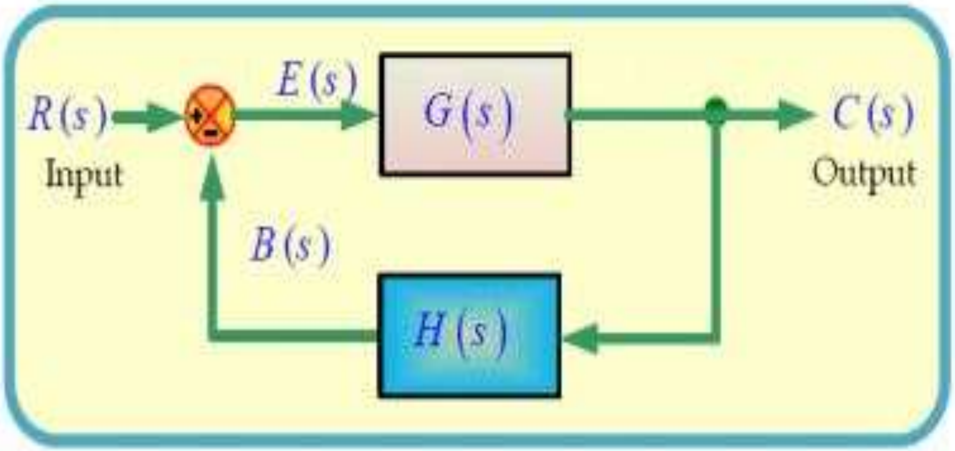
System
(*b*)

Summing junction
(*c*)

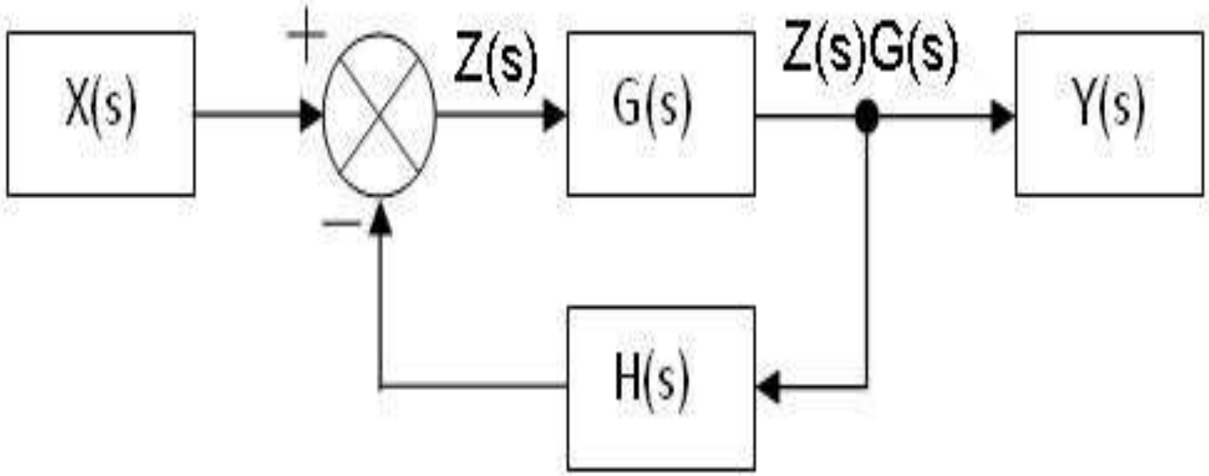
Pickoff point
(*d*)



- $G(s)$ ≡ Direct transfer function = Forward transfer function.
- $H(s)$ ≡ Feedback transfer function.
- $G(s)H(s)$ ≡ Open-loop transfer function.
- $C(s)/R(s)$ ≡ Closed-loop transfer function = Control ratio
- $C(s)/E(s)$ ≡ Feed-forward transfer function.



$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$$



$$\frac{Y(s)}{X(s)} = \frac{G(s)}{1 + G(s)H(s)}$$

Using this figure we write:

$$Y(s) = G(s)Z(s)$$

$$Z(s) = X(s) - H(s)Y(s)$$

Now, plug the second equation into the first to eliminate Z(s):

$$Y(s) = G(s)[X(s) - H(s)Y(s)]$$

Move all the terms with Y(s) to the left hand side, and keep the term with X(s) on the right hand side:

