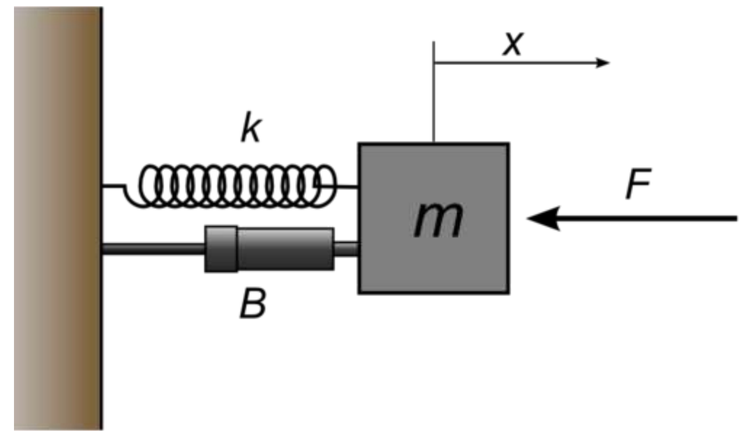
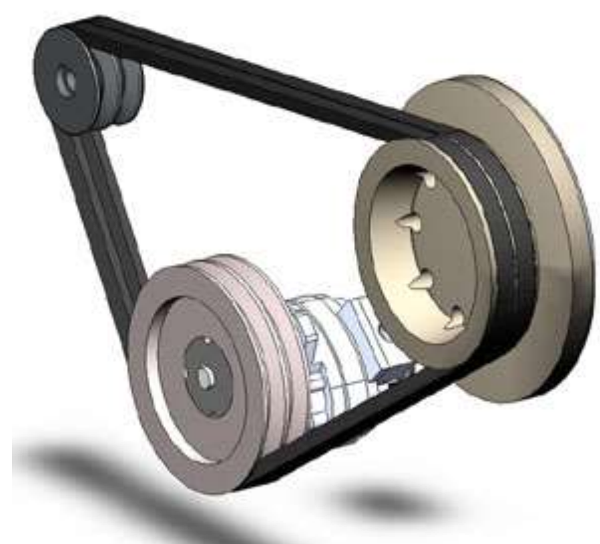
TRANSLATIONAL MECHANICAL SYSTEMS

Basic Types of Mechanical Systems

- Translational
  - Linear Motion



- Rotational
  - Rotational Motion



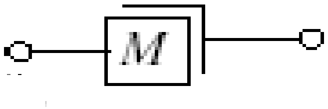
Basic Elements of Translational Mechanical Systems

Translational Spring

i)

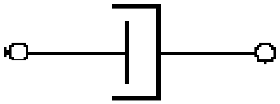
Translational Mass

ii)



Translational Damper

iii)

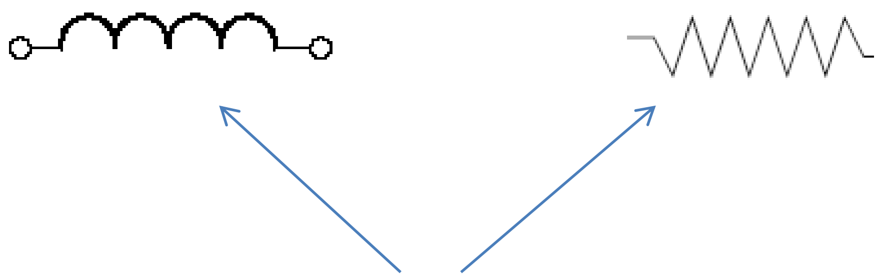


Translational Spring

- A translational spring is a mechanical element that can be deformed by an external force such that the deformation is directly proportional to the force applied to it.

Translational Spring

i)

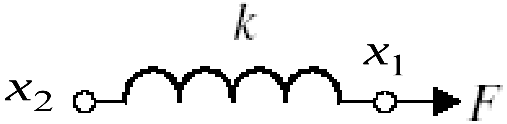


Circuit Symbols

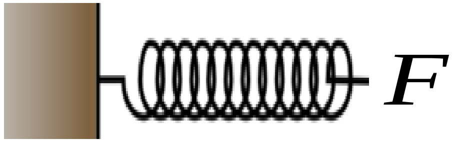
Translational Spring

Translational Spring

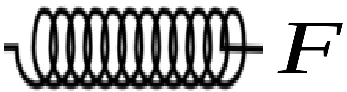
- If *F* is the applied force



- Then x_1 is the deformation if $x_2 = 0$



- Or $(x_1 - x_2)$ is the deformation.



- The equation of motion is given as

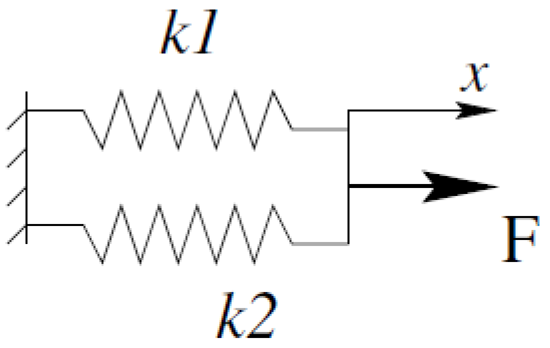
$$F = k(x_1 - x_2)$$

- Where k is stiffness of spring expressed in N/m

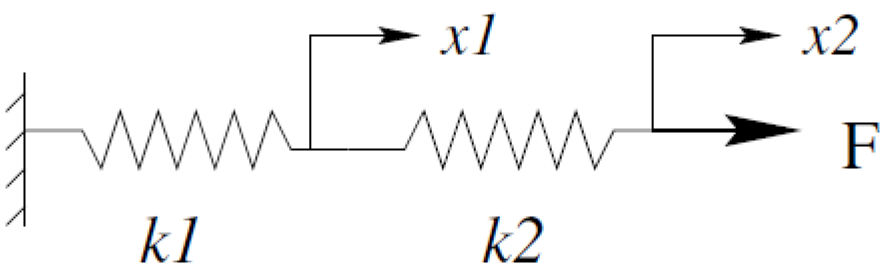
Translational Spring

- Given two springs with spring constant k_1 and k_2, obtain the equivalent spring constant k_{eq} for the two springs connected in:

(1) Parallel



(2) Series



Translational Spring

- The two springs have same displacement therefore:

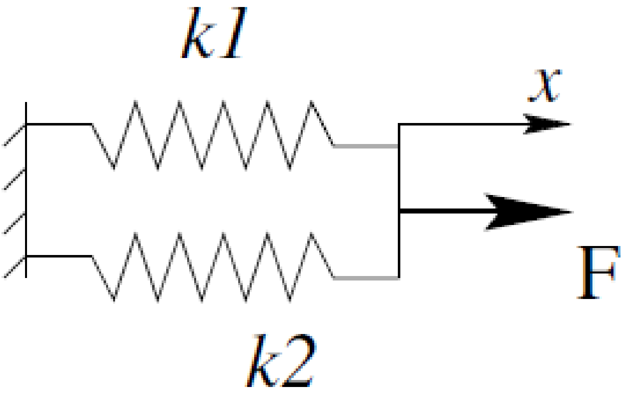
$$k_1 x + k_2 x = F$$

$$(k_1 + k_2)x = F$$

$$k_{eq} x = F$$

$$k_{eq} = k_1 + k_2$$

(1) Parallel



- If *n* springs are connected in parallel then:

