Equation of FM wave

$$s(t) = E_c \sin(\omega_c t + m_f \sin \omega_m t)$$

(or)

$$s(t) = E_c \sin\left(\omega_c t + \frac{\Delta f}{f_m} \sin \omega_m t\right)$$

(or)

$$s(t) = E_c \sin(2\pi f_c t + m_f \sin 2\pi f_m t)$$

i) m_f = Modulation Index $= \dfrac{\Delta f}{f_m}$

$= \dfrac{\text{Frequency deviation}}{\text{Modulation } \cancel{\text{index}} \text{ frequency}}$

ii) $\boxed{\text{Deviation Ratio}} = \dfrac{\text{Maximum deviation}}{\text{Max modulating frequency}}$ (limited to 75 kHz in FM)

iii) % Modulation $= \dfrac{\text{Actual frequency deviation}}{\text{Maximum allowed deviation}}$

iv) Bandwidth,

NBFM

$BW = 2 f_m$ (same as AM)

WBFM

$BW = 2(m_f + 1) f_m$

Power, of FM, $P = \frac{\left(\frac{E}{\sqrt{2}}\right)^2}{R}$

$P = \frac{E_c^2}{2R}$ | for $R = 1$

Carson's rule,

$BW = 2[\Delta f + f_{m(max)}]$

highest modulating frequency

Frequency ~~sensitivity~~ deviation, $\boxed{\Delta f = m_f \times f_m}$

Frequency sensitivity, $k_f = \frac{\Delta f}{E_m}$

modulating voltage //

Example: 1: A music signal with frequency components from 50Hz to 21000Hz is Frequency modulated. If the maximum allowed frequency deviation is 50 kHz.

i) What is the modulation index?

ii) What is the signal bandwidth? [Using Carson's rule]

Sol:- Maximum modulating frequency, $f_{m(max)} = 21000\ Hz = 21\ kHz$

Maximumum allowed frequency deviation, $\Delta f = 50\ kHz$.

i) Modulation index, $\boxed{m_f = \dfrac{\Delta f}{f_{m(max)}}}$

$$m_f = \frac{50\ kHz}{21\ kHz}$$

$$= \frac{50}{21}$$

$$= 2.38$$

$\boxed{m_f = 2.38}$ // Ans.

ii) Signal Bandwidth, $\boxed{BW = 2\left(\Delta f + f_{m(max)}\right)}$

(Using Carson's rule)

$$BW = 2\left(50\ kHz + 21\ kHz\right)$$

$$= 2 \times 71\ kHz$$

$$= 142\ kHz$$

$\boxed{BW = 142\ kHz}$ // Ans.

iii)

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Example 2: A 200 MHz carrier is frequency modulated by a 10 V (peak) signal of 10 kHz. The carrier frequency varies between 199.90 and 200.10 MHz.

Calculate:

i) Frequency sensitivity

ii) Modulation index

iii) Signal Bandwidth

Sol: i) Calculate carrier swing – total variation in frequency from lowest to highest point vice versa. to determine Δf

$$f_{High} = f_c + \Delta f$$
$$f_{low} = f_c - \Delta f$$

Given, $f_m = 10\,kHz$

$$f_{High} - f_{low} = f_c + \Delta f - (f_c - \Delta f)$$
$$f_{High} - f_{low} = \cancel{f_c} + \Delta f - \cancel{f_c} + \Delta f$$
$$\boxed{f_{High} - f_{Low} = 2\Delta f}$$

Given, $f_{High} = \cancel{199}\ 200.10\,MHz$

$f_{low} = 199.90\,MHz$

$$f_{High} - f_{low} = 2\Delta f$$

$$\Rightarrow 200.10 - 199.90 = 2\Delta f$$

$$\Rightarrow \Delta f = \frac{200.10 - 199.90}{2}$$

$$= 0.1\,MHz$$

$$= 100\,kHz$$

Frequency deviation, $\Delta f = 100\,kHz$

Amplitude,

$$E_m = 10V \quad (\text{peak})$$

$\therefore$ Frequency sensitivity, $\boxed{k_f = \frac{\Delta f}{E_m}}$

$$\Rightarrow k_f = \frac{100\ kHz}{10\ V} = 10\ kHz/V$$

$\cancel{\Rightarrow k_f = 100 f}$

$\therefore \boxed{k_f = 10\ kHz/V}$ // Ans.

ii) Modulation index, $\boxed{m_f = \frac{\Delta f}{f_m}}$

$$m_f = \frac{100\ kHz}{10\ kHz}$$

$\boxed{m_f = 10}$ // Ans.

iii) B.W = 2(frequency of modulating signal + frequency deviation)

$$= 2(f_m + \Delta f)$$

$$= 2(10 + 100)$$

$$= 2 \times 110 = 220\ kHz$$

$\boxed{BW = 220\ kHz}$ // Ans.//

Example 3: A sinusoidal modulating waveform of amplitude 5V and a frequency 2 kHz is applied to FM generator, which has a frequency sensity of 40 Hz / volt.

