# Chapter 3
# Angle Modulation

## Lecture Plan- (2nd June 2021)

- ▢ Introduction to angle modulation
- ▢ Classification of angle modulation
- ▢ Average and Instantaneous frequency
- ▢ Relationship between Phase modulation and Frequency Modulation
- ▢

## 1. Introduction to Angle Modulation

In our previous chapter, we have discussed the various types of Amplitude modulation systems in which the carrier amplitude is changed in accordance with the variation in the message signal magnitude .

- There is another method of modulating a sinusoidal carrier namely the angle modulation in which either frequency or phase of the carrier is varied according to the message signal, but the carrier amplitude is constant .

Note- *In Angle Modulation, the amplitude of carrier wave is maintained constant whereas the angle of the carrier wave is varied according to the message signal.*

## Advantages and disadvantages of angle modulation :

Angle modulation has several advantages over the amplitude modulation such as-

- noise reduction,
- improved system fidelity and
- more efficient use of power.

But there are some disadvantages too such as

- increased bandwidth and
- use of more complex circuits.

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## Applications

Angle modulation is being used for the following applications :

- Radio broadcasting
- TV sound transmission
- Two way mobile radio
- Cellular radio
- Microwave communication
- Satellite communication

## Principle of Angle Modulation

The principle of angle modulation can be stated as follows :

*The phase angle (Θ) of a sinusoidal carrier wave is varied with respect to time .*

An angle modulated wave can be expressed mathematically as :

$$s(t) = E_c \cos[\omega_c t + \theta(t)]$$

where, $E_c$ → peak carrier amplitude

$\omega_c$ → carrier frequency

$\theta(t)$ → instantaneous phase deviation

In angle modulation,

→ $\theta(t)$ is a function of modulating signal

∴ $$\theta(t) = F[e_m(t)]$$

Here, $e_m(t)$ is the modulating signal

$$e_m = E_m \sin(\omega_m t)$$

# Basic Definitions

Let the modulated carrier wave be expressed in general form as :

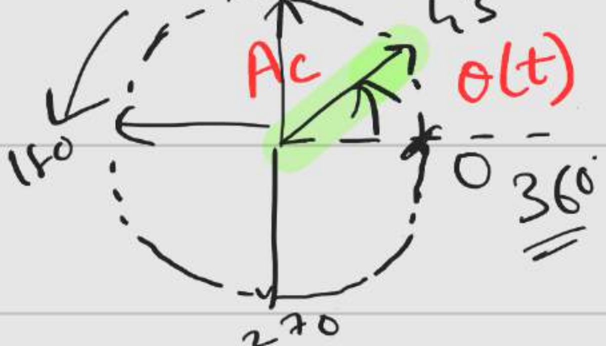
$$s(t) = E_c \cos \theta(t)$$

- $E_c$ : carrier amplitude
- $\theta(t)$ : angle at time 't' → Function of message signal

✓ Expression : $s(t)$ is a Modulated sinusoidal carrier at time 't'

**Note**: Complete oscillations occur whenever angle $\theta(t)$ changes by $2\pi$ radian.

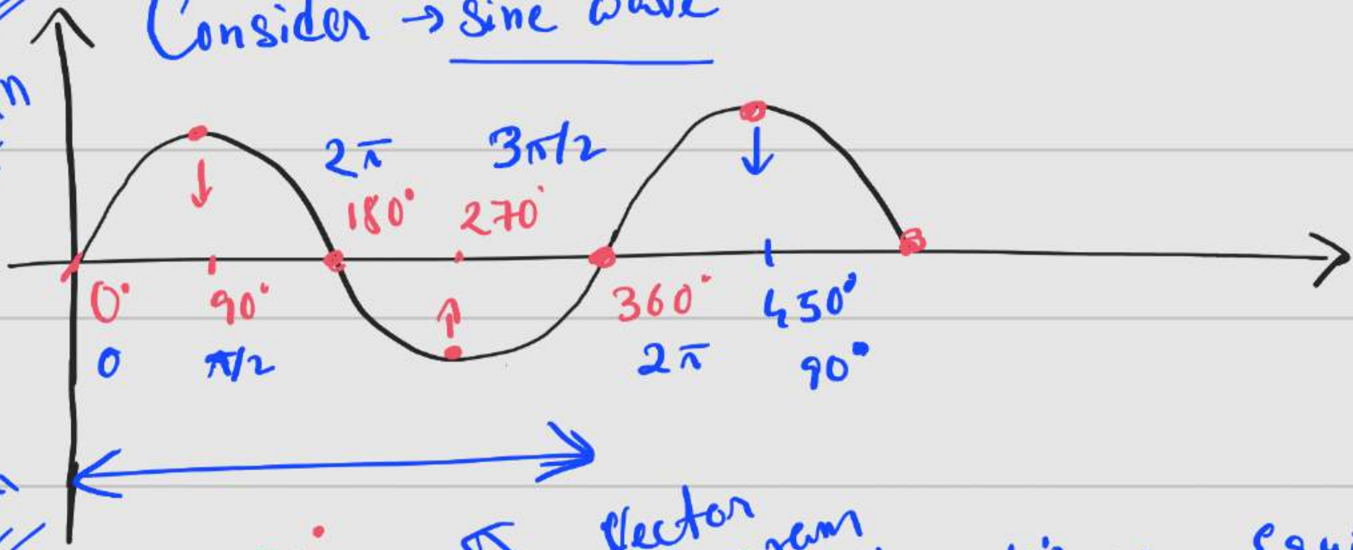
✓ Rotating phasor with angular velocity (rad/sec) $\frac{d\theta(t)}{dt}$ w.r.t $t$