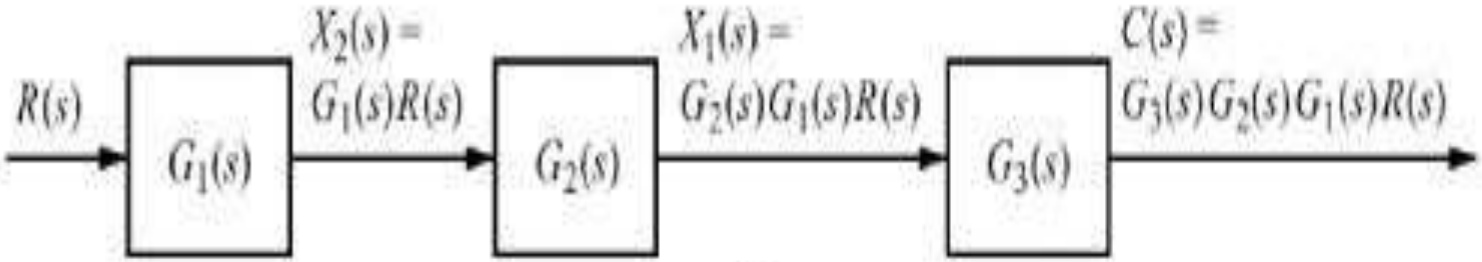
$$Y(s) + G(s)H(s)Y(s) = G(s)X(s)$$

Therefore,

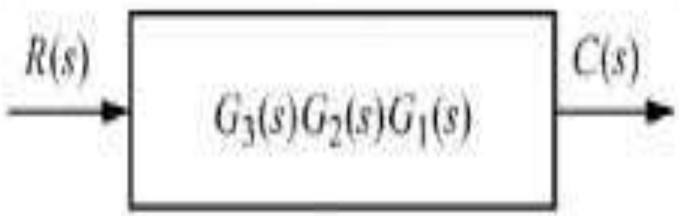
$$Y(s)(1 + G(s)H(s)) = G(s)X(s)$$

$$\Rightarrow \frac{Y(s)}{X(s)} = \frac{G(s)}{1 + G(s)H(s)}$$

Cascade (Series) Connections

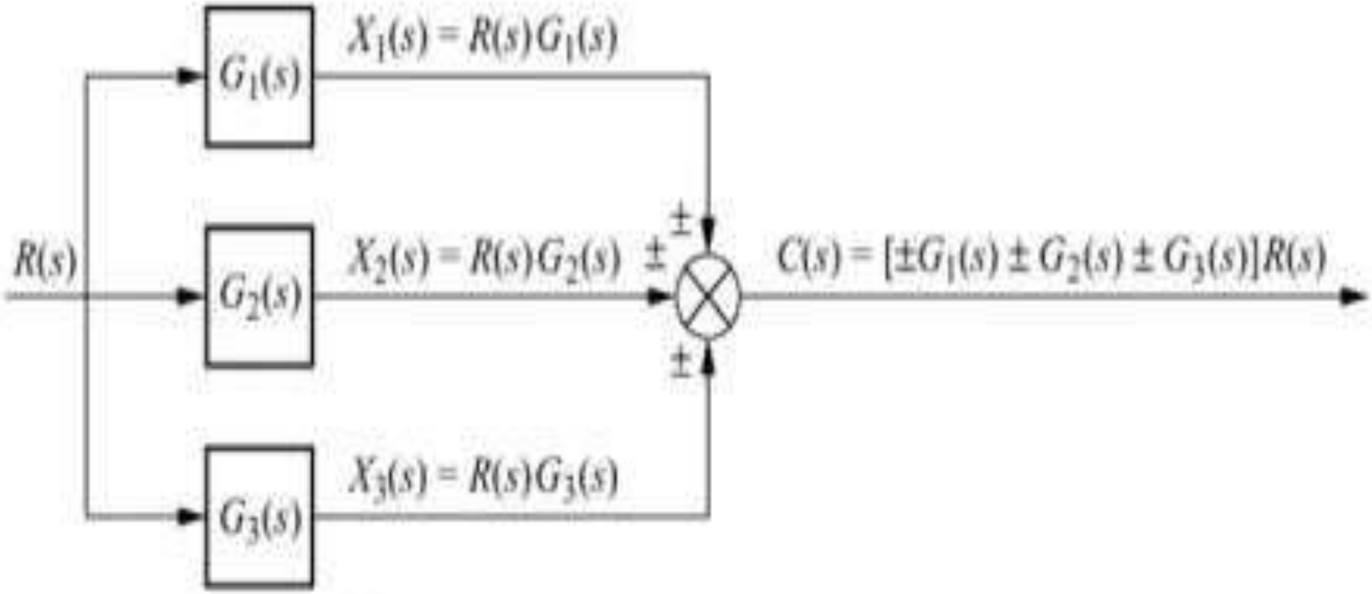


(a)

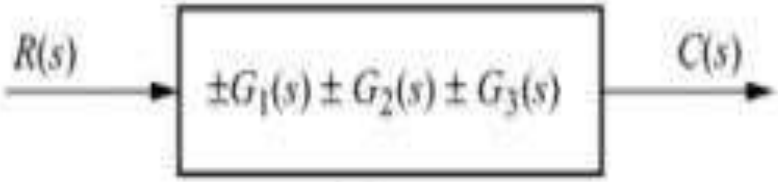


(b)

