RELIABILITY ENGINEERING

BE 8TH SEM (EE)

ASSIGNMENT 1

Part I : Basic Probability Theorem

1. The probability of two events A and B are $P(A) = 0.32$, $P(B) = 0.44$ and $P(A \cup B) = 0.58$
   (i) Are the events mutually exclusive ?
   (ii) Are they independent ?
   (iii) Calculate $P(A|B)$ and $P(B|A)$.

2. The probability of two events A and B are such that $P(A) = 0.4$, $P(A \cup B) = 0.8$; $P(A \cap B) = 0.2$. Determine
   (i) $P(B)$. (ii) $P(A|B)$ (iii). $P(B|A)$

3. For three events A, B and C, if $P(A) = 0.8$, $P(B) = 0.3$, $P(C) = 0.4$, $P(A|B \cap C) = 0.5$ and $P(B|C) = 0.6$
   (i) Determine whether events B and C are independent.
   (ii) Determine whether events B and C are mutually exclusive.
   (iii) Evaluate $P(A \cap B \cap C)$ and $P(B \cap C|A)$

4. Two events E_1 and E_2 are such that $P(E_1|E_2) = 0.4$, $P(E_1|\bar{E}_2) = 0.5$ and $P(E_2) = 0.6$. Find the value of $P(E_1)$.

5. The occurrence of a particular event A depends on two mutually exclusive events B_1 and B_2. If $P(B_1) = 0.4$ and $P(B_2) = 0.2$ then $P(A) = 0.34$ but if $P(B_1) = 0.2$ and $P(B_2) = 0.4$ then $P(A) = 0.32$. Evaluate $P(A|B_1)$ and $P(A|B_2)$.

6. A batch of 1000 piston rings manufactured in an engine manufacturing facility contains 40 defective ones. Two piston rings are randomly selected from the batch, one at a time, without replacement. If E_i denotes the event that the i^{th} piston ring selected is defective ($i = 1, 2$), determine the values of $P(E_1)$ and $P(E_2)$.
(Hints: apply total prob. th^m) [Ans. $P(E_1) = P(E_2) = 0.04$]

7. Two machines produce the total output of a factory. Machine 1 produces 70% and machine 2 produces 30% of the output. Five percent of the output of machine 1 is defective and 8% from machine 2. If a finished item is selected at random, what is the probability of it being defective?
(Hints: apply total prob. th^m) [Ans. 0.059]

rt II : Binomial Distribution

8. Ten engines undergoing testing. If the failure prob. for an individual engine is 0.10, what is the probability that more than two engines will fail the test?

9. The probability of an engine's failing during a 30-day acceptance test is 0.3 under adverse environmental conditions. Eight engines are included in such a test. What is the probability of the following:
(i) none will fail?
(ii) all will fail?
(iii) more than half will fail?

10. A thermal power plant buys four boilers. If the probability of a boiler functioning without failure for a year is 0·7, plot probability mass function (pmf) and distribution function (cdf) of the status of the boilers at the end of the year.

11. There is a 5% chance of dimensional distortion (considered as failure) in each of the forged connecting rods produced in an engine manufacturing plant.
(a) Determine the probability of finding exactly two connecting rods with dimensional distortion when 15 connecting rods are inspected. Assume that the connecting rods are independent w.r.t. dimensional distortions.
(b) Determine the probability that the number of connecting rods with dimensional distortion lies between 2 and 5 when 15 connecting rods are inspected.
[Ans. (a) 0·134752 (b) 0·170899]

PART III : Probability density function & distribution function.

12. Find the probability density function of a random variable X for which the cumulative distribution fn is given by

$$F_X(x) = 1 - e^{-5x}, \quad x > 0$$

13. For the probability density fn

$$f(x) = K x(1-x), \quad 0 \le x \le 1$$
$$= 0, \quad \text{otherwise}$$

Determine K and mean (μ).

14. The probability density function of the wind load acting on a water tank is given by

$$f_x(x) = \frac{3x}{50\times10^4}(100-x) \quad ; \quad 0 \leq x \leq 100 \text{ units}$$

$$= 0 \quad ; \text{ otherwise}$$

Find the probability of realizing the wind load to be less than 20 units OR greater than 80 units.

[Ans: 0.208].

15. The diameter of a rod produced on a machine during mass production with the specification of the nominal diameter as 20 mm, is known to follow the following distribution:

$$f_x(x) = 0 \quad ; x < 20 \text{ mm}.$$

$$= 15\, e^{-15(x-20)} \quad ; x \geq 20 \text{ mm}.$$

Determine (a) the probability distribution function of the diameter of the rod. (b) If rods with diameter larger than 20.2 mm are not acceptable, determine the portion of the rods that are accepted.

