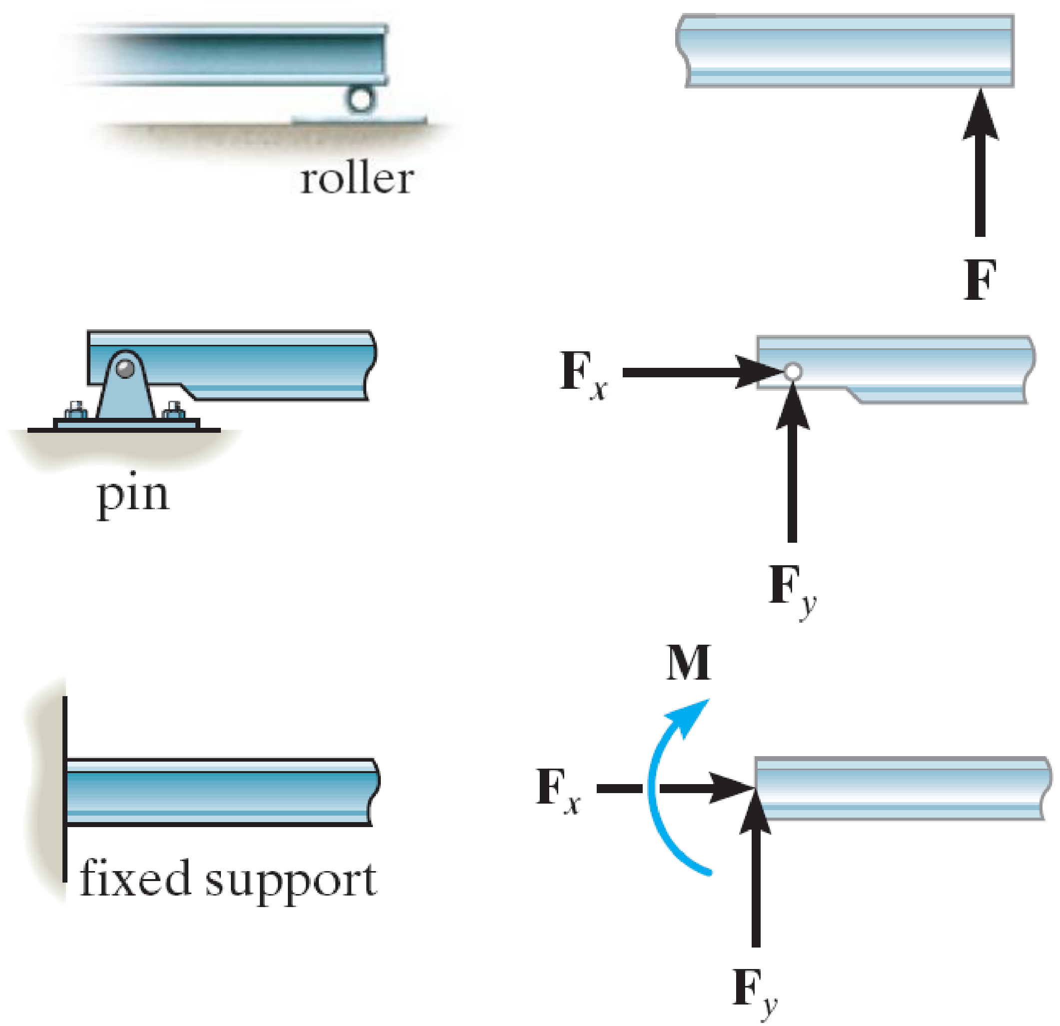
Rigid Body Equilibrium

Support Reactions

Prevention of Translation or Rotation of a body

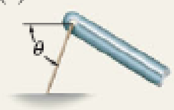
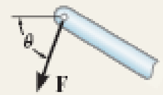
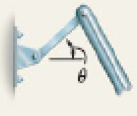
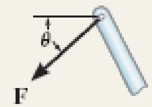
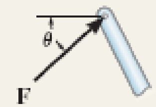
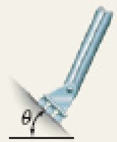
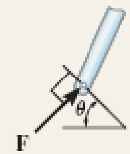
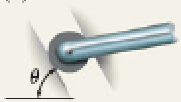
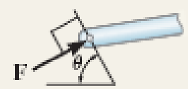
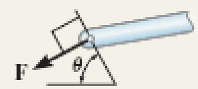
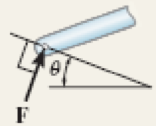
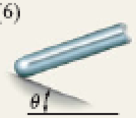
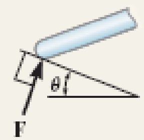
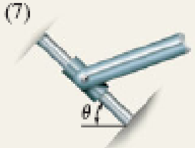
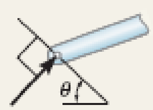
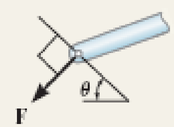
Restraints



Rigid Body Equilibrium

Various Supports

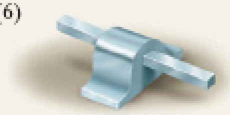
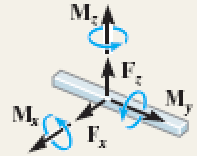
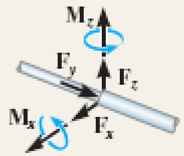
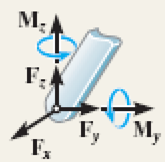
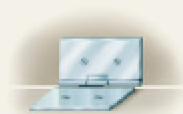