Parallel Connections

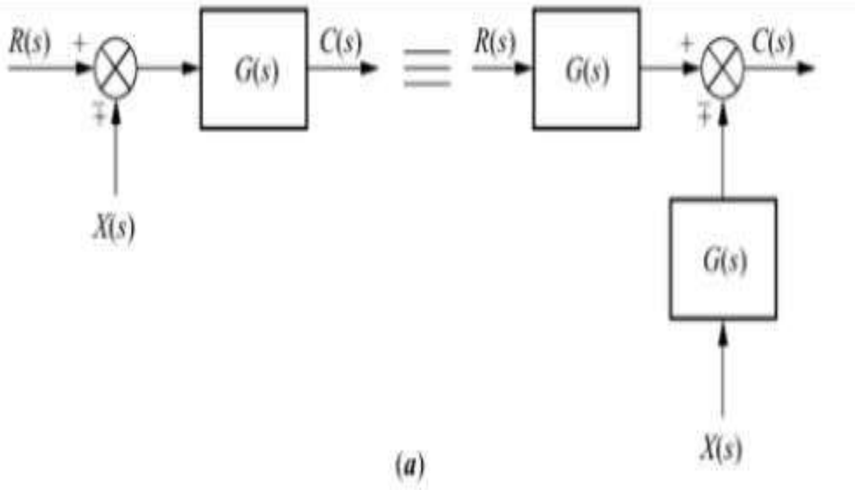


(a)

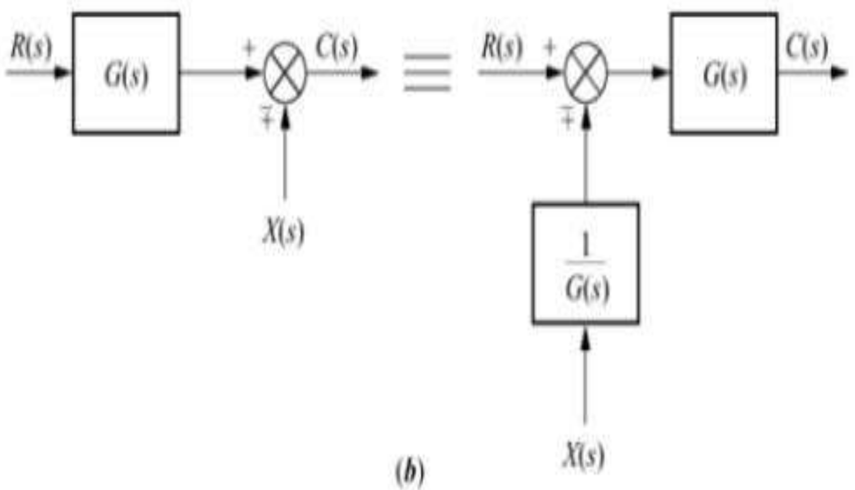


(b)