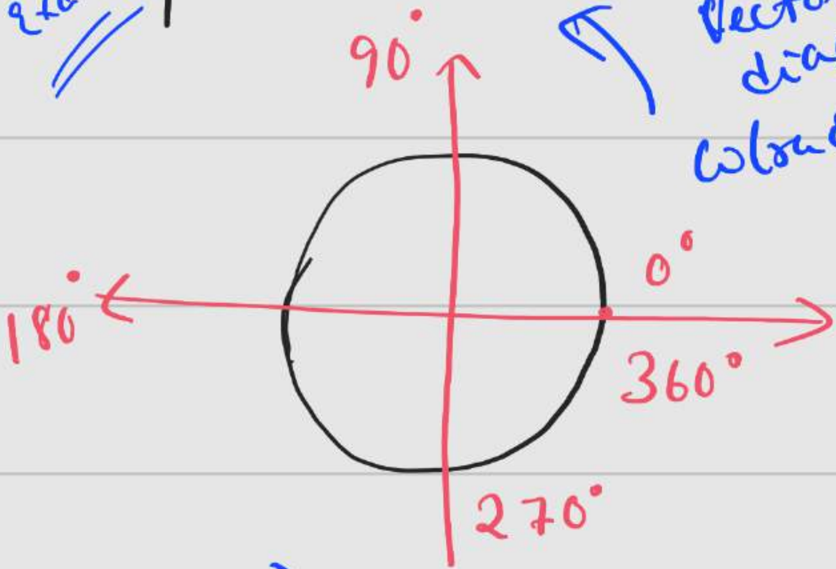


**Explain**

## Consider → sine wave

N/B need in exam




Vector diagram ω(rad/s)

