Unit-I & 2 Chapter 1 & 2

Assignment

1. Distinguish between
   (a) Reliability and quality
   (b) Product cost vs product reliability

Ans: (a) Differentiate between Reliability and Quality.

Quality of a device or a product is the degree of conformance to applicable specification and workmanship standards. It is not concerned with the element of time and environment.

1. Quality is associated with the manufacture whereas reliability is primarily associated with the design.
2. Reliability is the ability of a unit to maintain its quality under specified conditions for a specified time.
3. One can build a reliable complex system using less reliable elements but it is impossible to construct a "good" quality system from a "poor" quality elements.

(b) Product cost vs product reliability:

If the producer increases the reliability of his product, he will increase the cost of the design and/or production of the product. However, a low production and design cost does not imply a low overall product cost. The overall product cost should not be calculated as merely the cost of the product cost when it leaves the shipping dock, but as the total cost of the product through its lifetime. This includes warranty and replacement cost for defective products, cost incurred by loss of customers due to defective product, loss of subsequent sales, etc. By increasing product reliability one may increase the initial product cost, but it will most likely decreases the support cost. An optimum minimal total product cost can be determined and implemented by calculating the optimum reliability for such a product.

2 Explain in detail:

(a) Binomial and Poisson distributions.

Ans: Binomial Distribution.

The binomial distribution is one of the most widely used discrete random variable distribution in reliability and quality inspection. It has application in reliability engineering e.g. when one is dealing with situation in which an event is either a success or a failure.

The pdf of the distribution is given by

$$p(X=x) = \binom{n}{x} p^x (1-p)^{n-x} \quad x = 0, 1, 2, \ldots n.$$

$$\binom{n}{x} = \frac{n!}{x!(n-x)!}$$

where n = number of trials ; x = number of successes;

p = single trail probability of success.

The reliability function, $R(k)$, (i.e, at least k out of n items are good) is given by

$$R(k) = \sum_{x=k}^{n} \binom{n}{x} p^x (1-p)^{n-x}$$

* Poisson Distribution:

Although the Poission distribution can be used in a manner similar to the bionomial distribution it is used to deal with events in which the sample size is unknown. This is also a discrete random variable distribution whose pdf is given by

$$P(X=x) = \frac{(\lambda t)^x e^{-\lambda t}}{x!} \quad \text{for } x = 0, 1, 2 \ldots\ldots$$

where. λ = constant failure rate,

x = is the number of events. In other words, $P(X=x)$

is the probability of exactly x failures occuring in time t. Therefore, the reliability Poisson distribution, R(k) (the probability of k or fewer failures) is given by

$$R(k) = \sum_{x=0}^{k} \frac{(\lambda t)^x e^{-\lambda t}}{x!}$$

This distribution can be used to determine the number of spares required for the reliability of standby redundant system during a given mission.

(b) Expected mean and standard deviation for continuous and distributions.

For a continuous random variable this becomes the corresponding integral involving the probability density function.

$$\mu = E(X) = \int_{-\infty}^{+\infty} x f(x)\, dx \quad \text{——— (1)}$$

we know, that the variance of a discrete random variable is the expectation of $(x_1 - \mu)^2$. This carries over to a continuous random variable and becomes.

$$\sigma_x^2 = E\left[(x-\mu)^2\right] = \int_{-\infty}^{+\infty} (x-\mu)^2 f(x)\, dx. \quad \text{— (2)}$$

The alternative form given by equation.

$$\sigma_x^2 = E(x^2) - \mu_x^2 \quad \text{——— (3)}$$

still holds and is generally faster for calculations. For continuous random variables

$$E(X^2) = \int_{-\infty}^{+\infty} x^2 f(x)\, dx \quad — (3)$$

* For any discrete random variable, equation gives that the expected mean is $E(R) = \mu$ (or μ_R) $= \Sigma$ (number of "successes") (probability of that number of successes) for all possible results.

