- Principles of Telecommunication
- Module 3- Angle Modulation
- Lecture plan- Part 3

2.Mathematical expression of Single-tone Phase Modulated Wave

- Phase modulation is another type of angle modulation. The PM wave is obtained by varying the *phase angle Φ of a carrier in proportion with the amplitude of the modulating voltage.*

$$e_c = E_c \sin(\omega_c t + \phi)$$

- If the carrier voltage is expressed as under:

$$s(t) = e_{PM} = E_c \sin(\omega_c t + \phi_m \sin \omega_m t)$$

Then, the PM wave can be expressed as under:

Here, Φm = Maximum phase change corresponding to the maximum amplitude of the modulating signal.
For the sake of uniformity, let us modify the above

$$s(t) = e_{PM} = E_c \sin(\omega_c t + m_p \sin \omega_m t)$$

equation as under:

where, mp = Φm = Modulation index of PM = phase deviation.

The FM and PM waves look identical when their modulation index are identical. However, if we change the modulating frequency fm, then mf will change but there is no change in the value of mp.

Deviation Sensitivity of PM

$$x(t) = E_m \cos(\omega_m t),$$

For the modulating signal :

$$\theta(t) = k_p\, x(t) \text{ rad}$$

the instantaneous phase deviation is given by,
Where,kp is the deviation sensitivity of phase modulation.

$$k_p = \frac{\text{rad}}{\text{V}}$$

Thus,

3. Single Tone Frequency Modulation

For the single tone frequency modulation,i.e the modulating signal x(t) be a sinusoidal signal of amplitude Em and frequency fm .

Therefore, $x(t) = E_m \cos(2\pi f_m t)$

The unmodulated carrier is represented by the expression :

$e_c = E_c \sin(\omega_c t + \varphi)$

Instantaneous frequency of an FM wave

In FM, the frequency f of the FM wave varies in accordance with the modulating voltage .

Thus,
Where $\Delta f = k_f E_m$ and it is called as **frequency deviation** .

$$f_i(t) = f_c + k_f x(t) = f_c + k_f . E_m \cos(2\pi f_m t)$$

$$\text{or} \qquad f_i(t) = f_c + \Delta f \cos(2\pi f_m t)$$

Frequency Deviation (Δf)

Frequency deviation Δf represents the maximum departure of the instantaneous frequency $f_i(t)$ of the FM wave from the carrier frequency f_c .

Since $\Delta f = k_f E_m$, the frequency deviation is proportional to the amplitude of modulating voltage (E_m) and it is independent of the modulating frequency f_m .

Maximum frequency of FM Wave

The maximum frequency of FM wave is given by :

$$f_{max} = f_c \pm \Delta f$$

Mathematical Expression for FM

FM wave is a sine wave having a constant amplitude and a variable instantaneous frequency .

As the instantaneous frequency is changing continuously, the angular velocity ω of an FM wave is the function of ω_c and ω_m .

Therefore, the FM wave is represented by,

$$s(t) = E_c \sin[F(\omega_c, \omega_m)]$$

$$s(t) = E_c \sin\theta(t)$$

$$\text{Where} \qquad \theta(t) = F(\omega_c, \omega_m)$$

As shown in fig.1, Ec sin Θ(t) is a rotating vector . If Ec is rotating at a constant velocity ‘ω’, then we could have written that Θ(t)=ωt .