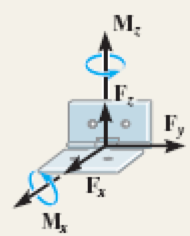
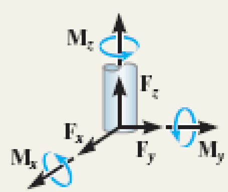
2-D Force Systems

Types of Connection	Reaction	Number of Unknowns
(1)  cable		One unknown. The reaction is a tension force which acts away from the member in the direction of the cable.
(2)  weightless link	 or 	One unknown. The reaction is a force which acts along the axis of the link.
(3)  roller		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.
(4)  roller or pin in confined smooth slot	 or 	One unknown. The reaction is a force which acts perpendicular to the slot.
(5)  rocker		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.
(6)  smooth contacting surface		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.
(7)  member pin connected to collar on smooth rod	 or 	One unknown. The reaction is a force which acts perpendicular to the rod.

Rigid Body Equilibrium

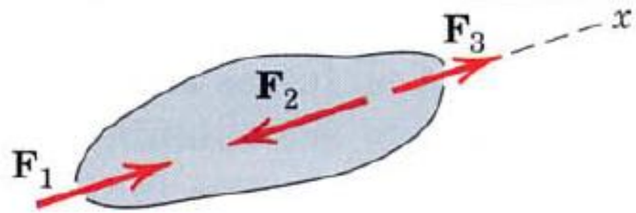
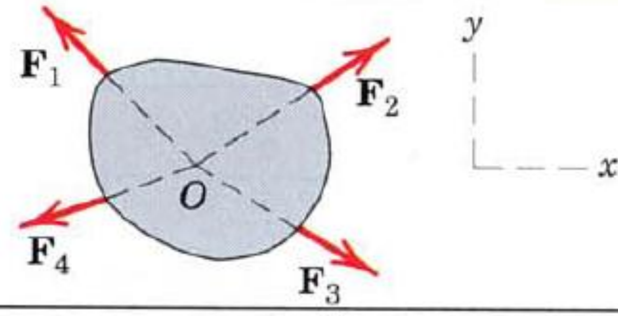
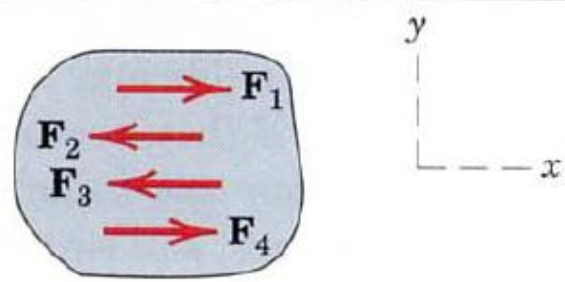
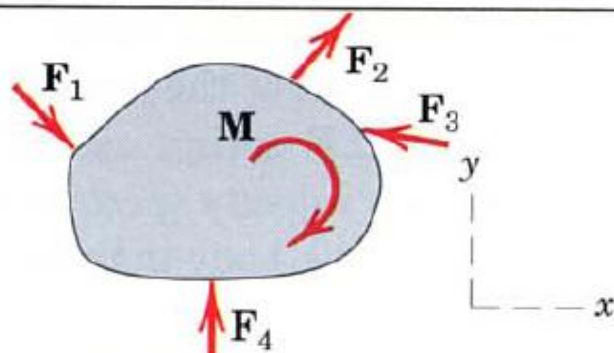
Various Supports