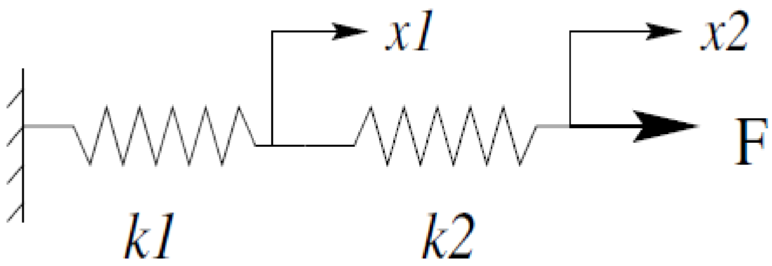
$$k_{eq} = k_1 + k_2 + \cdots + k_n$$

Translational Spring

- The forces on two springs are same, *F*, however displacements are different therefore:

(2) Series

$$k_1 x_1 = k_2 x_2 = F$$



$$x_1 = \frac{F}{k_1} \qquad x_2 = \frac{F}{k_2}$$

- Since the total displacement is $x = x_1 + x_2$, and we have $F = k_{eq} x$

$$x = x_1 + x_2 \Rightarrow \frac{F}{k_{eq}} = \frac{F}{k_1} + \frac{F}{k_2}$$

Translational Spring

$$\frac{F}{k_{eq}} = \frac{F}{k_1} + \frac{F}{k_2}$$

- Then we can obtain

$$k_{eq} = \frac{1}{\dfrac{1}{k_1} + \dfrac{1}{k_2}} = \frac{k_1 k_2}{k_1 + k_2}$$

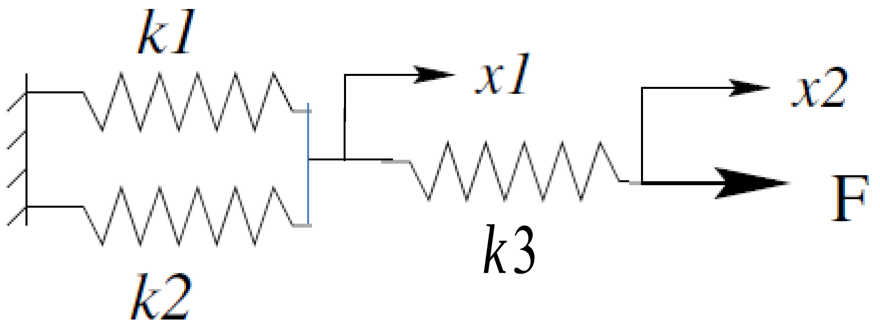
- If *n* springs are connected in series then:

$$k_{eq} = \frac{k_1 k_2 \cdots k_n}{k_1 + k_2 + \cdots + k_n}$$

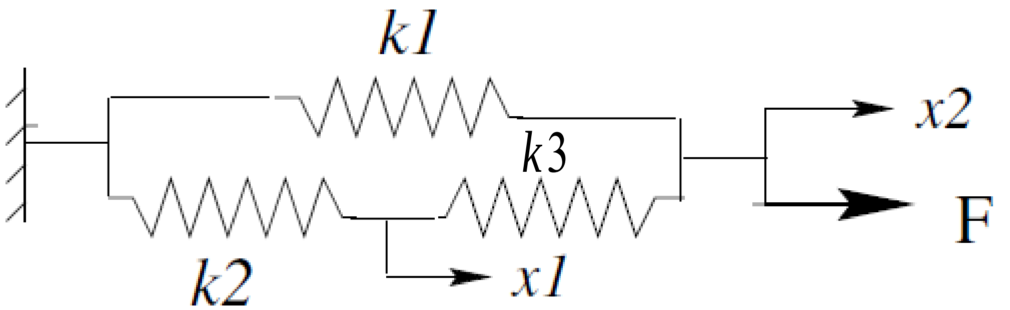
Translational Spring

- **Exercise:** Obtain the equivalent stiffness for the following spring networks.

i)



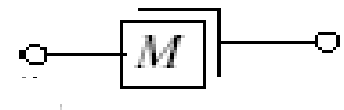
ii)

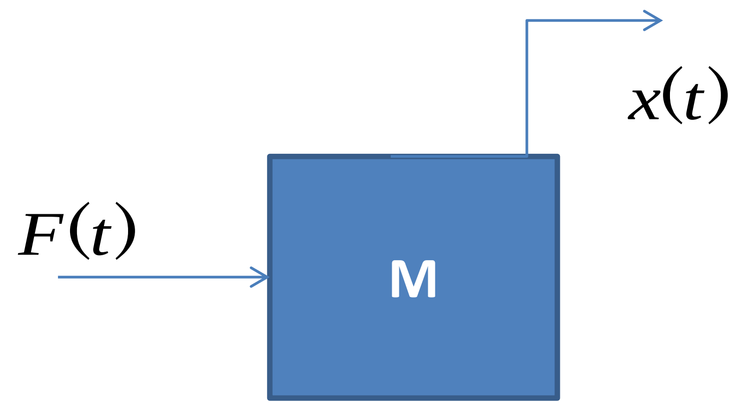


Translational Mass

- Translational Mass is an inertia element.
- A mechanical system without mass does not exist.
- If a force *F* is applied to a mass and it is displaced to *x* meters then the relation b/w force and displacements is given by Newton's law.

ii) **Translational Mass**





$$F = M\ddot{x}$$

Translational Damper

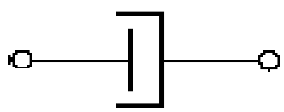
- When the viscosity or drag is not negligible in a system, we often model them with the damping force.

- All the materials exhibit the property of damping to some extent.

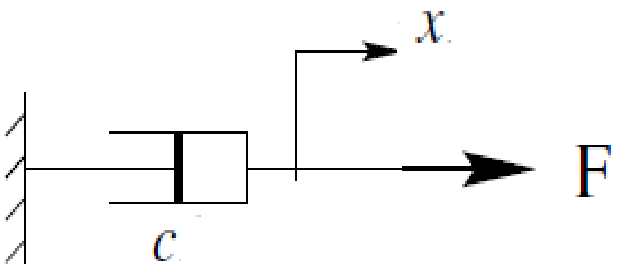
- If damping in the system is not enough then extra elements (e.g. Dashpot) are added to increase damping.