For the binomial distribution for the probability of r "successes" in n trails is given by.

$$P_r[R = r] = {}_nC_r (1-p)^{n-r} p^r$$

Then $$\mu = \sum_{r=0}^{n} (r) P_r[R = r] = \left[\sum_{r=0}^{n} (r)({}_nC_r)^{(n-r)} p^r\right.$$

If the algebra is followed through the result is

$$\mu = np \quad — (1)$$

Thus, the mean value of the binomial distribution is the product of the number of trials and the probability of "success" in a single trial. This seems to be intuitively correct.

For any discrete probability distribution,

$$\sigma^2 = E(r-\mu)^2 = \sum_{r=0}^{n} (r-\mu)^2 \, Pr[R=r]$$

Substituting for the probability for the binomial distribution and following through the algebra gives.

$$\sigma^2 = np(1-p)$$

or

$$\sigma^2 = npq \quad \text{—— (2)}$$

The standard deviation is always given by the square root of the corresponding variance, so the standard deviation for the binomial distribution is

$$\sigma = \sqrt{npq} \quad \text{—— (3)}$$

* For discrete distribution (Poisson distribution).

Mean and Variance for the Poisson Distribution.

Since the Poisson distribution is discrete, the mean and variance can be found from the previous general relation.

$$\mu = E(R) = \sum_{\text{all } r} (r)(Pr[R=r])$$

For any discrete probability distribution,

$$\sigma^2 = E(r-\mu)^2 = \sum_{r=0}^{n} (r-\mu)^2 \, Pr[R=r]$$

Substituting for the probability for the binomial distribution and following through the algebra gives.

$$\sigma^2 = np(1-p)$$

or

$$\sigma^2 = npq \quad \text{———} \quad (2)$$

The standard deviation is always given by the square root of the corresponding variance, so the standard deviation for the binomial distribution is

$$\sigma = \sqrt{npq} \quad \text{———} \quad (3)$$

(c) Importance of Weibull and Normal distribution in reliability - draw typical graphs showing failure density function, cummulative failure distribution and hazard rate for this distribution.

Ans:. The Weibull and normal distribution has one very important property; the distribution has no specific characteristic shape. Depending upon the values of the parameter in to its reliability function, it can be shaped to represent many distribution as well as. shaped to fit set of experiment data that cannot be characterized as a particular distribution other than as a Weibull distribution with certain shaping parameters.

For this reason the Weibull distribution has a very important role to play in the statistical analysis of experimental data. The great adaptability of the Weibull distribution can be seen by - Weibull reliability functions.

(a) Failure reliability function.

The failure density function of the Weibull distribution is defined as -

$$f(t) = \frac{\beta t^{\beta-1}}{\alpha^{\beta}} \exp\left[-\left(\frac{t}{\alpha}\right)^{\beta}\right] \quad \text{—— (i)}$$

where $t \geq 0$, $B > 0$ and $\alpha > 0$

(b) Cummulative failure distribution.

It is defined as -

$$Q(t) = 1 - R(t)$$

$$Q(t) = 1 - \exp\left[-\left(\frac{t}{\alpha}\right)^{\beta}\right] \quad \text{—— } ②$$

(c) Hazard rate

It is defined as -

$$\lambda(t) = f(t)/R(t)$$

$$\lambda(t) = \frac{B\, t^{B-1}}{\alpha^{B}} \quad \text{—— } ③$$

Two particular cases that can be deduced from the Weibull distribution; $\beta = 1$ and $\beta = 2$

$\beta = 1$

$$(1) \Rightarrow f(t) = \frac{1}{\alpha} \exp\left[-\frac{t}{\alpha}\right] \quad \text{— (iv)}$$

$$(iii) \Rightarrow \lambda(t) = \frac{1}{\alpha} \quad \text{— (v)}$$

(iv) and (v) are identical to those for exponential distribution if $\alpha = \frac{1}{\lambda}$. i.e, the value of α represent the mean time to failure (MTTF).