3-D Force Systems

Types of Connection	Reaction	Number of Unknowns
(6)  single journal bearing with square shaft		Five unknowns. The reactions are two force and three couple-moment components. *Note*: The couple moments are generally not applied if the body is supported elsewhere. See the examples.
(7)  single thrust bearing		Five unknowns. The reactions are three force and two couple-moment components. *Note*: The couple moments are generally not applied if the body is supported elsewhere. See the examples.
(8)  single smooth pin		Five unknowns. The reactions are three force and two couple-moment components. *Note*: The couple moments are generally not applied if the body is supported elsewhere. See the examples.
(9)  single hinge		Five unknowns. The reactions are three force and two couple-moment components. *Note*: The couple moments are generally not applied if the body is supported elsewhere. See the examples.
(10)  fixed support		Six unknowns. The reactions are three force and three couple-moment components.

Rigid Body Equilibrium

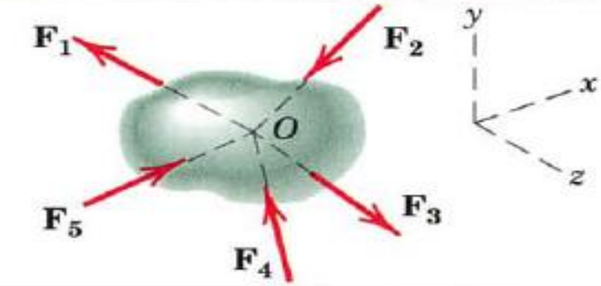
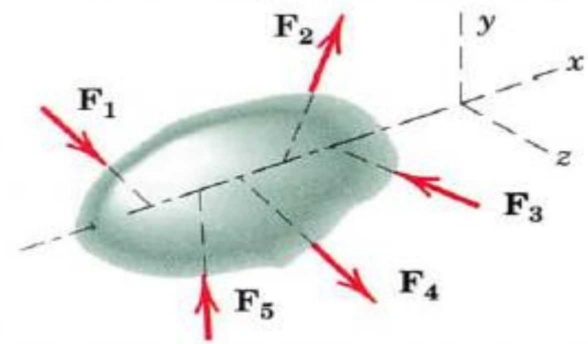
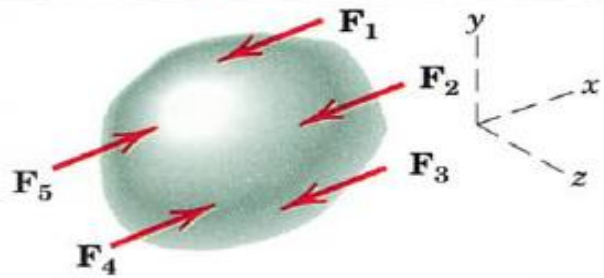
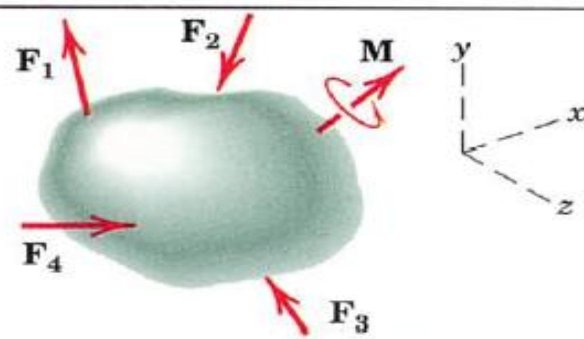
Categories in 2-D