$\sin(0°) = 0$ equilibrium

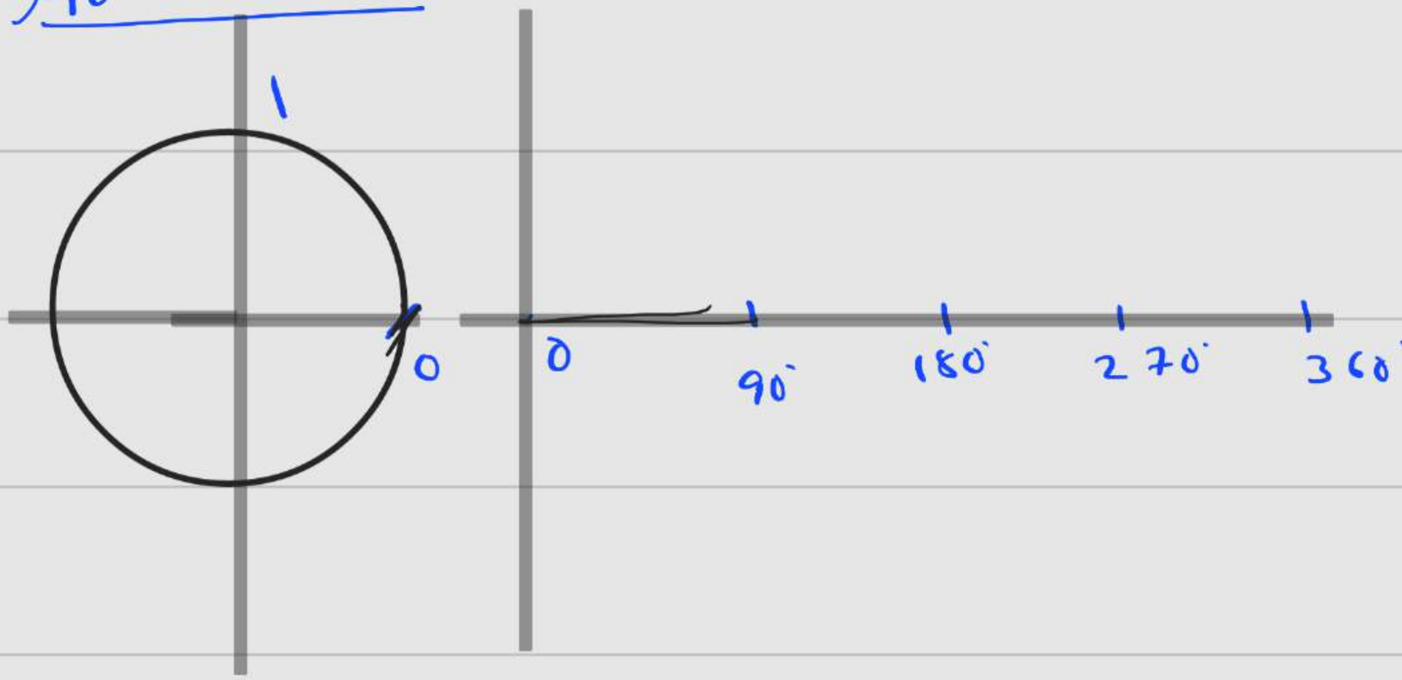
$\sin(90°) = 1$ max

$\sin(180°) = 0$ (equ)