Calculate:

i) frequency deviation

2) Modulation index

3) Bandwidth

Sol:) Given, Amplitude of modulating signal,

given

$$E_m = 5V$$

Frequency of modulating signal,

$$f_m = 2kHz = 2000 Hz$$

& Frequency sensitivity, $k_f = 40 Hz/volt$.

i) Frequency deviation, $\boxed{\Delta f = k_f E_m}$

$$\Delta f = 40 \frac{Hz}{Volt} \times 5 Volt$$

$$\boxed{\Delta f = 200 Hz.}$$ // Ans.

ii) Modulation index, $\boxed{m_f = \frac{\Delta f}{f_m}}$

$$m_f = \frac{200 Hz}{2000 Hz} = 0.1$$

∵ Modulation index is less than 1. It is NBFB.

iii) <u>Bandwidth</u>

Since formula for NBFB is same as that of AM wave.

$$\boxed{BW = 2f_m}$$

$$B.W = 2 \times 2\ kHz$$
$$= 4\ kHz$$

// Ans.

Example 4: An FM wave is given by

$$s(t) = 20 \cos\left(8\pi \times 10^{6} t + 9 \sin(2\pi \times 10^{3} t)\right)$$

Calculate :

i) Frequency deviation

ii) Bandwidth

Sol:-) Given equation of an FM wave as

$$s(t) = 20 \cos(\underbrace{8\pi \times 10^6 t}_{2\pi f_c} + \underbrace{9}_{m_f} \sin(\underbrace{2\pi \times 10^3 t}_{2\pi f_m}))$$

We know standard equation of an FM wave as

$$s(t) = E_c \cos\left(2\pi f_c + \underbrace{m_f}_{\frac{\Delta f}{f_m}} \sin(2\pi f_m t)\right)$$

On comparing two equations,

Frequency of carrier signal, $f_c = \cancel{4 MHz}\ 4 \times 10^6 = 4 MHz$

$$\left[\text{Hint: } \cancel{2\pi} f_c = \cancel{8\pi}^{4} \times 10^{6} \quad f_c = 4 \times 10^{6} = 4 MHz \right]$$

Frequency of message signal, $f_m = 1 \times 10^3 Hz = 1 kHz$

$$\left[\text{Hint: } \cancel{2\pi} f_m = \cancel{2\pi} \times 10^3 \quad f_m = 10^3 = 1 KHz \right]$$

Modulation index, $m_f = 9$

Here, the value of modulation index is greater than 1. Hence, it is Wide Band FM.

i) We know the formula for modulation index

$$\boxed{m_f = \frac{\Delta f}{f_m}}$$

$$\Delta f = m_f \times f_m$$

$$= 9 \times 1 \text{ kHz}$$

$$= 9 \text{ kHz}$$

∴ Frequency deviation, $\Delta f = 9 \text{ KHz}$

ii) Formula for Bandwidth of Wide Band FM

$$\boxed{BW = 2(m_f + 1) f_m}$$

$$BW = 2(9+1) \text{ kHz} = 20 \text{ kHz}.$$

//

EXAMPLE 4.18. **A carrier wave of frequency 1 GHz and amplitude 3 volts is frequency modulated (FM) by a sinusoidal modulating signal frequency of 500 Hz and of peak amplitude 1 volt. The frequency deviation Δf is 1 kHz. The level of the modulating waveform (signal) is changed to 5 volt peak and the modulating frequency is changed to 2 kHz. Obtain the expression for the new modulated waveform (FM).**

Solution : We know that the FM wave is given by the expression

$$s(t) = A\cos[2\pi f_c t + m_f \sin(2\pi f_m t)]$$

where m_f = Modulation index of FM wave = $\frac{\Delta f}{f_m}$

and Δf = frequency deviation = $k_f A_m$

k_f = Sensitivity of Frequency modulator

A_m = Amplitude of the modulating signal

Given that f_c = 1 MHz = 1×10^6 Hz

A = 3 volt

A_m = 1 volt

and Δf = 1 kHz

Therefore, k_f can be found as

$$k_f = \frac{\Delta f}{f_m} = \frac{1\times10^3}{1} = 10^3 \text{ Hz/volt}$$

Now, for the second case, we have

when, A_m = 5 volt and f_m = 2 kHz

Modulation Index will be

$$m_f = \frac{\Delta f}{f_m} = \frac{10^3\times5}{2\times10^3} = 2.5$$

The desired FM signal can be expressed by

$$s(t) = A\cos[2\pi f_c t + m_f \sin(2\pi f_m t)]$$

Substituting all the values, we get

$$s(t) = 3\cos\left[2\pi 10^6 t + 2.5\sin(2\pi\times2\times10^3 t)\right]$$

or

$$s(t) = 3\cos\left[2\pi 10^6 t + 2.5\sin(4\pi\times10^3 t)\right]$$

EXAMPLE 4.25. A carrier is frequency modulated (FM) by a sinusoidal modulating signal x(t) of frequency 2 kHz, it results in a frequency deviation Δf of 5 kHz. Find the bandwidth occupied by the FM waveform. The amplitude of the modulating sinusoid is increased by a factor of 3 and its frequency lowered by 1 kHz. Find the new bandwidth.

Solution : Given that

$$f_m = 2 \text{ kHz} \text{ and}$$
$$\Delta f = 5 \text{ kHz}$$

Hence, the bandwidth of the FM signal will be

$$BW = 2(\Delta f + f_m)$$

or $$BW = 2(5 + 2)$$

or $$BW = 14 \text{ kHz}$$

When the amplitude of the modulating signal is trippled, then the frequency deviation increases three times.

Therefore, $$\Delta f = 3 \times 5 \text{ kHz} = 15 \text{ kHz}$$

Also, $$f_m = 1 \text{ kHz}$$

The new bandwidth will be

$$BW' = 2(\Delta f + f_m) = 2(15 + 1)$$

or $$BW' = 32 \text{ kHz}$$ **Ans.**