CATEGORIES OF EQUILIBRIUM IN TWO DIMENSIONS		
Force System	**Free-Body Diagram**	**Independent Equations**
1. Collinear		$\Sigma F_x = 0$
2. Concurrent at a point		$\Sigma F_x = 0$ $\Sigma F_y = 0$
3. Parallel		$\Sigma F_x = 0$ $\Sigma M_z = 0$
4. General		$\Sigma F_x = 0$ $\Sigma M_z = 0$ $\Sigma F_y = 0$

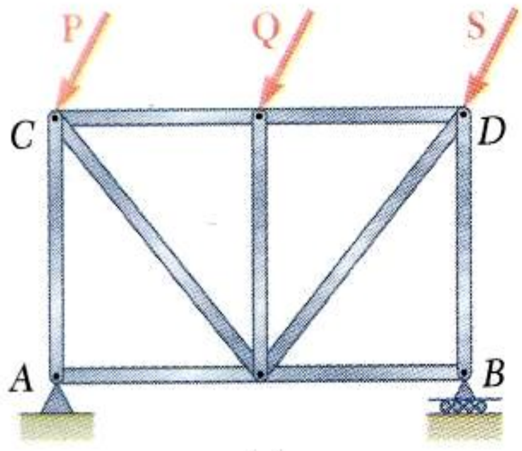
Rigid Body Equilibrium

Categories

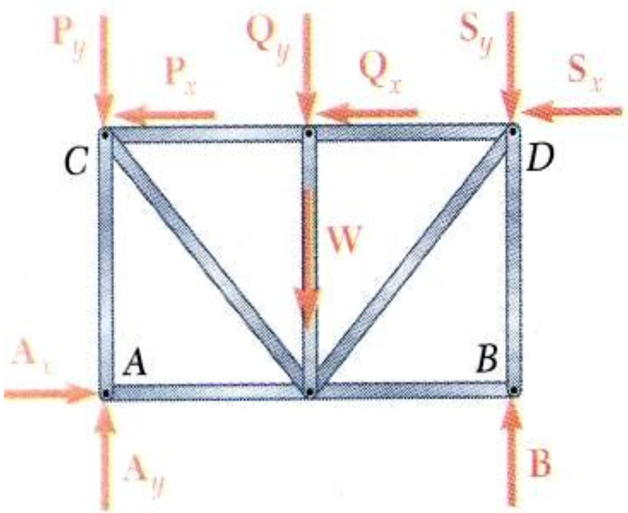
in 3-D

CATEGORIES OF EQUILIBRIUM IN THREE DIMENSIONS		
Force System	**Free-Body Diagram**	**Independent Equations**
1. Concurrent at a point		$\Sigma F_x = 0$ $\Sigma F_y = 0$ $\Sigma F_z = 0$
2. Concurrent with a line		$\Sigma F_x = 0$ $\Sigma M_y = 0$ $\Sigma F_y = 0$ $\Sigma M_z = 0$ $\Sigma F_z = 0$
3. Parallel		$\Sigma F_x = 0$ $\Sigma M_y = 0$ $\Sigma M_z = 0$
4. General		$\Sigma F_x = 0$ $\Sigma M_x = 0$ $\Sigma F_y = 0$ $\Sigma M_y = 0$ $\Sigma F_z = 0$ $\Sigma M_z = 0$

Equilibrium of a Rigid Body in Two Dimensions



(a)



(b)

- For all forces and moments acting on a two-dimensional structure,

$$F_z = 0 \quad M_x = M_y = 0 \quad M_z = M_O$$

- Equations of equilibrium become

$$\sum F_x = 0 \quad \sum F_y = 0 \quad \sum M_A = 0$$

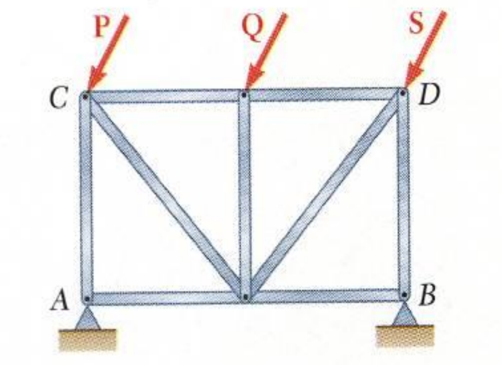
where A is any point in the plane of the structure.