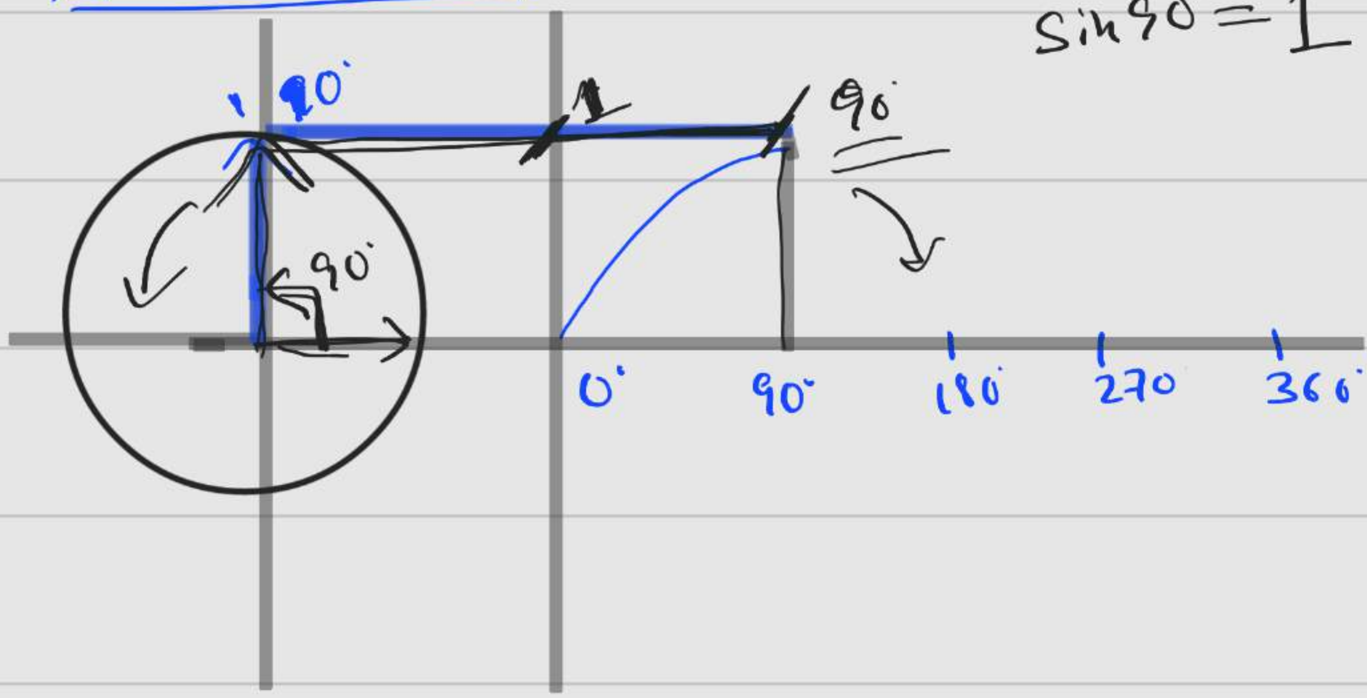
$\sin(270°) = \text{min}$

$\sin(360°) = \text{equ}$

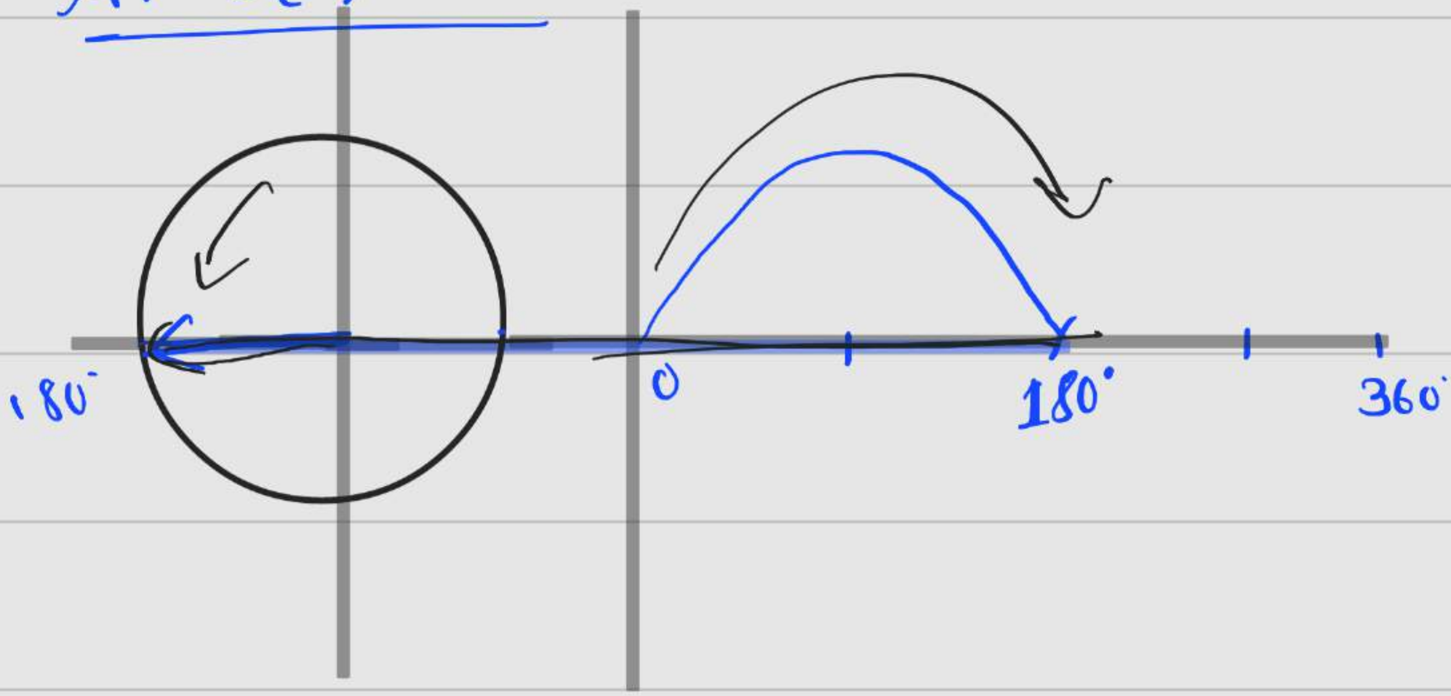
## At θ(t) = 0



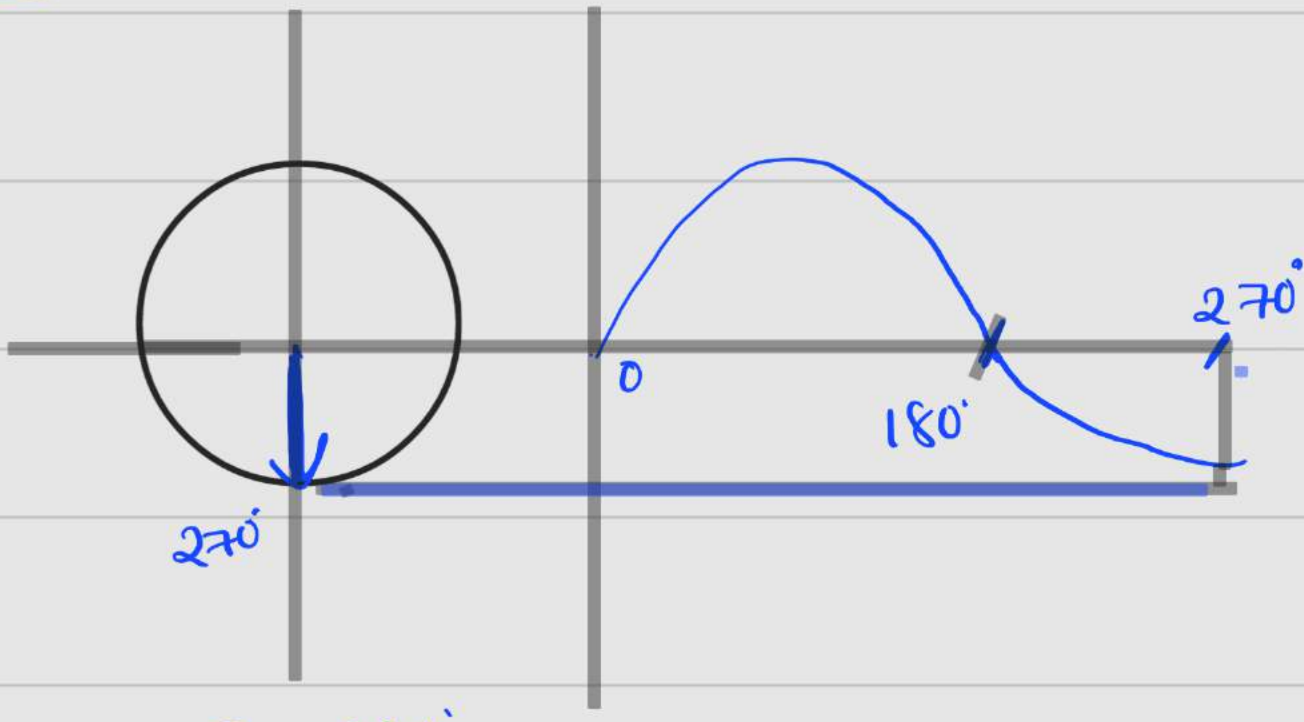
## At $\theta(t) = 90^\circ$



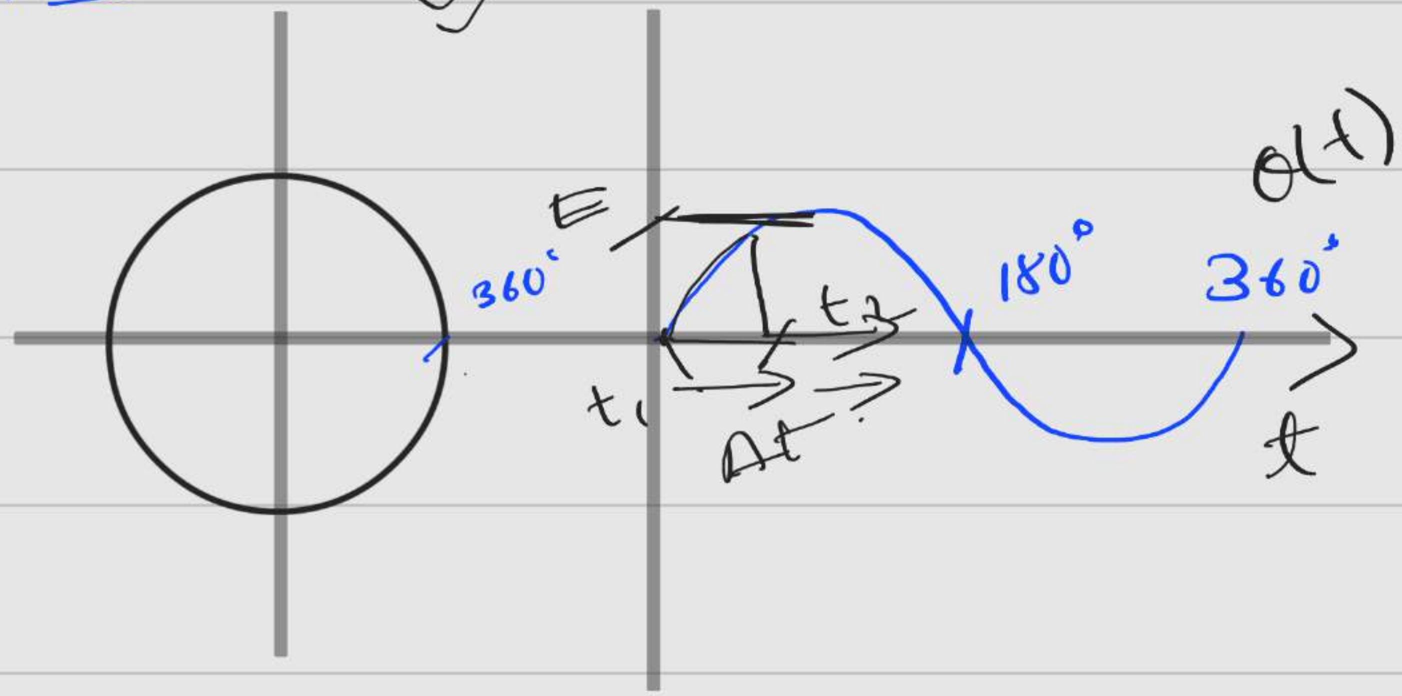

## At $\theta(t) = 180^\circ$



## At $\theta(t) = 270°$




## At $\theta(t) = 360°$

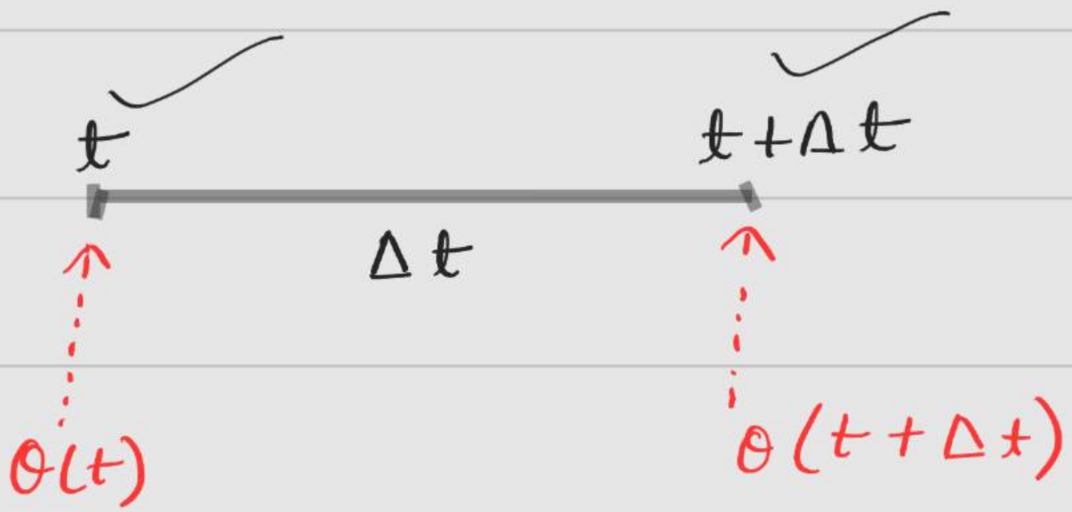



- A full cycle of a Sine wave wave can be represented by rotation of a phaser through 360 degree.

Instantaneous value of Sine wave is at any point is equal to vertical distance from the tip of phaser to the horizontal axis.

Back to topic