$\beta = 2$

$$(I) \Rightarrow f(t) = \frac{2t}{\alpha^2} \exp\left[-\frac{t^2}{\alpha^2}\right] \quad \text{— (VI)}$$

$$(iii) \Rightarrow \lambda(t) = \frac{2t}{\alpha^2} \quad \text{— (VII)}$$

(VI) and (VII) are identical to those for the Rayleigh distribution.

It concludes that Weibull distribution can be made to fit, or approximate to, a number of distributions. It can be scaled and shaped varying its shaping parameter, show in figures

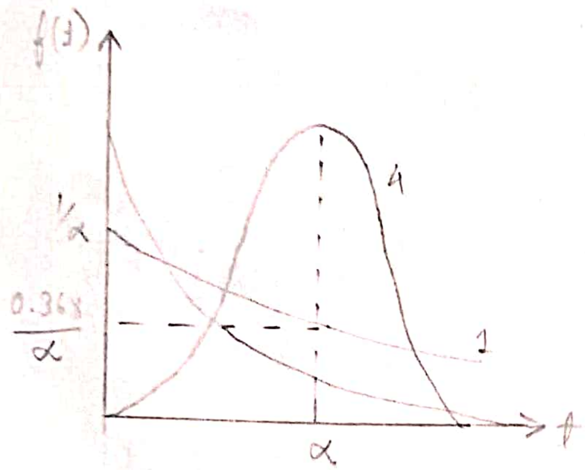

fig (a) Failure density function.

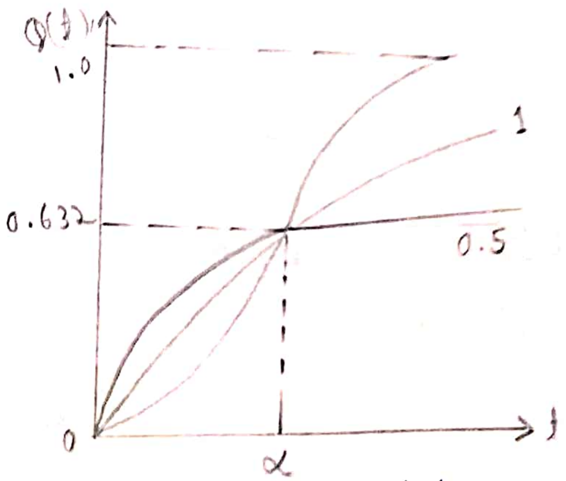

fig (b) Cumulative failure distribution.

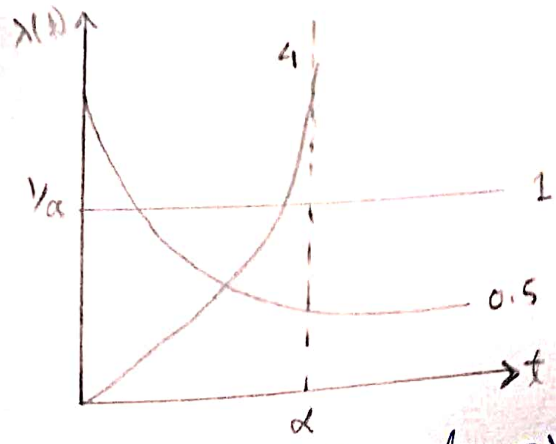

fig.(c) Hazard rate, Parameters = values of β.

(3) For a total number of tools produced in a certain process, 5 % of the tools are defective. Find the probability in a sample of 40 tools choosen at random, exactly three will be defective. Calculate using (a) Binomial distribution and (b) Poisson distribution as an approximation.

Solⁿ:

(a) For the binomial distribution with $n = 40$, $p = 0.05$,

$$Pr[R = 3] = {}_{40}C_3 (0.05)^3 (0.95)^{37}$$

$$= \frac{(40)(39)(38)}{(3)(2)(1)} (0.05)^3 (0.95)^{37}$$

$$= 0.185$$

(b) For the Poisson distribution, $\mu = (n)(p) = (40)(0.05)$
$= 2.00$

$$Pr[R = 3] = \frac{(2.00)^3 e^{-2.00}}{(3)(2)(1)} = 0.180.$$