- The 3 equations can be solved for no more than 3 unknowns.

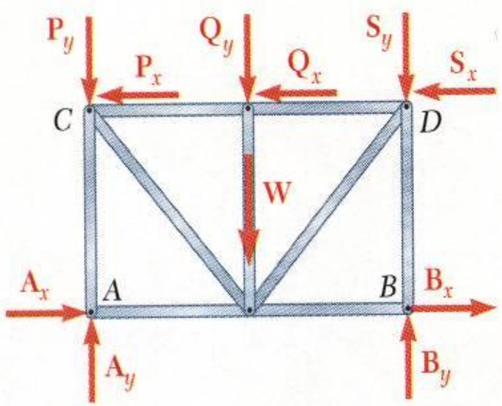
- The 3 equations can not be augmented with additional equations, but they can be replaced

$$\sum F_x = 0 \quad \sum M_A = 0 \quad \sum M_B = 0$$

Statically Indeterminate Reactions

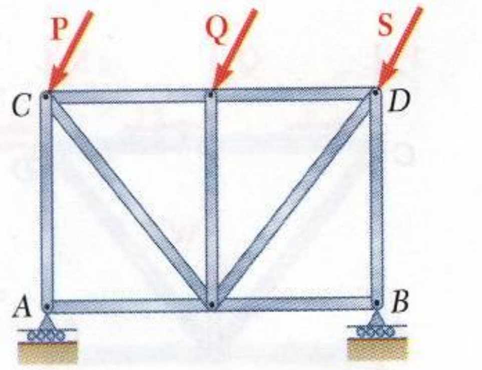


(a)

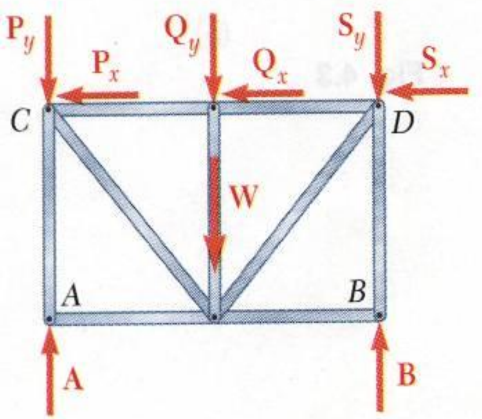


(b)

- More unknowns than equations: Statically Indeterminate

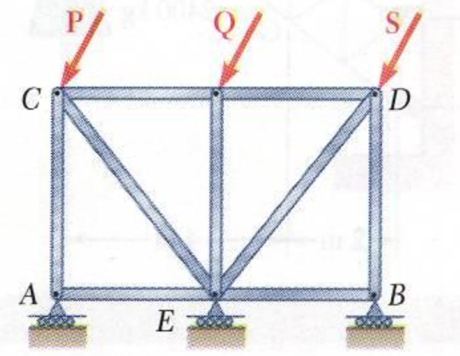


(a)

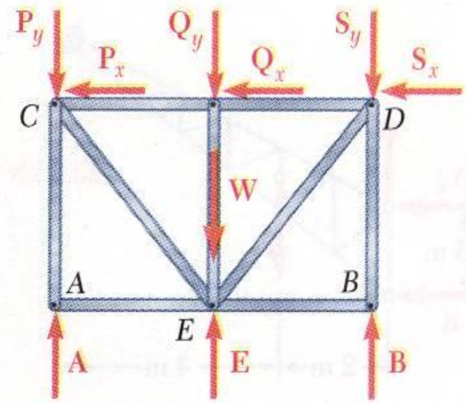


(b)