[Ans (a) $F_x(x) = 0$ for $x < 20$ mm, $F_x(x) = 1 - e^{-15(x-20)}$, $x \geq 20$
(b) 95.0213%]

PART IV : Reliability block diagram

16. In a thermal power plant, four components – the feed water pump, boiler, turbine, and generator – are connected in series as shown in Fig. 1. The probabilities that the different components functions successfully are also indicated in the figure. The plant generates power only when each of the four components is functional. Determine the prob. that the four components are functional and power is generated in the plant. (Assume that failure of a compt is independent)

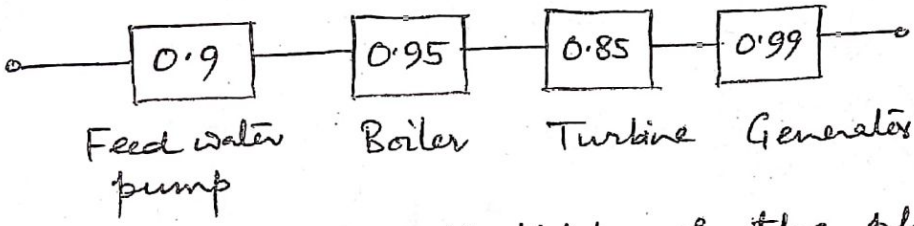

Fig. 1

To increase the reliability of the plant, the following two schemes are considered (Fig 2 & 3).

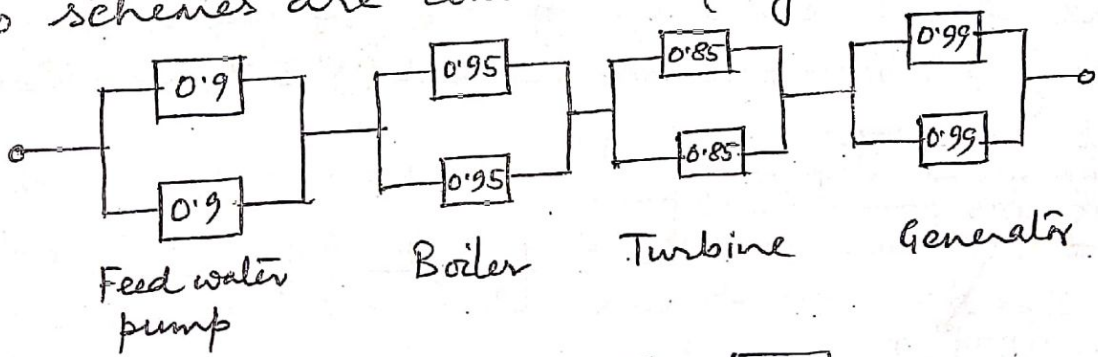

Fig 2

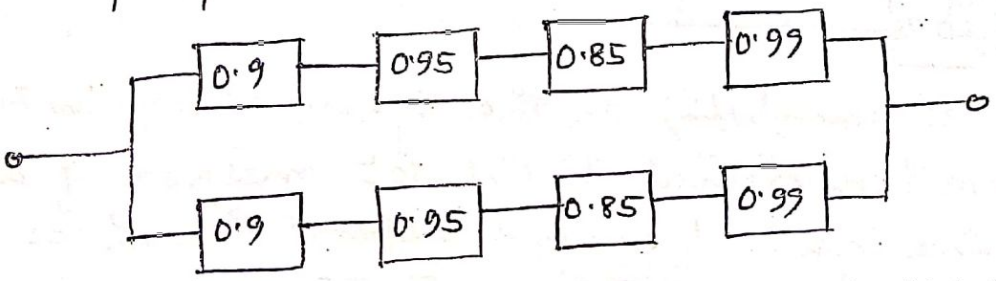

Fig 3

Determine and compare the probability that the power plant generates power for both cases.

* Refer page nos 98, 100 of the book: Reliability Evaluation of Engg. Systems (Billinton & Allan) and solve the following problems:

17. Problem NO. 1 (pp: 98)
18. " 2 (pp: 99)
19. " 4 (pp: 99)
20. " 5 (pp: 99)
21. " 6 (pp. 99)
22. " 7 (pp. 99)
23. " 8 (pp. 99)
24. " 9 (pp. 100)

PART V: Reliability evaluation of complex systems.

25. Determine the reliability of the system shown in Fig 4 using (a) Decomposition method (b) Boolean method and (c) cut-set method.

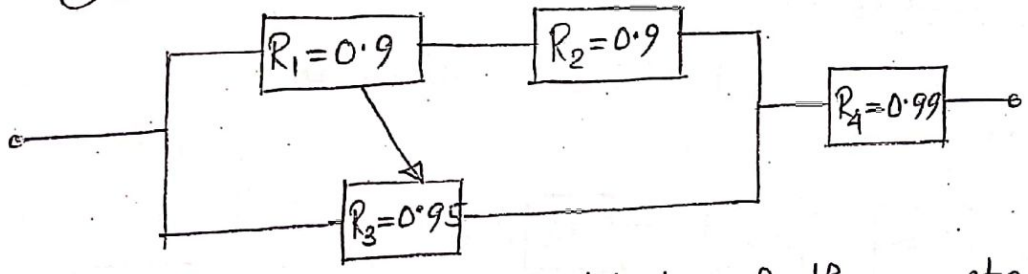

Fig. 4

26. Determine the reliability of the system shown in Fig 5 using (a) Decomposition method (b) Cut-set method if all the components are identical and independent with a reliability of 0.8.