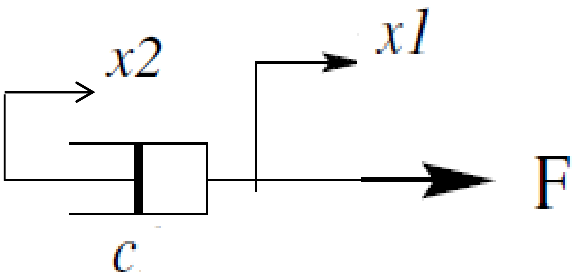
Translational Damper

iii)



Translational Damper





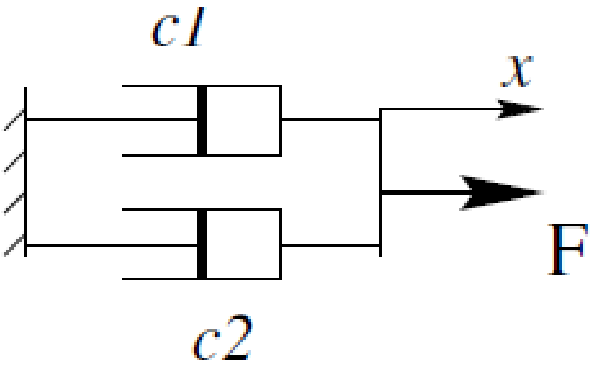
$$F = C\dot{x}$$

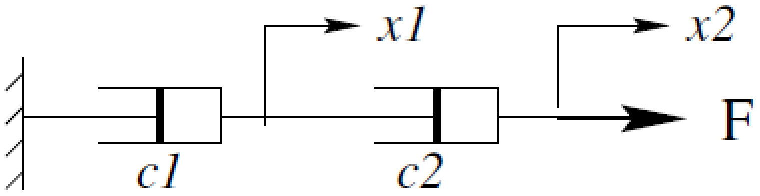
$$F = C(\dot{x}_1 - \dot{x}_2)$$

- Where *C* is damping coefficient (*N/ms*$^{-1}$).

Translational Damper

- Translational Dampers in series and parallel.



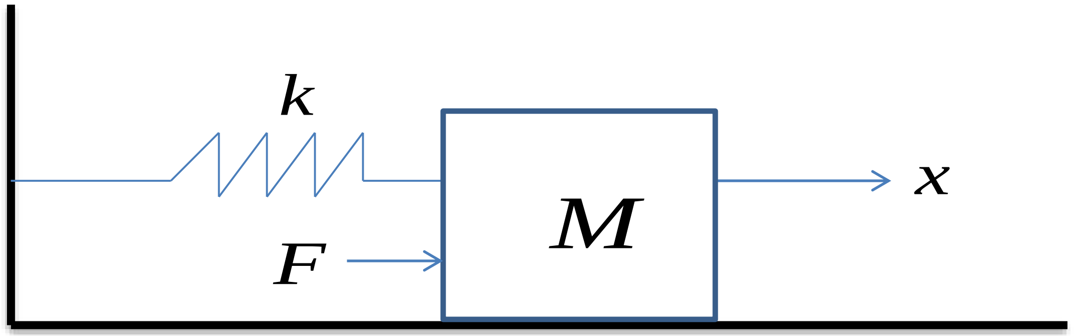


$$C_{eq} = C_1 + C_2$$

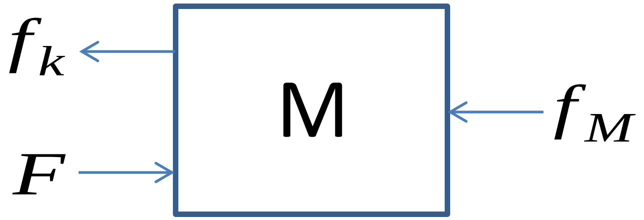
$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$$

Example-1

- Consider the following system (friction is negligible)

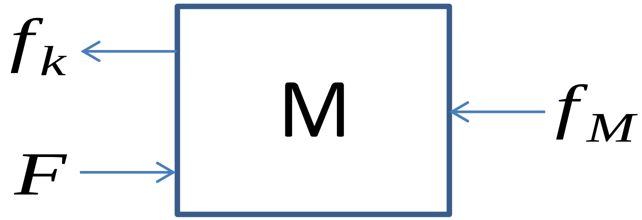


- Free Body Diagram



- Where f_k and f_M are force applied by the spring and inertial force respectively.

Example-1



$$F = f_k + f_M$$

- Then the differential equation of the system is:

$$F = M\ddot{x} + kx$$

- Taking the Laplace Transform of both sides and ignoring initial conditions we get

$$F(s) = Ms^2 X(s) + kX(s)$$

Example-1

$$F(s) = Ms^2 X(s) + kX(s)$$

- The transfer function of the system is

$$\frac{X(s)}{F(s)} = \frac{1}{Ms^2 + k}$$

- if

$$M = 1000\,kg$$

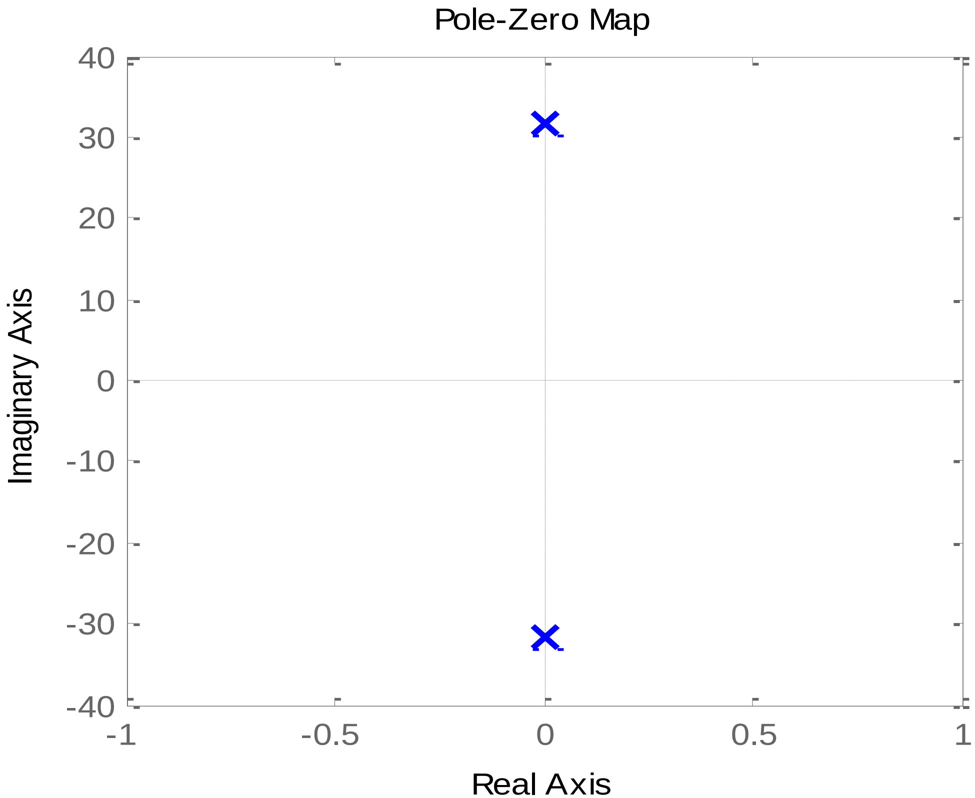
$$k = 2000\,Nm^{-1}$$

$$\frac{X(s)}{F(s)} = \frac{0.001}{s^2 + 2}$$

Example-1

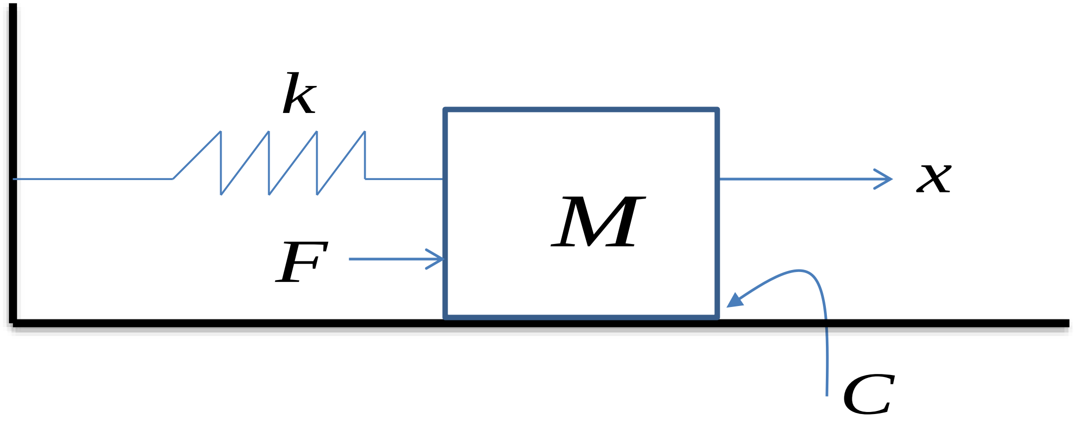
$$\frac{X(s)}{F(s)} = \frac{0.001}{s^2 + 2}$$