Block Diagram Algebra for Summing Junctions

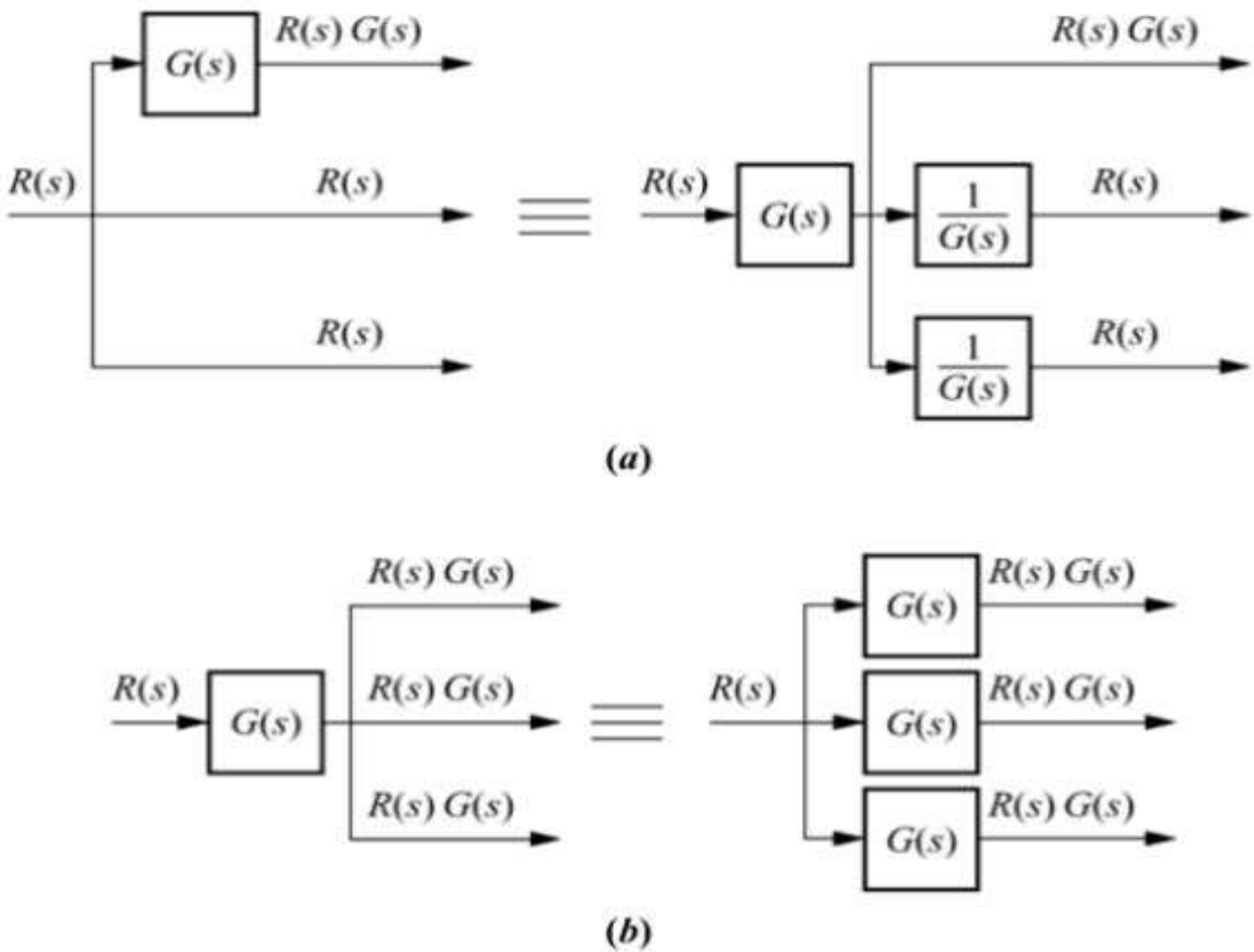


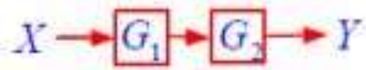
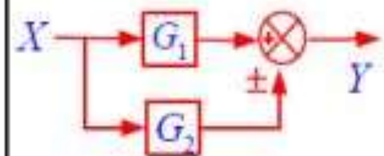
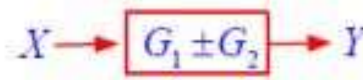
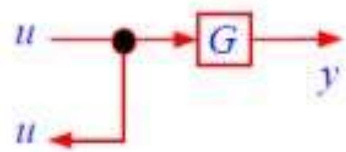
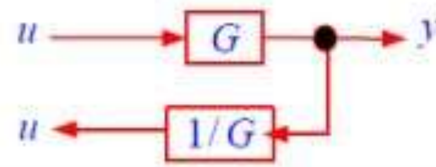
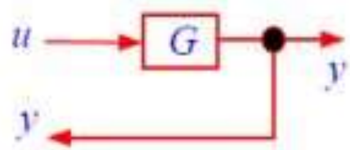
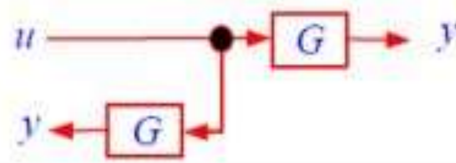
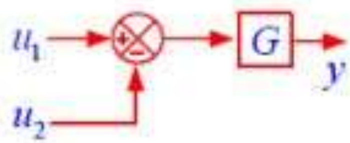
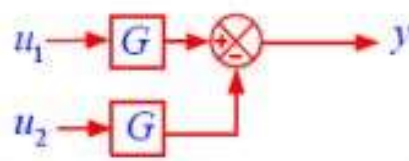
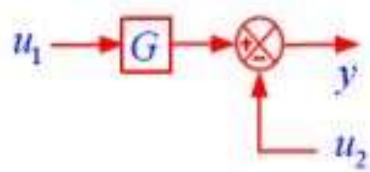
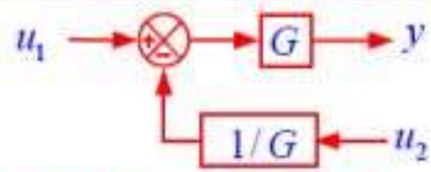
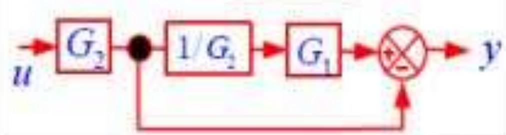
$$C = G(+R \pm X)$$
$$= +GR \pm GX$$