- Fewer unknowns than equations, partially constrained



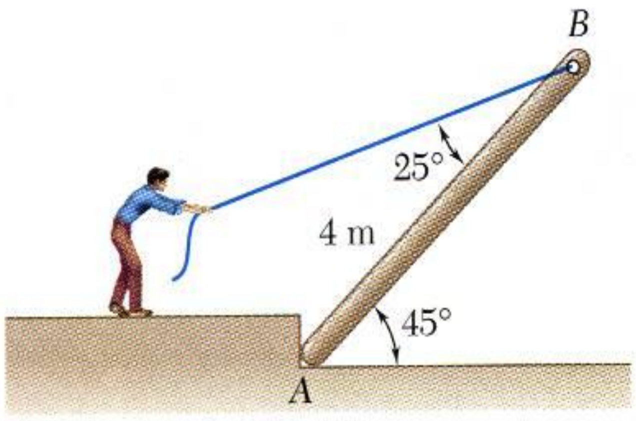
(a)



(b)

- Equal number unknowns and equations but improperly constrained

Rigid Body Equilibrium: Example



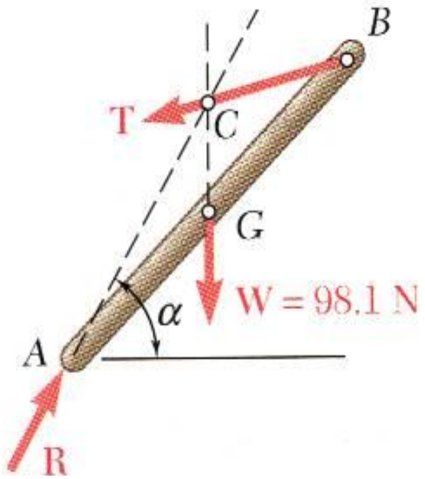
A man raises a 10 kg joist, of length 4 m, by pulling on a rope.

Find the **tension in the rope** and the **reaction at A**.

Solution:

- Create a **free-body diagram** of the joist.
  - The joist is a **3 force body** acted upon by the **rope**, its **weight**, and the **reaction at A**.
- The **three forces** must be **concurrent** for **static equilibrium**.
  - Reaction **R** must pass through the intersection of the lines of action of the weight and rope forces.
  - Determine the direction of the reaction force **R**.
- Utilize a **force triangle** to determine the magnitude of the reaction force **R**.

Rigid Body Equilibrium: Example



- Create a **free-body diagram** of the joist
- Determine the direction of the reaction force **R**

$$AF = AB\cos 45 = (4\,\text{m})\cos 45 = 2.828\,\text{m}$$

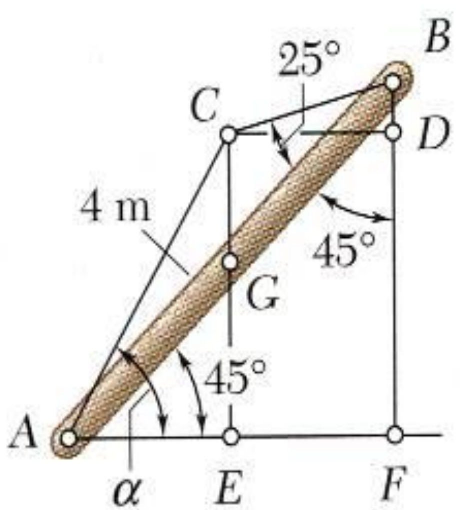
$$CD = AE = \tfrac{1}{2}AF = 1.414\,\text{m}$$