- The pole-zero map of the system is

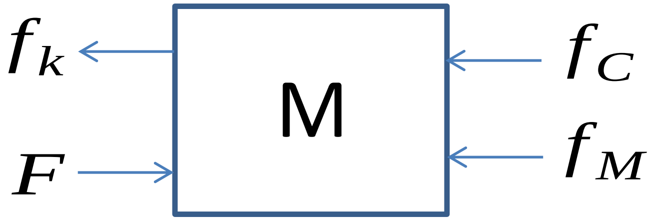


Example-2

- Consider the following system



- Free Body Diagram



$$F = f_k + f_M + f_C$$

Example-2

Differential equation of the system is:

$$F = M\ddot{x} + C\dot{x} + kx$$

Taking the Laplace Transform of both sides and ignoring Initial conditions we get

$$F(s) = Ms^2 X(s) + CsX(s) + kX(s)$$

$$\frac{X(s)}{F(s)} = \frac{1}{Ms^2 + Cs + k}$$

Example-2

$$\frac{X(s)}{F(s)} = \frac{1}{Ms^2 + Cs + k}$$

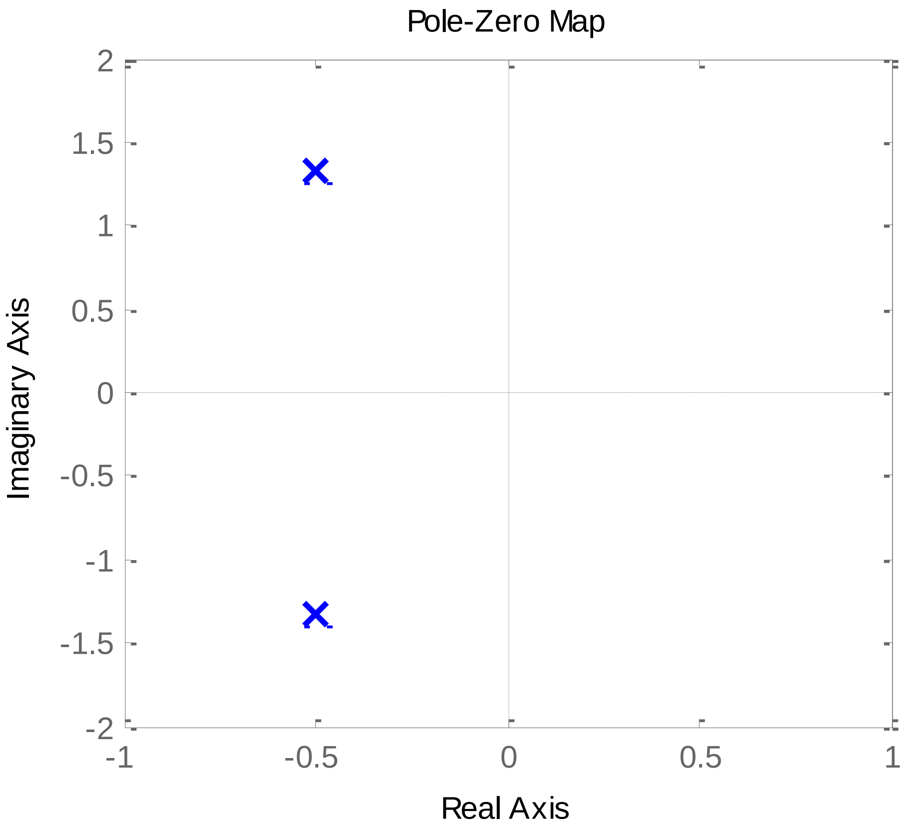
- if

$$M = 1000\,kg$$

$$k = 2000\,Nm^{-1}$$

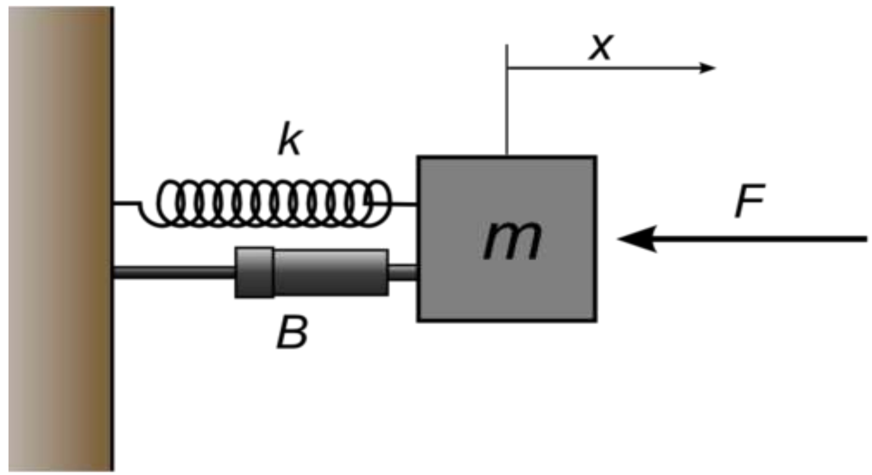
$$C = 1000\,N/ms^{-1}$$

$$\frac{X(s)}{F(s)} = \frac{0.001}{s^2 + s + 1000}$$

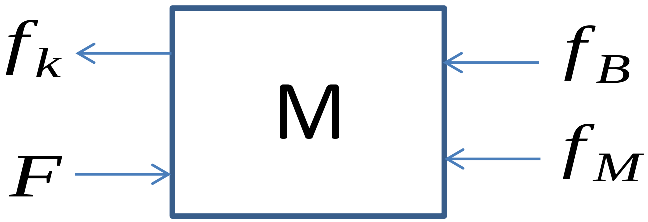


Example-3

- Consider the following system



- Free Body Diagram (same as example-2)