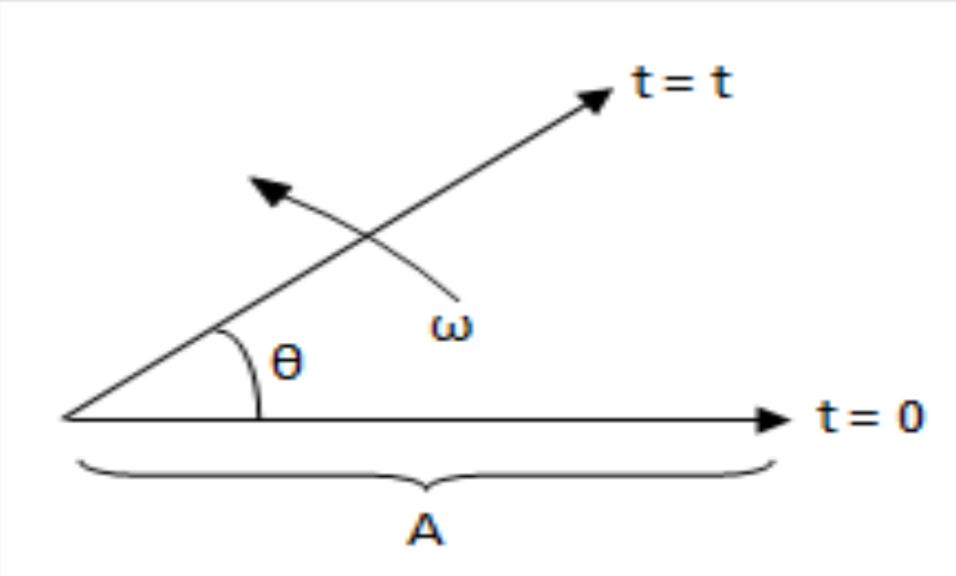

Fig.1 : Frequency modulated vector

But, in FM , this velocity is not constant. In fact, it is changing continuously.

The angular velocity of FM wave is given as,

$$\omega = [\omega_c + kE_m \cos\omega_m t]$$

Hence to find $\omega(t)$, we must integrate ω with respect to time.

Therefore, $$\theta(t) = \int \omega\, dt = \int [\omega_c + kE_m \cos\omega_m t] dt$$

or $$\theta(t) = \omega_c \int [1 + \frac{kE_m}{\omega_c} \cos\omega_m t]\, dt$$

$$= \omega_c [t + \frac{kE_m \sin\omega_m t}{\omega_c \omega_m}]$$

$$= \omega_c t + \frac{kE_m \omega_c \sin\omega_m t}{\omega_c \omega_m}$$

or $$\theta(t) = \omega_c t + \frac{kE_m \sin\omega_m t}{f_m}$$

As per the definition, $\Delta f = kE_m$

Thus, $$\theta(t) = \omega_c t + \frac{\Delta f \sin\omega_m t}{f_m}$$

Substituting this value of $\theta(t)$ in the equation of $s(t)$, we get the equation for the FM wave as under ;

$$s(t) = E_c \sin[\omega_c t + \frac{\Delta f}{f_m} \sin\omega_m t]$$

$$\text{But } \frac{\Delta f}{f_m} = m_f$$

i. e. the modulation index of FM wave. Hence, the equation for FM wave is given as :

$$s(t) = E_c \sin[\omega_c t + m_f \sin\omega_m t]$$

This is the expression for FM wave, where, m_f represents the modulation index.

Modulation Index

The modulation index of an FM wave is defined as under :

- The modulation index is very important in FM because it decides the bandwidth of the FM wave

$$m_f = \frac{\text{Frequency deviation}}{\text{Modulating frequency}}$$

or $$m_f = \frac{\Delta f}{f_m}$$

which we will discuss later .

- The modulation index also decides the number of sidebands having significant amplitudes .
- *In AM, the maximum value of the modulation index m is 1 .*
- *But, for FM, the modulation index can be greater than 1.*

Deviation Ratio

- In FM broadcasting, the maximum value of deviation is limited to 75 kHz. The maximum modulating frequency is also limited to 15 kHz.

- The modulation index corresponding to the maximum deviation and maximum modulating frequency is called as the deviation ratio .

Percentage Modulation of FM Wave

The percent modulation is defined as the ratio of the actual frequency deviation produced by the modulating signal to the maximum allowable frequency deviation . Thus,

$$Deviation\ Ratio = \frac{Maximum\ deviation}{Maximum\ Modulating\ frequency}$$

$$\% \, Modulation = \frac{Actual\ Frequency\ deviation}{Maximum\ allowed\ Deviation}$$