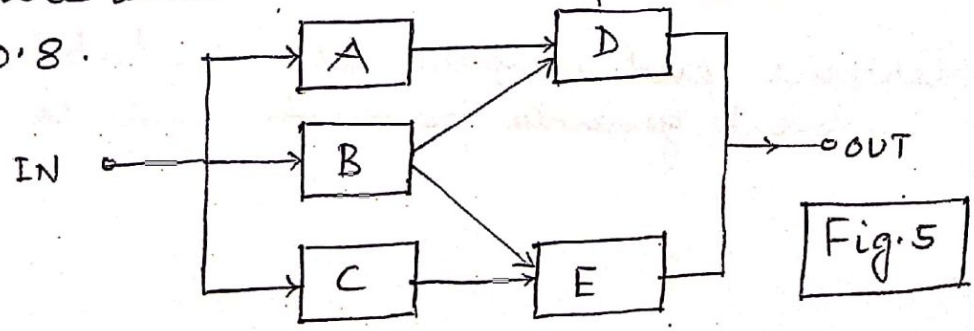

Fig. 5

27. Calculate the system reliability if all the individual components have a reliability of 0·9 in Fig. 6.

[Ans. 0·968266]

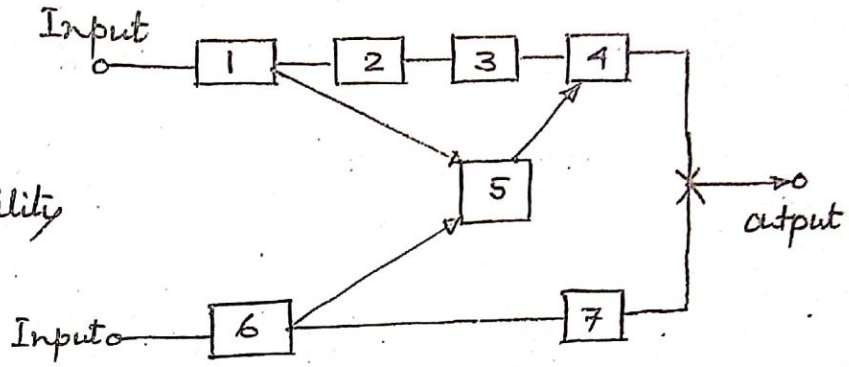

Fig. 6

28. Deduce minimal cut sets by constructing incidence matrix for the system shown in Fig. 7.

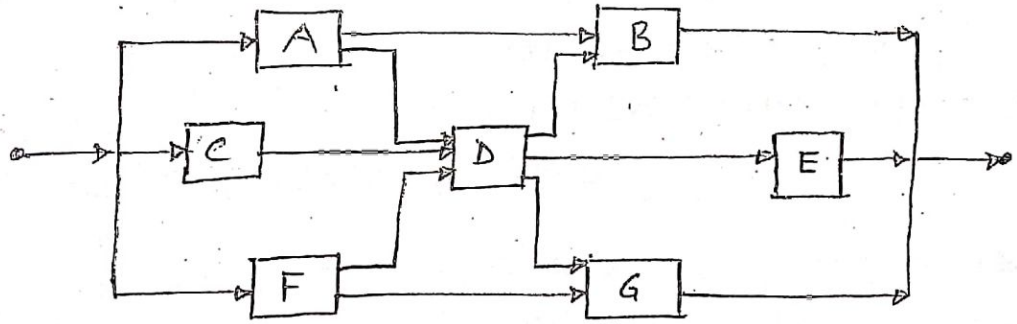

Fig 7

29. Develop an expression for the reliability of the syst shown in Fig. 8. Calculate the system reliability if all components have a reliability of 0·9. Estimate the system reliability using the minimal cut-set technique.

[Ans. 0·966588, > 0·964]

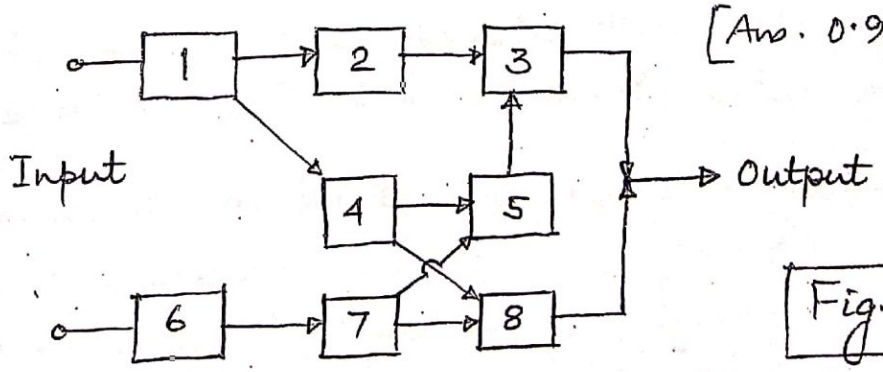

Fig. 8

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