$$BD = CD\cot(45+20) = (1.414\,\text{m})\tan 20 = 0.515\,\text{m}$$

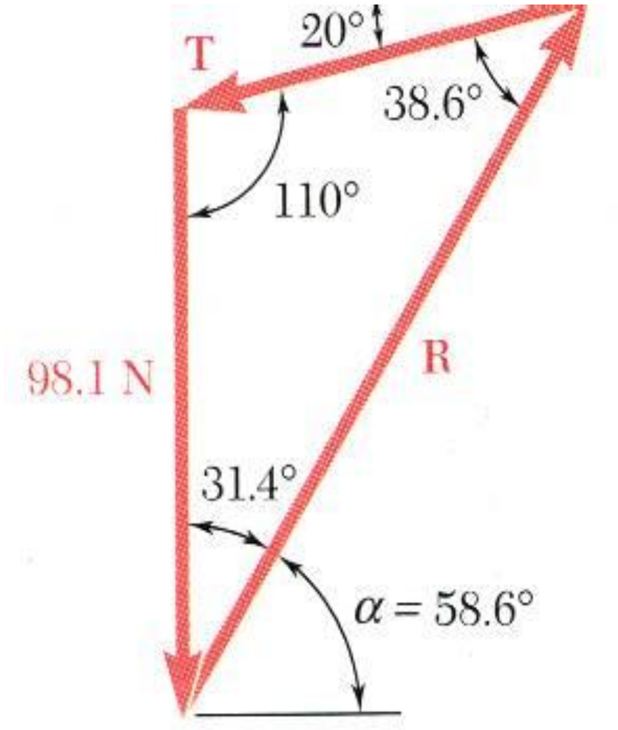
$$CE = BF - BD = (2.828 - 0.515)\,\text{m} = 2.313\,\text{m}$$

$$\tan\alpha = \frac{CE}{AE} = \frac{2.313}{1.414} = 1.636$$

$$\boxed{\alpha = 58.6^\circ}$$



Rigid Body Equilibrium: Example



- Determine the magnitude of the reaction force **R**.

$$\frac{T}{\sin 31.4^\circ} = \frac{R}{\sin 110^\circ} = \frac{98.1\,\text{N}}{\sin 38.6^\circ}$$

$$T = 81.9\,\text{N}$$

$$R = 147.8\,\text{N}$$

Engineering Structure

- Any **connected system of members** to **transfer** the **loads** and **safely withstand** them