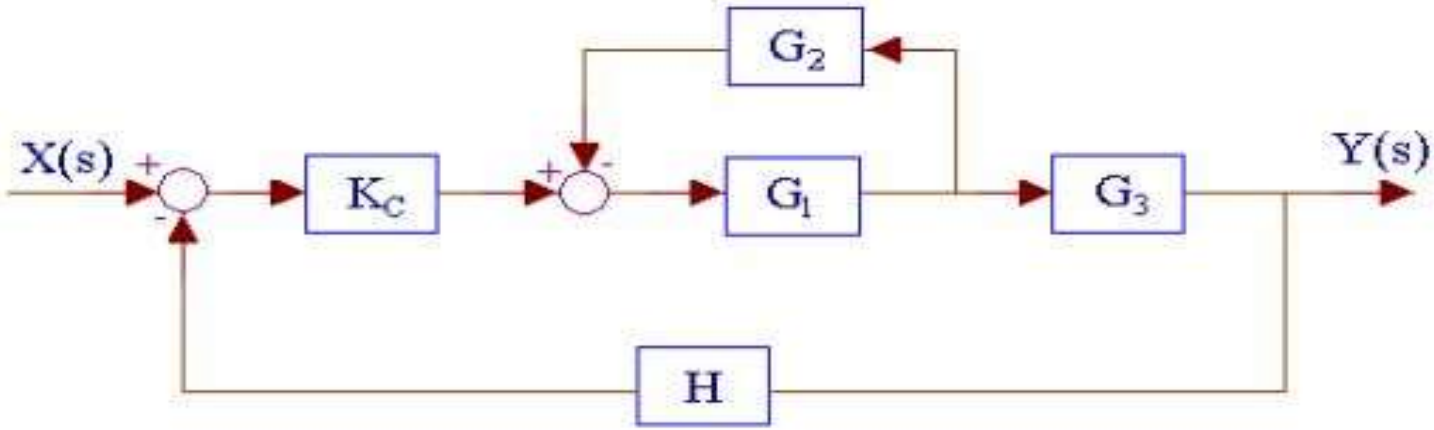


$$C = GR \pm X$$
$$= G(+R \pm X/G)$$

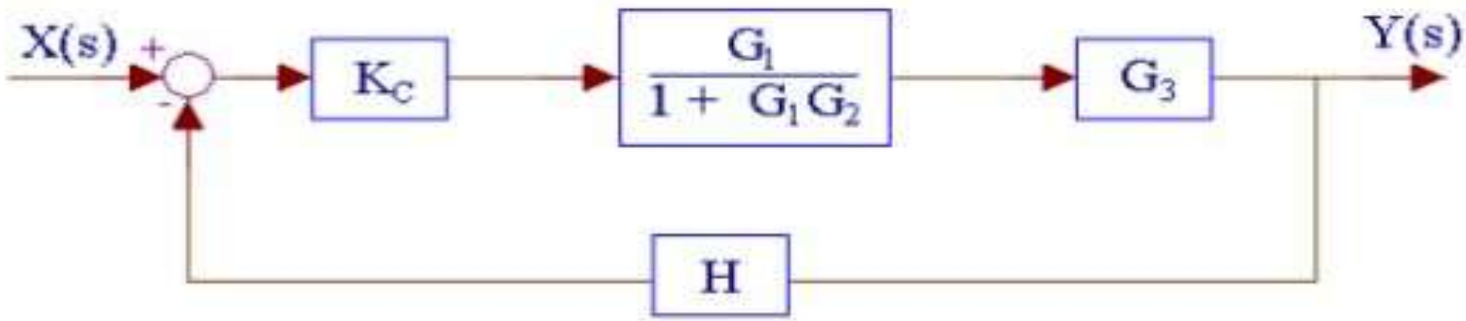
Block Diagram Algebra for Branch Point



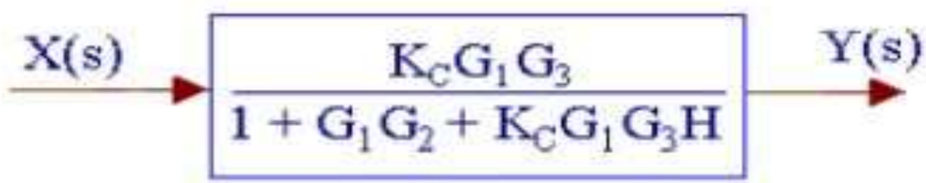
	Manipulation	Original Block Diagram	Equivalent Block Diagram	Equation
1	Combining Blocks in Cascade			$Y=(G_1G_2)X$
2	Combining Blocks in Parallel; or Eliminating a Forward Loop			$Y=(G_1 \pm G_2)X$
3	Moving a pickoff point behind a block			$y = G u$ $u = \frac{1}{G} y$
4	Moving a pickoff point ahead of a block			$y = Gu$
5	Moving a summing point behind a block			$e_2 = G(u_1 - u_2)$
6	Moving a summing point ahead of a block		 	$y = Gu_1 - u_2$ $y = (G_1 - G_2)u$