Consider small time interval, $\Delta t$




## Average frequency —

Over a small interval from $t$ to $t+\Delta t$

$$f_{\Delta t}(t) = \frac{\theta(t+\Delta t) - \theta(t)}{2\pi \Delta t}$$

[Hint: To derive above expression)

We know, $s(t) = A\cos\theta$

$= A\cos\omega t$

$= A\cos 2\pi f t$

So; $\theta = 2\pi f$

$f = \frac{\theta}{2\pi f}$ ]

# Instantaneous frequency

✓ On taking limits of $f_{\Delta t}$ $\Delta t \to 0$

$$f_i(t) = \lim_{\Delta t \to 0} f_{\Delta t}(t)$$

$$= \lim_{\Delta t \to 0} \left[ \frac{\theta(t+\Delta t) - \theta(t)}{2\pi \Delta t} \right]$$

$$= \frac{1}{2\pi} \lim_{\Delta t \to 0} \frac{\theta(t+\Delta t) - \theta(t)}{\Delta t}$$

$d\theta(t)$

$$= \frac{d\theta(t)}{dt}$$

→ Angular velocity of rotating phasor

derivative of angle w.r.t 't'

$\theta(t) = 2\pi f$

So, we can write,

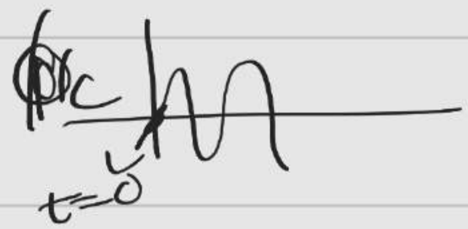
$$f_i(t) = \frac{1}{2\pi}\frac{d\theta(t)}{dt}$$

Case 1 : For unmodulated carrier, i.e, $x(t) = 0$

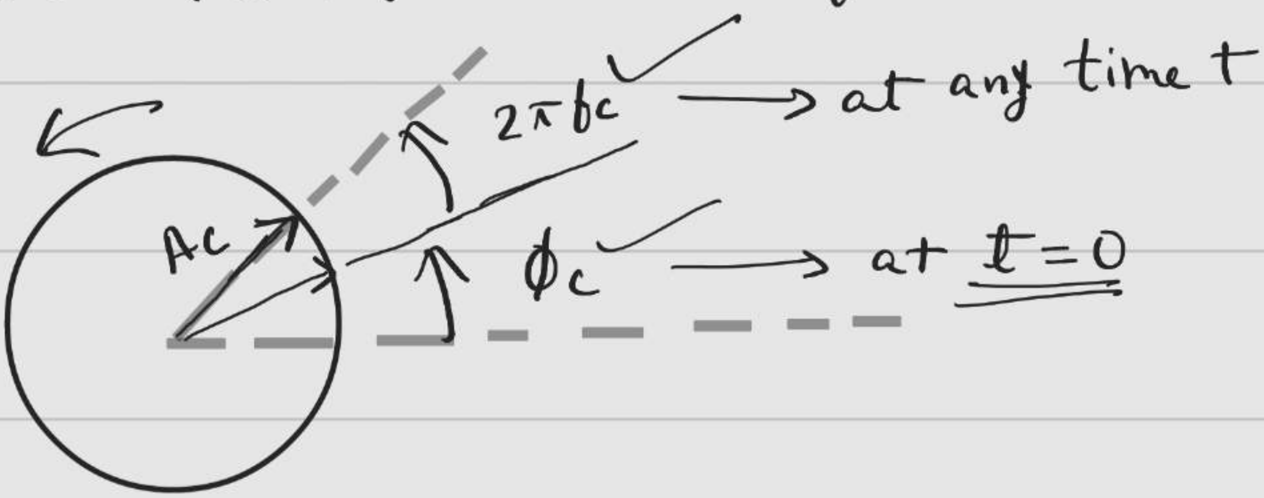
$$\theta_c(t) = \theta(t) = 2\pi f_c t + \phi_c$$

$2\pi f_c t$ → Constant angular velocity

$\phi_c$ → angle of unmodulated carrier at time $t = 0$



**Note-** The phasor will rotate at its constant angular velocity of $2\pi f_c$ and $\phi_c$ represents value of $\theta(t)$ at $t=0$.