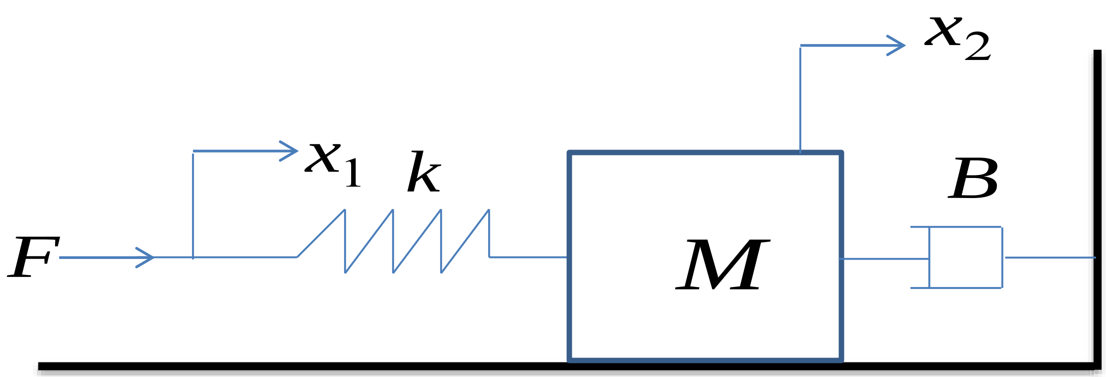


$$\frac{X(s)}{F(s)} = \frac{1}{Ms^2 + Bs + k}$$

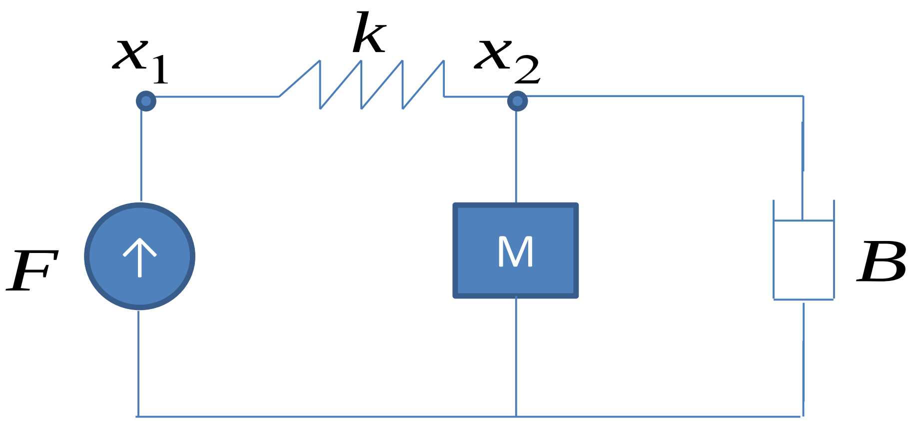
$$F = f_k + f_M + f_B$$

Example-4

- Consider the following system

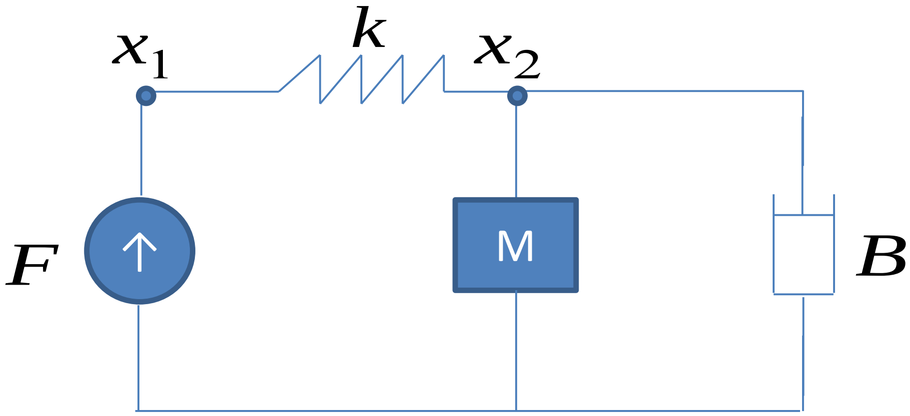


- Mechanical Network



Example-4

- Mechanical Network



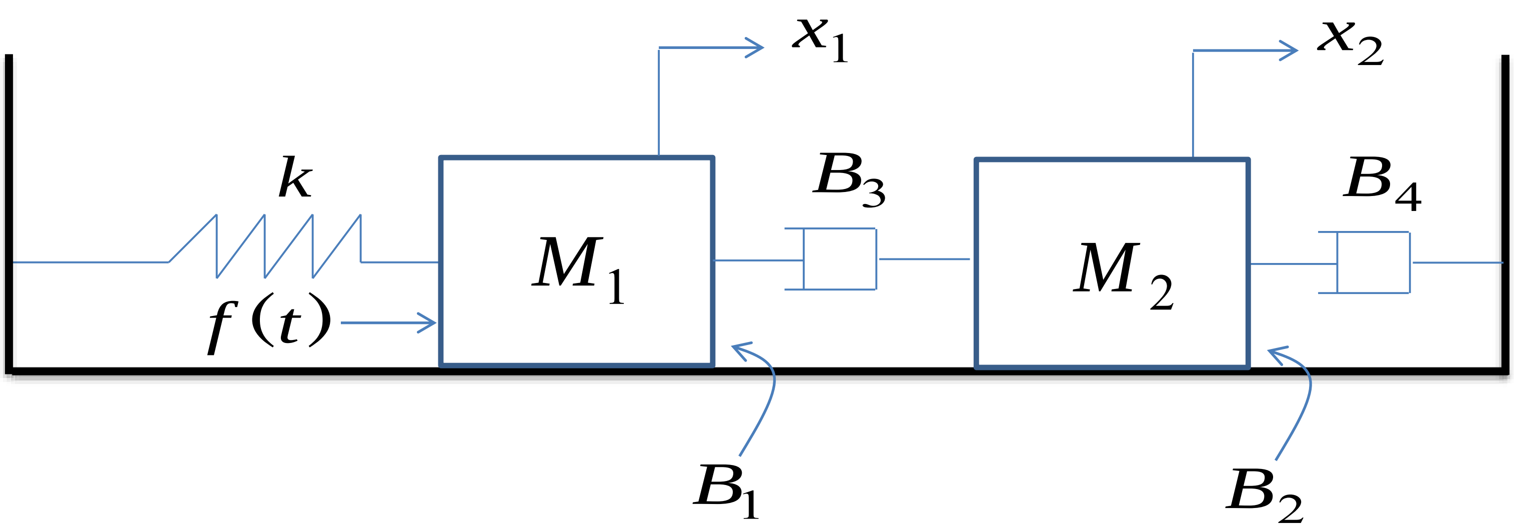
At node x_1

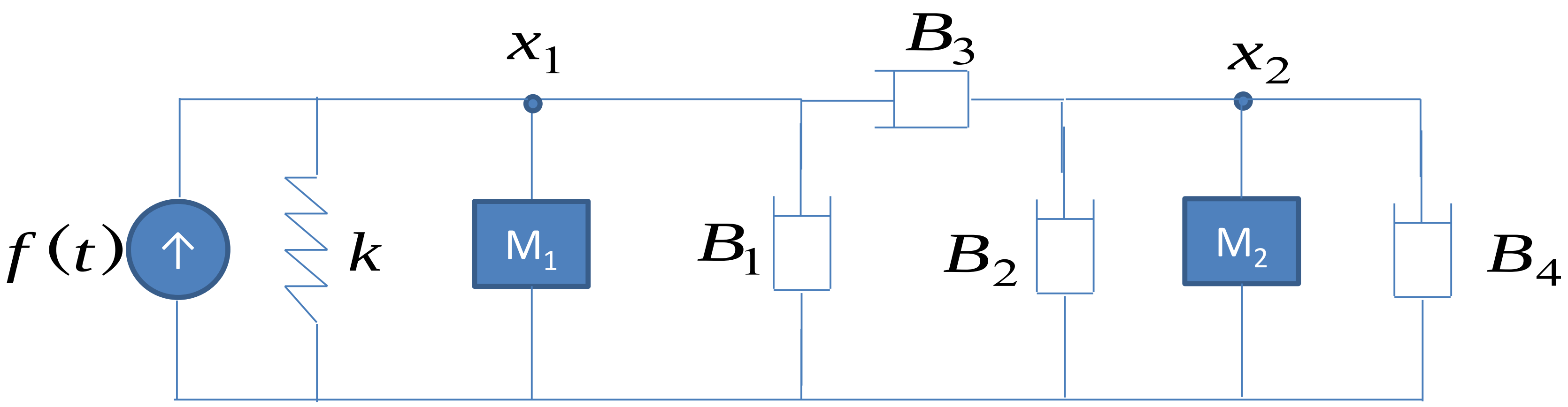
$$F = k(x_1 - x_2)$$

At node x_2

$$0 = k(x_2 - x_1) + M\ddot{x}_2 + B\dot{x}_2$$

Example-5





Example-6

- Find the transfer function of the mechanical translational system given in Figure-1.