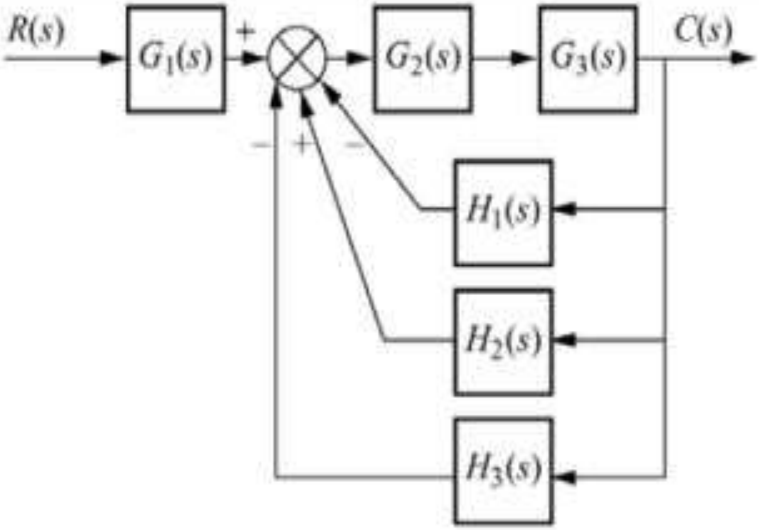
X(s)
+
-
K_C
+
-
G_2
G_1
G_3
Y(s)
H



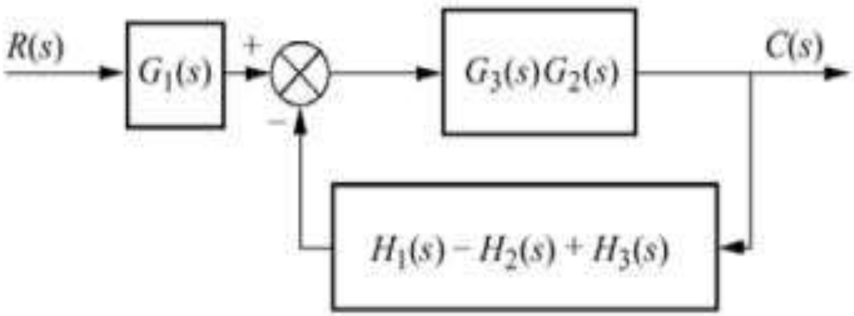
X(s)
+
-
K_C
$\frac{G_1}{1 + G_1 G_2}$
G_3
Y(s)
H



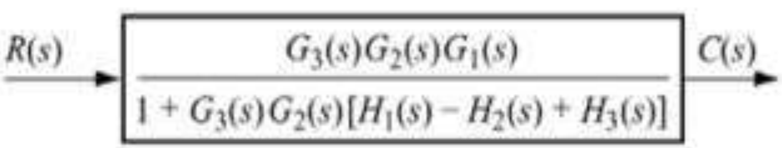
X(s)
$\frac{K_C G_1 G_3}{1 + G_1 G_2 + K_C G_1 G_3 H}$
Y(s)



(a)



(b)



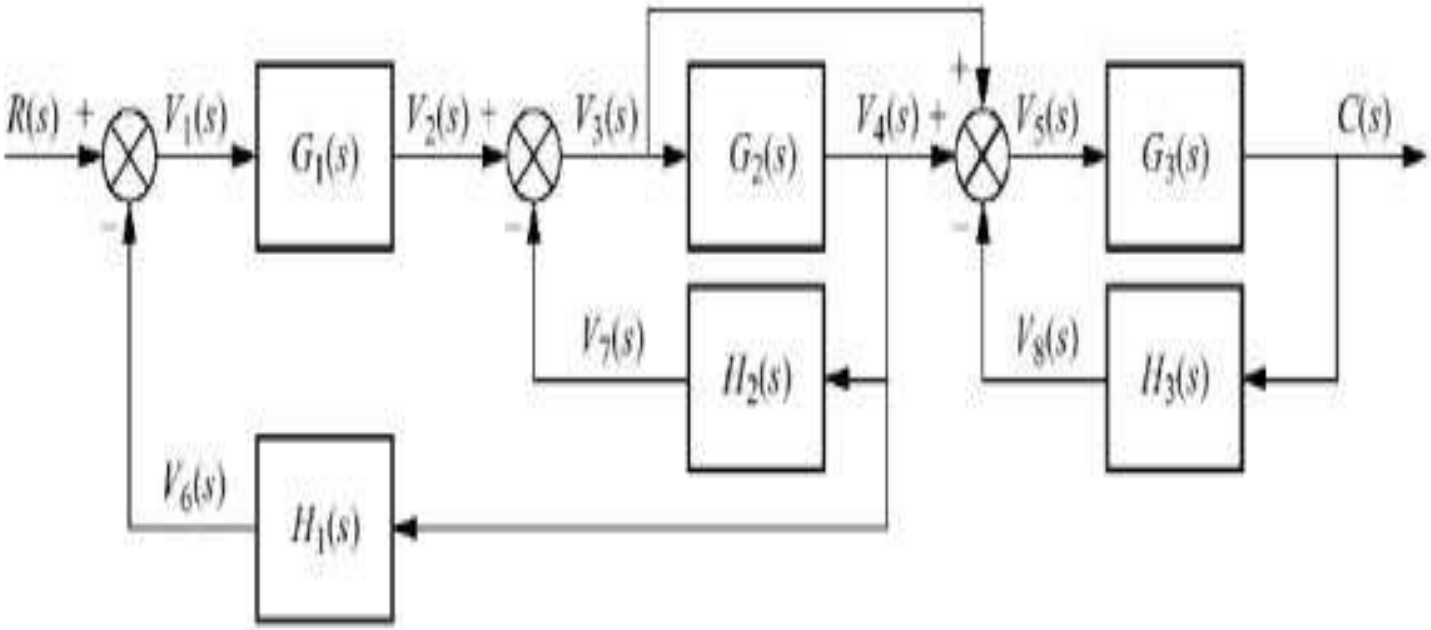
(c)