Case 2: After modulation

$$x(t) = A_m \cos 2\pi f_m t$$

Angle modulation can be expressed as-

$$s(t) = A_c \cos\theta(t)$$

angle $\theta(t)$ is varied with $x(t)$

## 2. Classification

Angle Modulation

- (a) Phase Modulation (PM)
- (b) Frequency Modulation (FM)

# a) Phase Modulation

Instantaneous angle $\theta(t)$ is varied linearly with $x(t)$

$$\theta(t) = 2\pi f_c t + k_p x(t)$$

- $\theta(t)$ → rad
- $2\pi f_c t$ → angle of unmodulated carrier / angular argument of unmodulated carrier
- $k_p$ → Phase sensitivity factor (rad/volt)
- $k_p x(t)$ → (1)

**Note:** Set $\phi_c = 0$ (for convenience)

$x(t) \approx$ voltage waveform

So, Phase modulated wave can be expressed as:

$\cos(\checkmark)$

$$s(t) = E_c \cos\left[2\pi f_c t + k_p x(t)\right]$$

$$E_c \cos\left[\omega_c t + \theta(t)\right]$$

General exp of Angle modulated

## b) Frequency Modulation

Instantaneous frequency $f_i(t)$ is varied linearly with $x(t)$

$$f_i(t) = f_c + k_f\, x(t) \quad \text{--- (i)}$$

$f_c$ → frequency of unmodulated carrier

$k_f$ → frequency sensitivity factor of modulator (Hz/volt)

Now, since $f_i(t) = \frac{1}{2\pi}\frac{d\theta(t)}{dt}$

$$\theta(t) = 2\pi \int_0^t f_i(t)\, dt \quad \text{--- (ii)}$$

Putting eqn (1) to (2)

Also; $\theta(t) = 2\pi f_c(t) + 2\pi k_f \int_0^t x(t)\,dt$

∴ Expression for FM wave,

$$s(t) = E_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t x(t)\,dt\right]$$

## Frequency variation in FM and PM waves

To understand the difference between FM and PM, we first consider the FM wave.

- [ ] For FM, the frequency changes with the change in modulating signal value.
- [ ] 

It remains constant during interval t2 at its maximum value and during interval t5 at its minimum value.

Now, let us look at the PM wave.

- The frequency of PM wave changes and only if the magnitude of modulating signal changes.

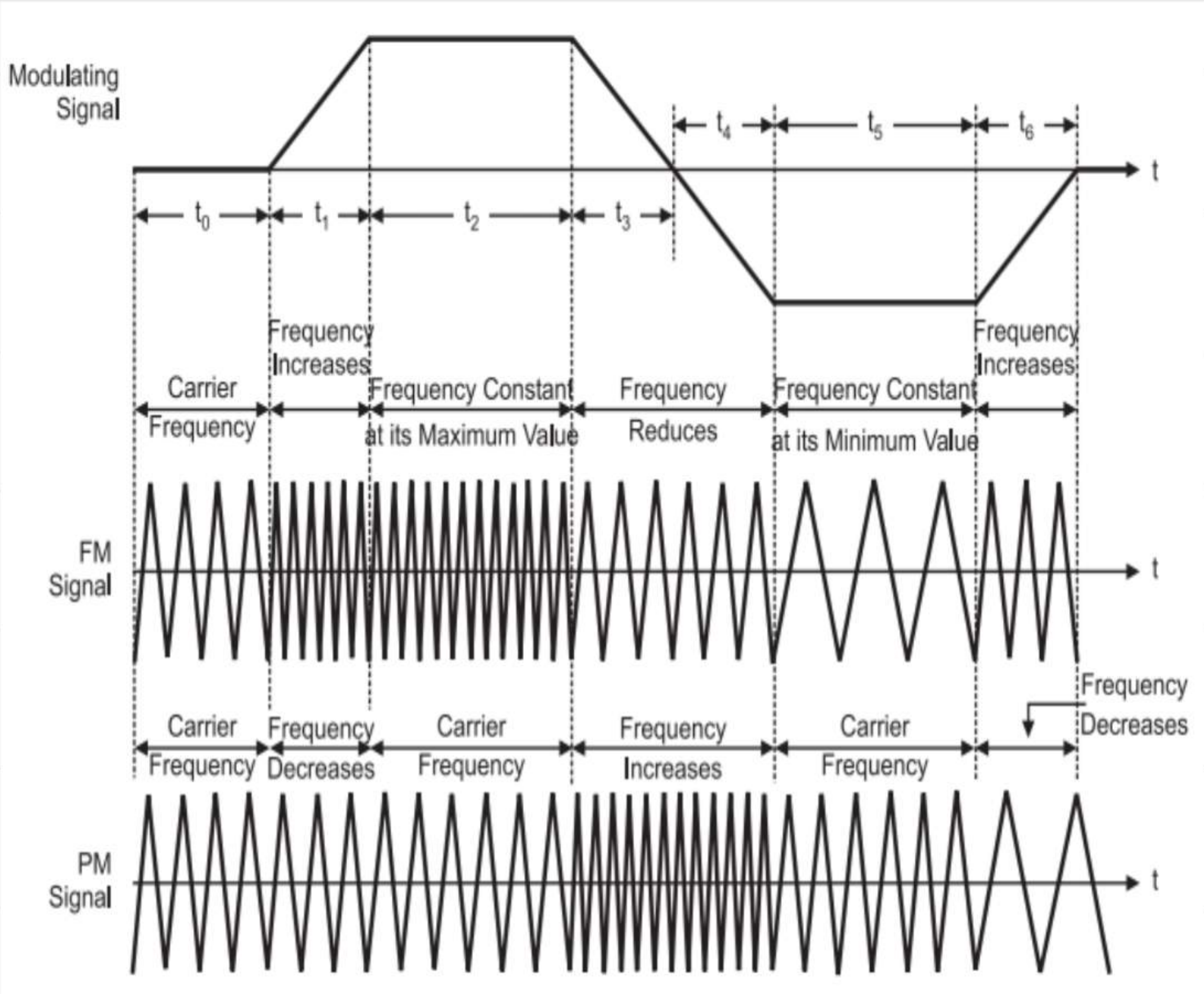
- Thus, during the intervals t1, t3, t4, t6, the frequency of PM wave changes. But during the intervals t2 and t5 the frequency of PM remains constant at the carrier frequency value with some phase shift.