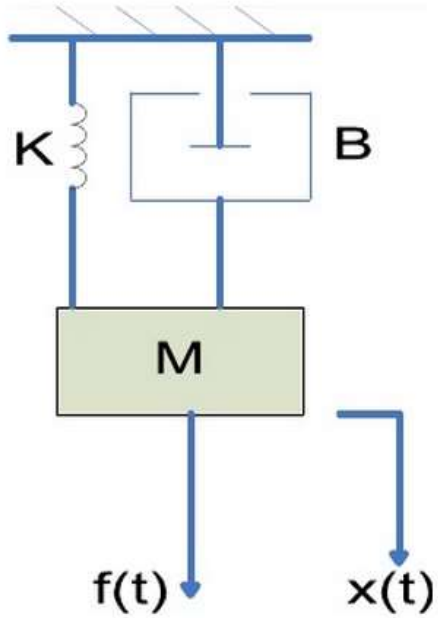
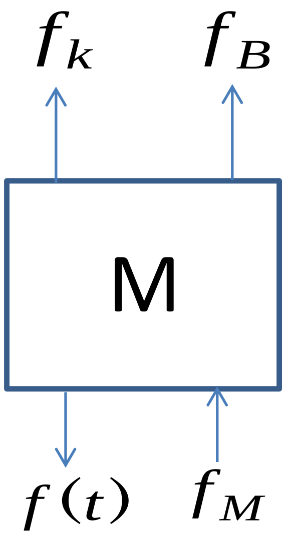


Figure-1

Free Body Diagram

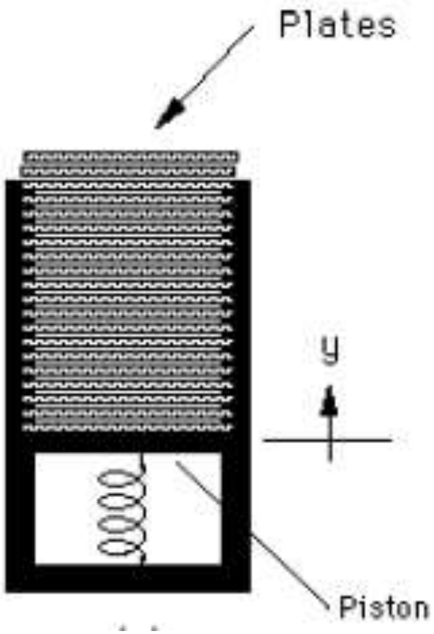


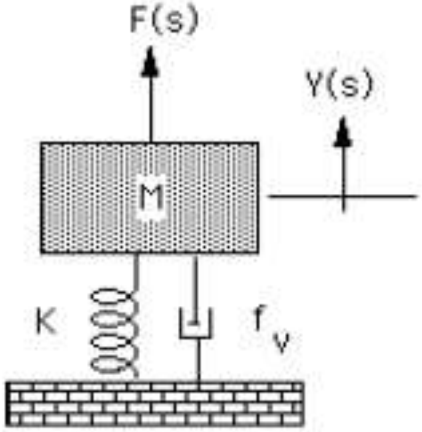
$$f(t) = f_k + f_M + f_B$$

$$\frac{X(s)}{F(s)} = \frac{1}{Ms^2 + Bs + k}$$

Example-7

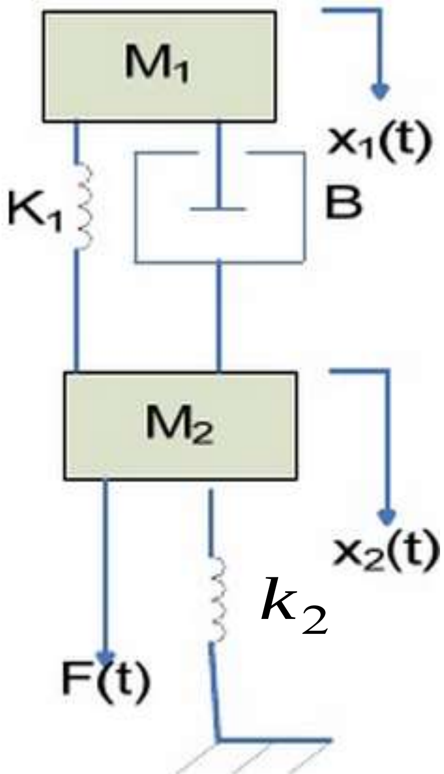
- Restaurant plate dispenser



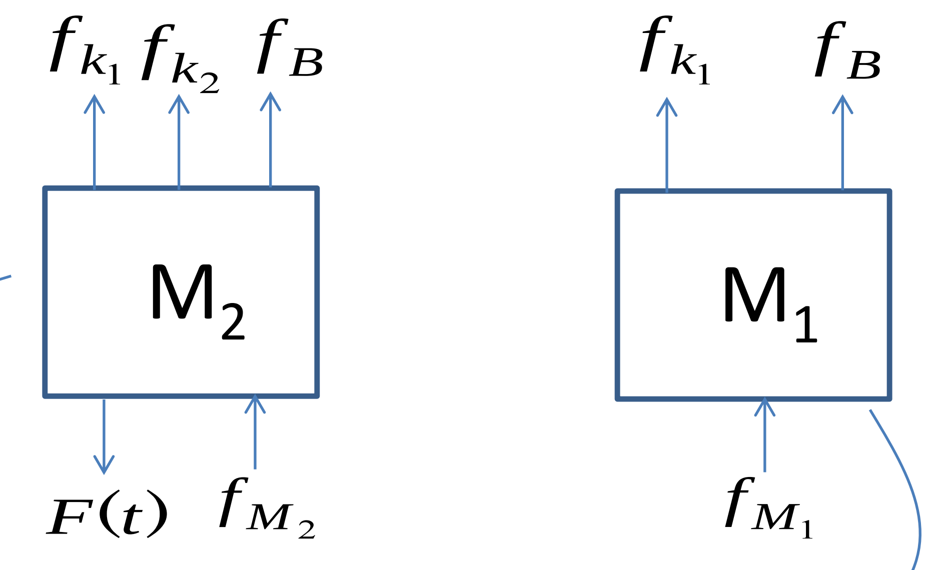


Example-8

- Find the transfer function $X_2(s)/F(s)$ of the following system.