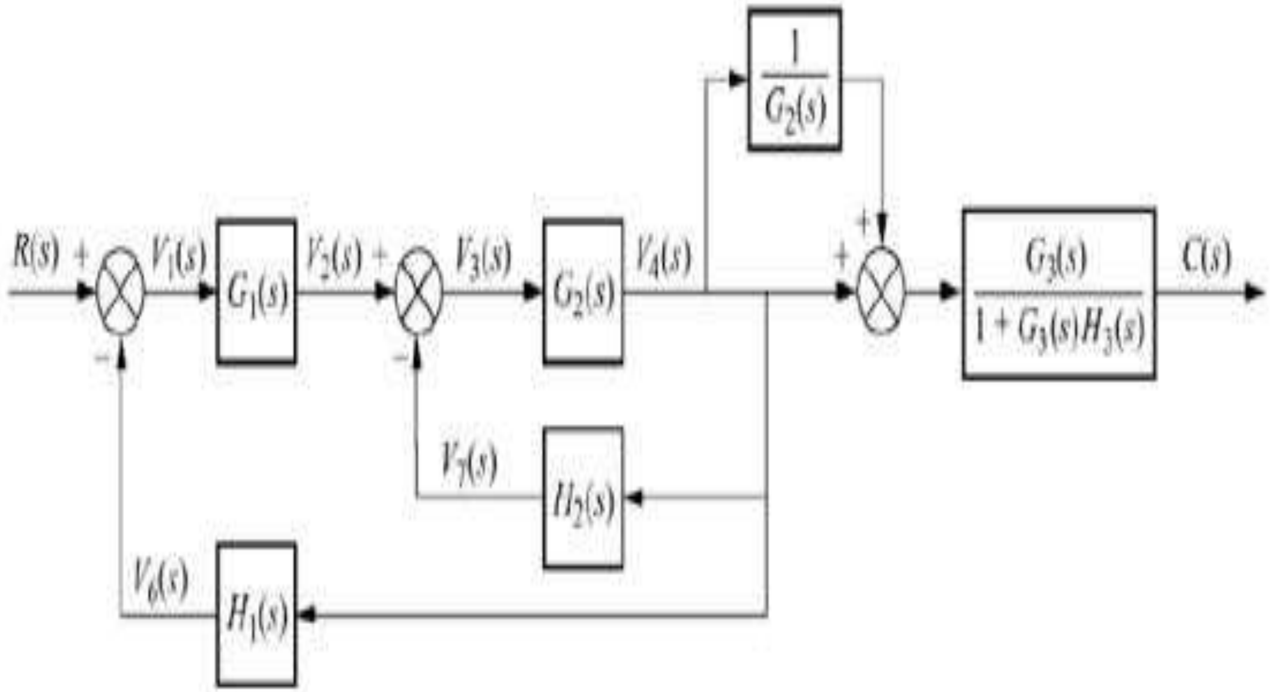
G_1 and G_2 are in series

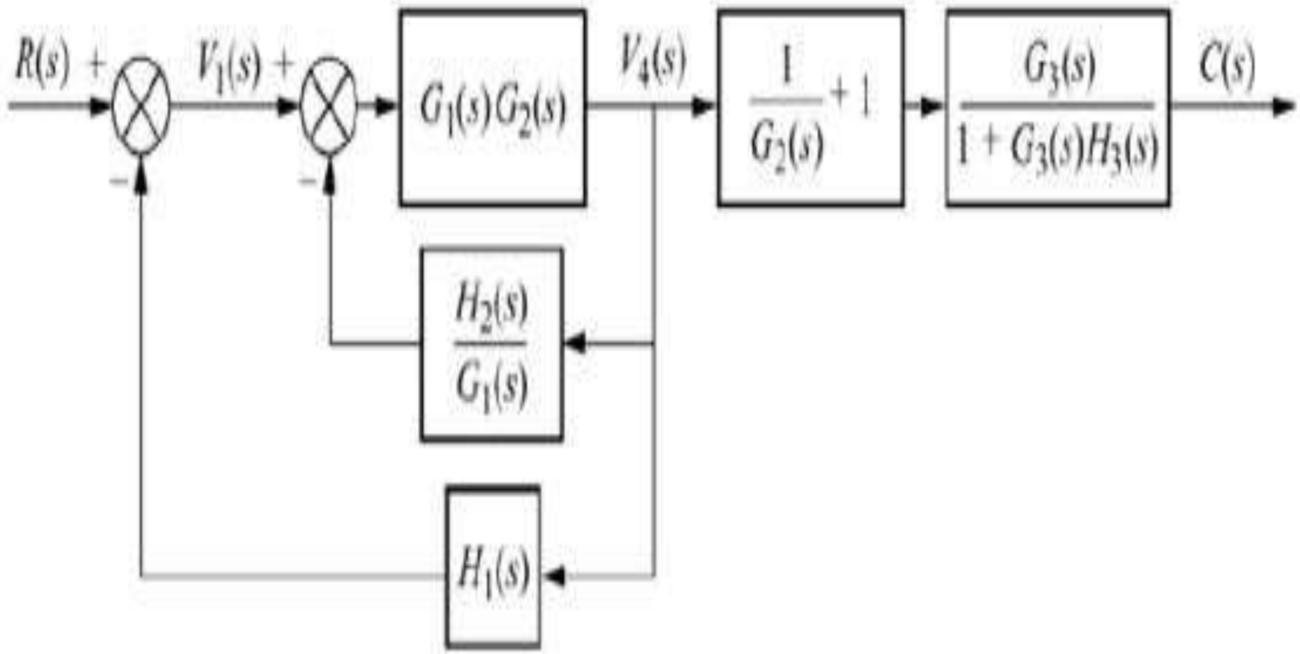
H_1 and H_2 and H_3 are in parallel

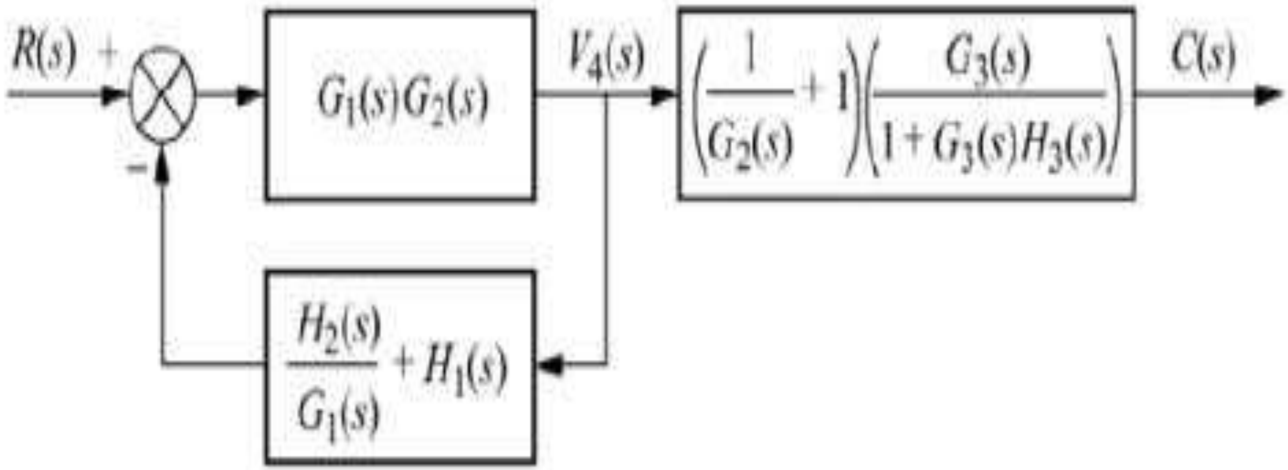
G_1 is in series with the feedback configuration.

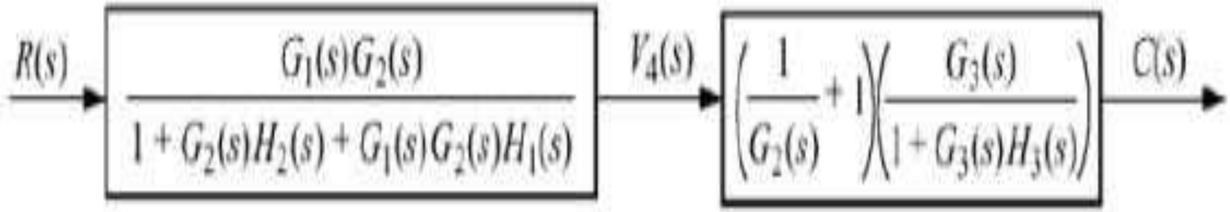
$$\frac{C(s)}{R(s)} = G_1\left[\frac{G_3G_2}{1+G_3G_2(H_1 - H_2 + H_3)}\right]$$

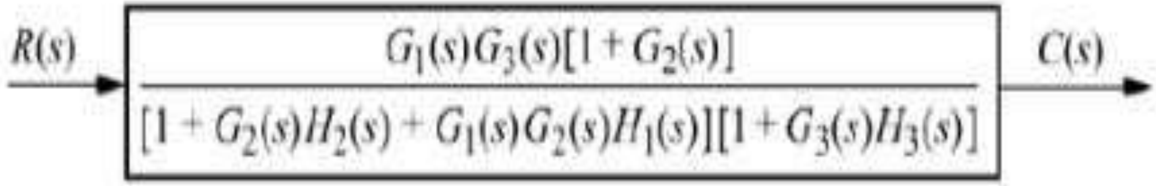

R(s)
+
V1(s)
G1(s)
V2(s)
+
V3(s)
G2(s)
V4(s)
+
+
V5(s)
G3(s)
C(s)
V7(s)
H2(s)
V8(s)
H3(s)
V6(s)
H1(s)


R(s)
+
V1(s)
G1(s)
V2(s)
+
V3(s)
G2(s)
V4(s)
1
G2(s)
+
+
G3(s)
1 + G3(s)H3(s)
C(s)
−
−
V7(s)
H2(s)
V6(s)
H1(s)


R(s)
+
−
V1(s)
+
−
G1(s)G2(s)
V4(s)
1/G2(s) + 1
G3(s) / (1 + G3(s)H3(s))
C(s)
H2(s)/G1(s)
H1(s)


R(s)
+
−
G1(s)G2(s)
V4(s)
(1/G2(s) + 1)(G3(s)/(1 + G3(s)H3(s)))
C(s)
H2(s)/G1(s) + H1(s)


R(s)
G1(s)G2(s) / (1 + G2(s)H2(s) + G1(s)G2(s)H1(s))
V4(s)
(1/G2(s) + 1)(G3(s)/(1 + G3(s)H3(s)))
C(s)


R(s)
G1(s)G3(s)[1 + G2(s)] / ([1 + G2(s)H2(s) + G1(s)G2(s)H1(s)][1 + G3(s)H3(s)])
C(s)