**Note-**

Phase modulation produces frequency modulation.

But, a very important point may be noted here that the frequency modulation (FM) is produced by PM if and only if the phase shift is being varied.

Let us understand this point by considering figure 1 below.

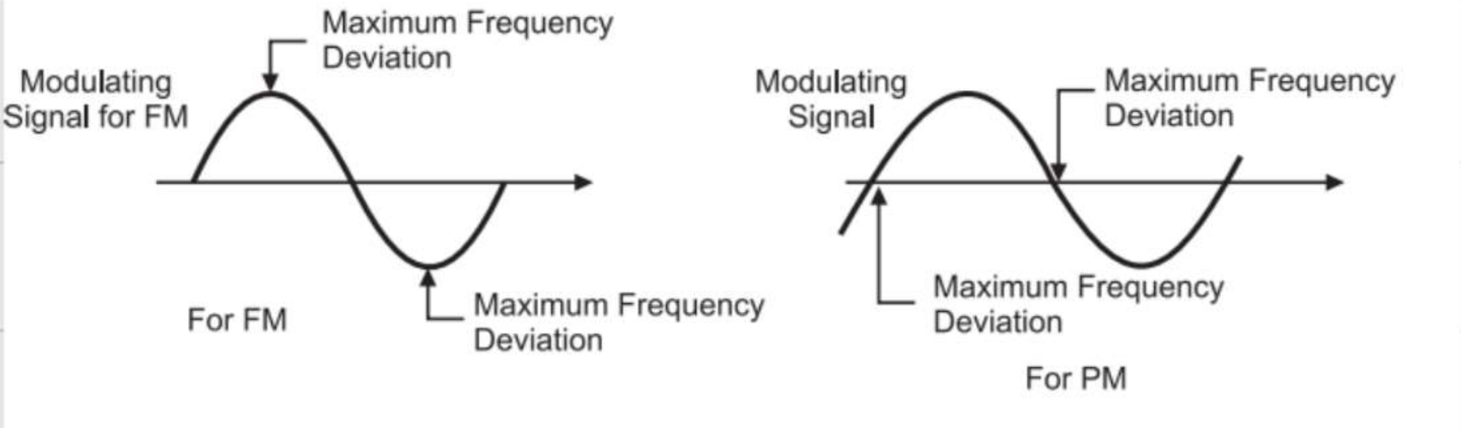


Modulating Signal
t
$t_0$
$t_1$
$t_2$
$t_3$
$t_4$
$t_5$
$t_6$
Carrier Frequency
Frequency Increases
Frequency Constant at its Maximum Value
Frequency Reduces
Frequency Constant at its Minimum Value
Frequency Increases
FM Signal
Carrier Frequency
Frequency Decreases
Carrier Frequency
Frequency Increases
Carrier Frequency
Frequency Decreases
PM Signal

# Maximum frequency deviation

- The maximum frequency deviation in FM occurs at the point where the rate of change of modulating signal is maximum that means the maximum frequency corresponding to peak values of modulating signal.

- But, in PM, the maximum frequency deviation occurs at the point deviation occurs near the zero crossing points of the modulating signal.




(a) Maximum Deviations Points for FM (b) Maximum Deviations Points for PM

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Thus, although the phase modulation produce FM, the maximum deviation occurs at different points of a modulating signal