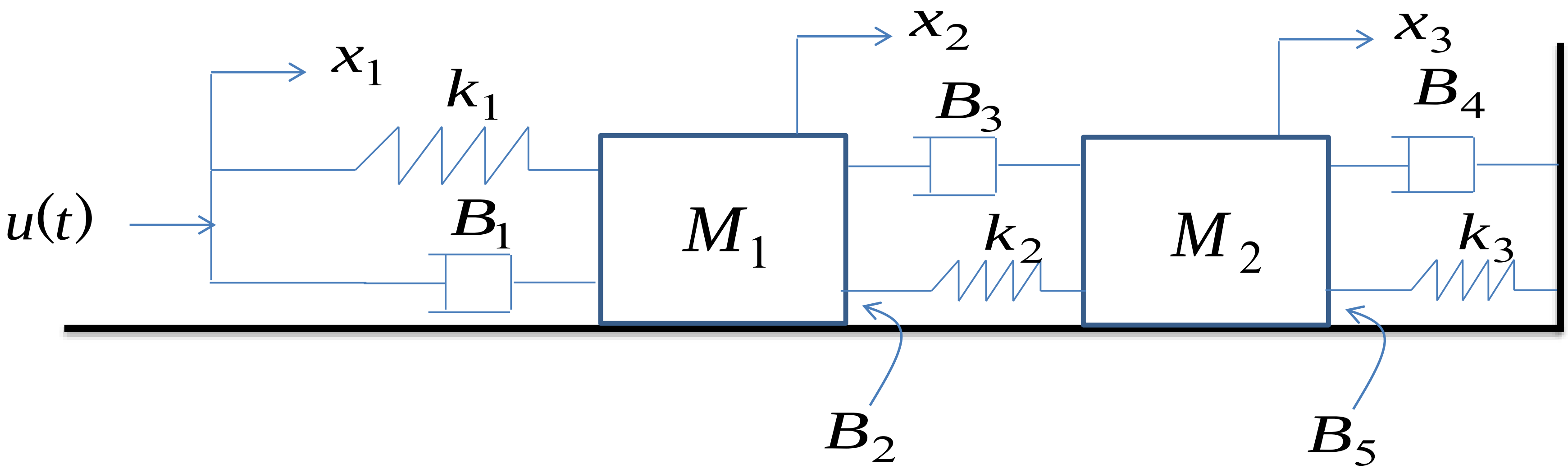
Free Body Diagram



$$F(t) = f_{k_1} + f_{k_2} + f_{M_2} + f_B$$

$$0 = f_{k_1} + f_{M_1} + f_B$$

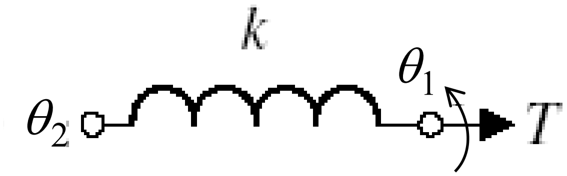
Example-09



ROTATIONAL MECHANICAL SYSTEMS

Basic Elements of Rotational Mechanical Systems

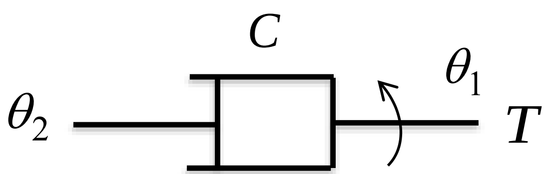
Rotational Spring



$$T = k(\theta_1 - \theta_2)$$

Basic Elements of Rotational Mechanical Systems

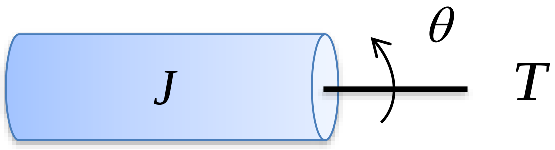
Rotational Damper



$$T = C(\dot{\theta}_1 - \dot{\theta}_2)$$

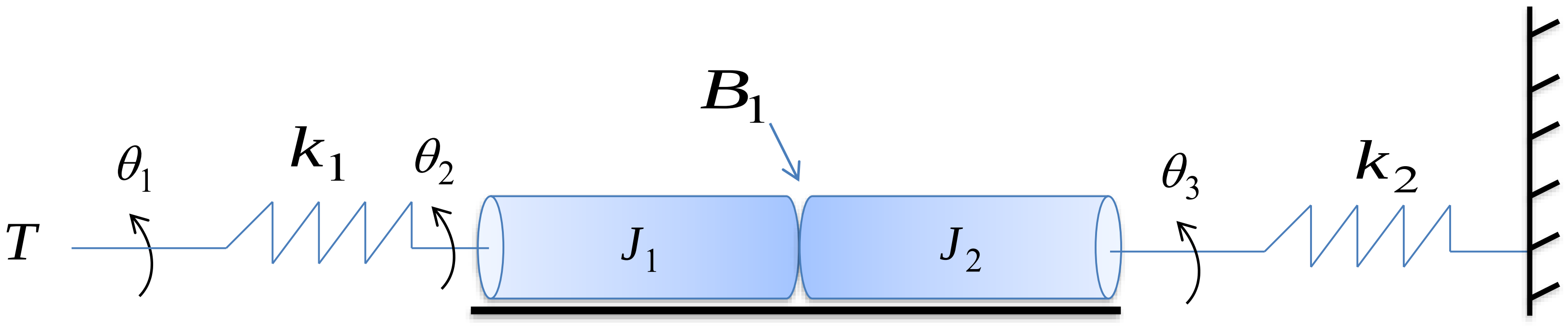
Basic Elements of Rotational Mechanical Systems

Moment of Inertia

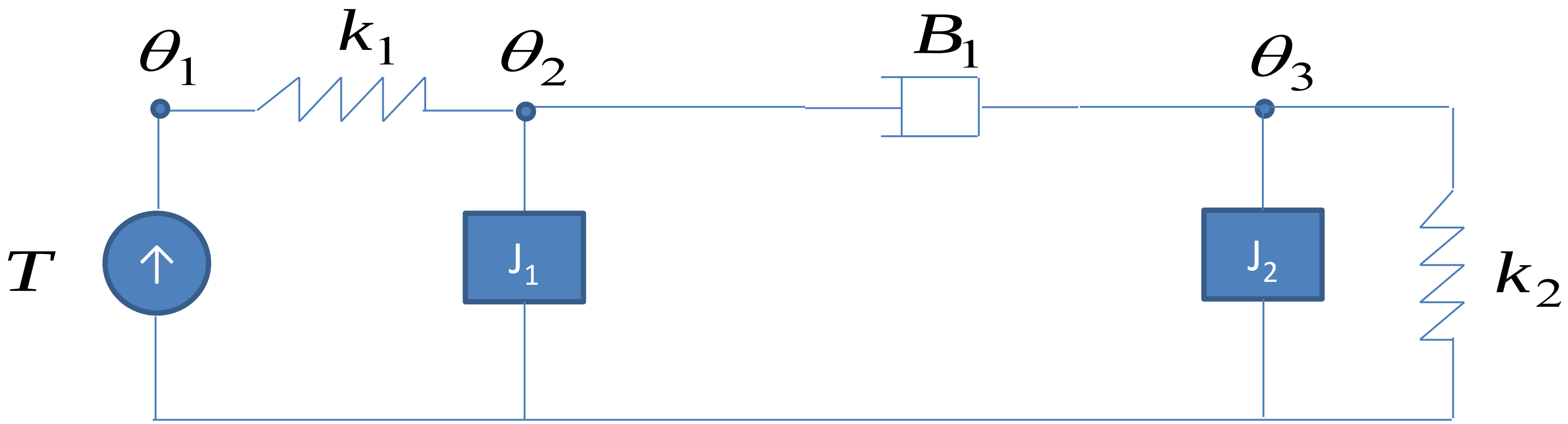


$$T = J\ddot{\theta}$$

Example-1

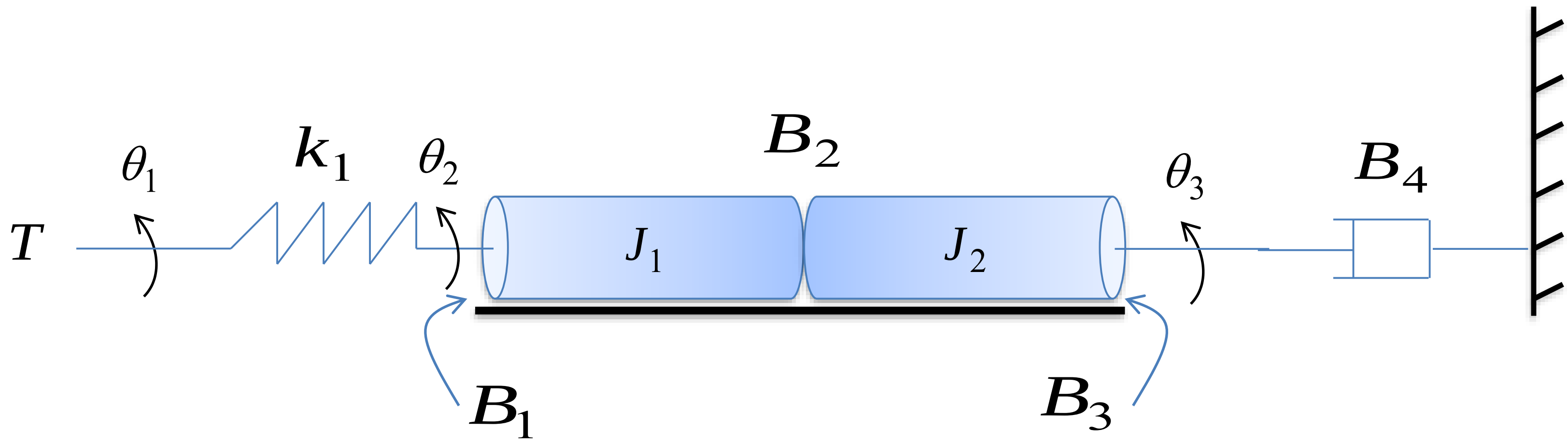


B_1
θ_1
k_1
θ_2
θ_3
k_2
T
J_1
J_2

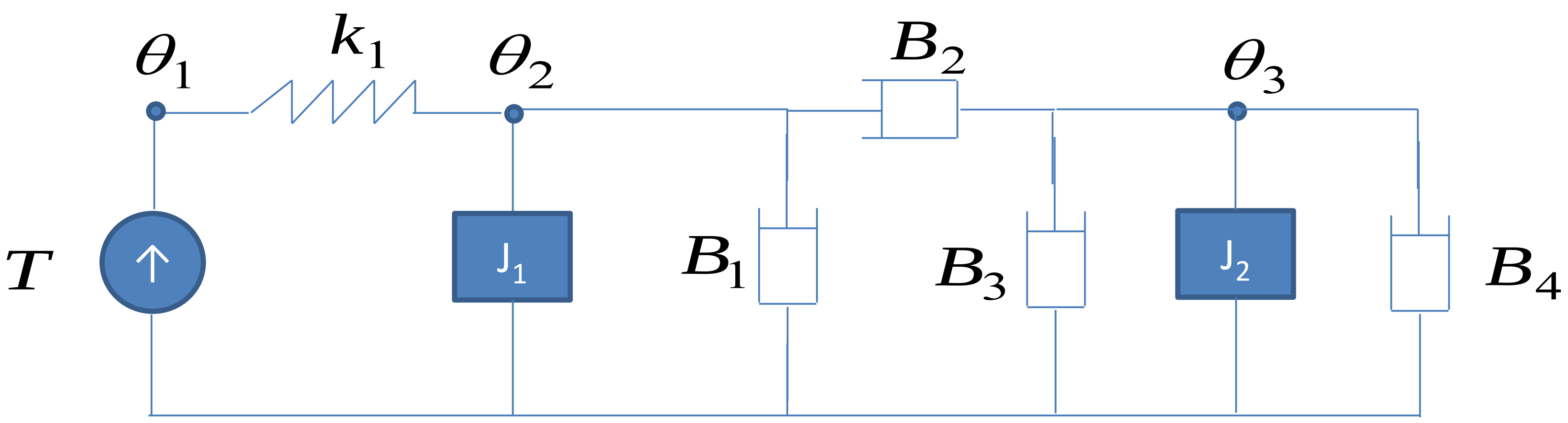


θ_1
k_1
θ_2
B_1
θ_3
T
J_1
J_2
k_2

Example-2

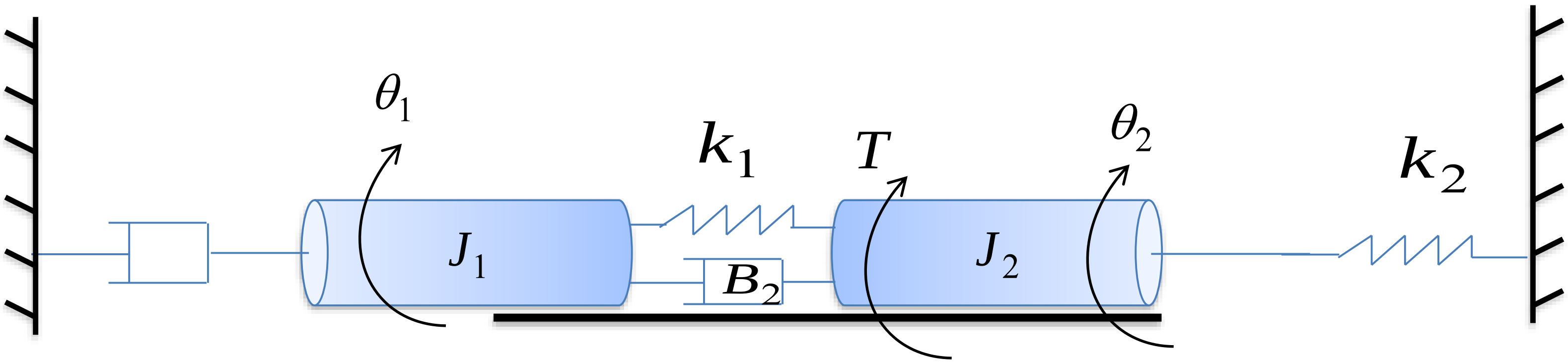


T
θ_1
k_1
θ_2
B_2
J_1
J_2
θ_3
B_4
B_1
B_3



θ_1
k_1
θ_2
B_2
θ_3
T
J_1
B_1
B_3
J_2
B_4

Example-3



Example-4