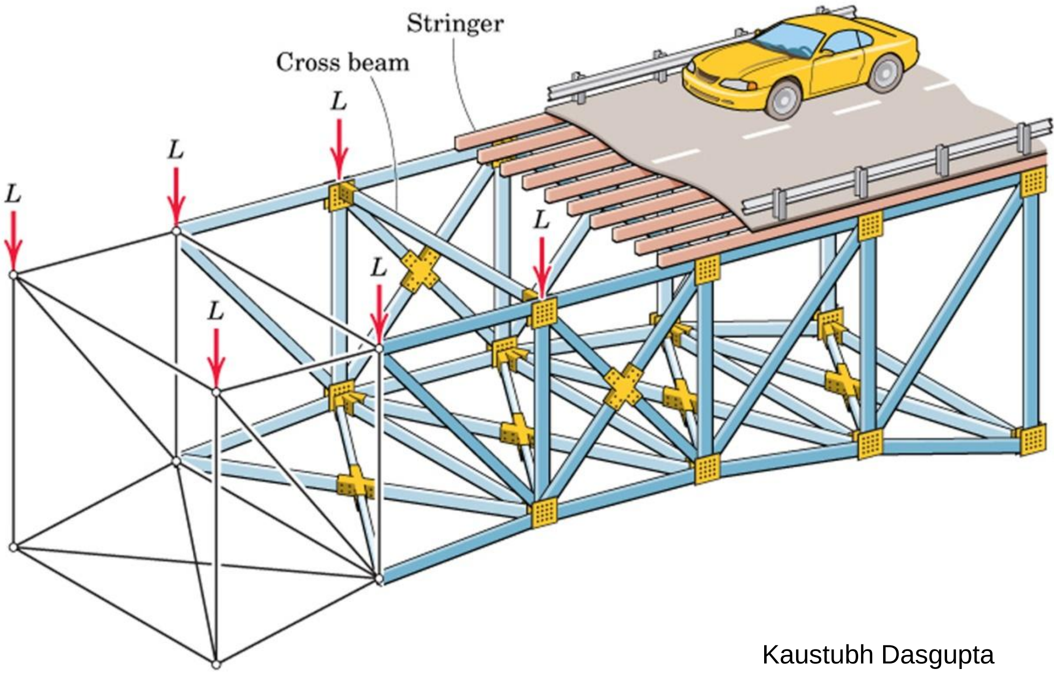
Structural Analysis

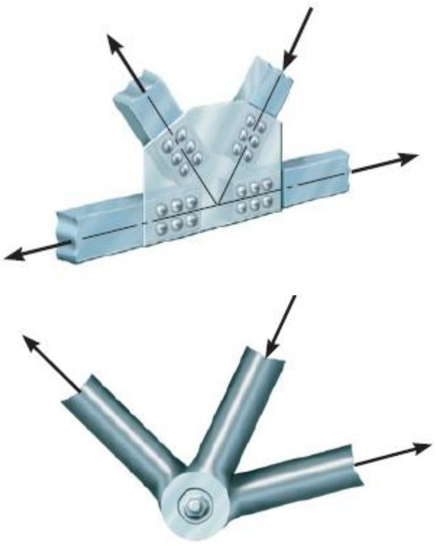
Structural Analysis

ME101→ Trusses/Frames/Machines/Beams/Cables

→ Statically Determinate Structures

To determine the internal forces in the structure, dismember the structure and analyze separate free body diagrams of individual members or combination of members.





Structural Analysis :: Plane Truss

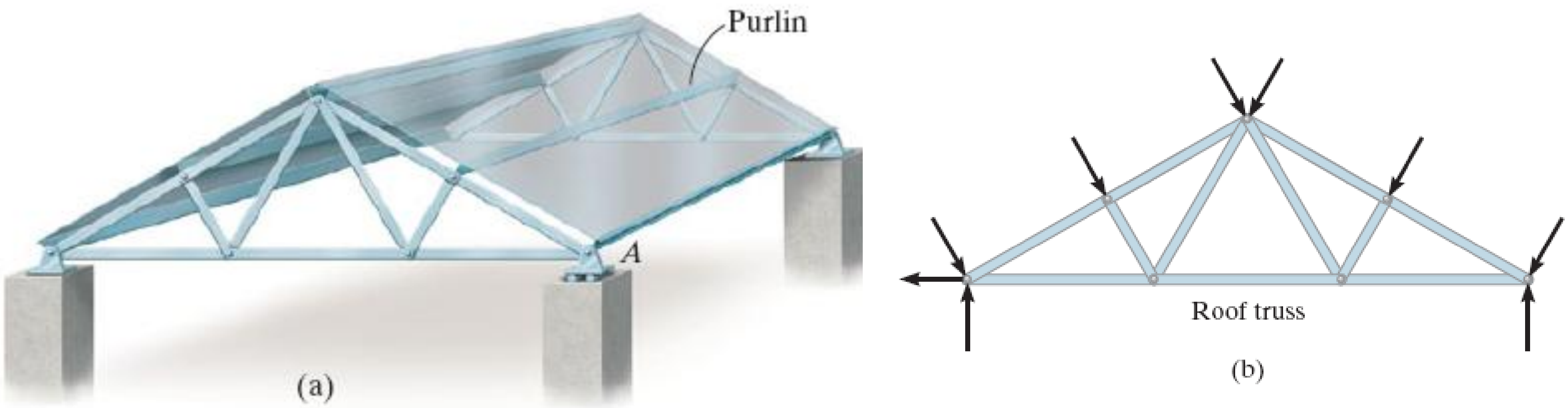
Structural Analysis :: Plane Truss

Structural Analysis: Plane Truss

Truss: A framework composed of members joined at their ends to form a rigid structure

– Joints (Connections): Welded, Riveted, Bolted, Pinned

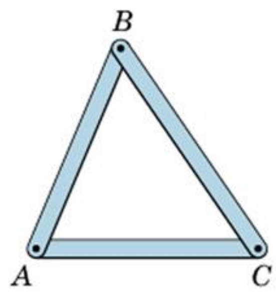
Plane Truss: Members lie in a single plane



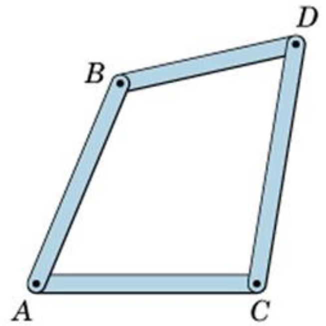
Structural Analysis: Plane Truss

Simple Trusses

Basic Element of a Plane Truss is the Triangle



- Three bars joined by pins at their ends → Rigid Frame
  - Non-collapsible and deformation of members due to induced internal strains is negligible

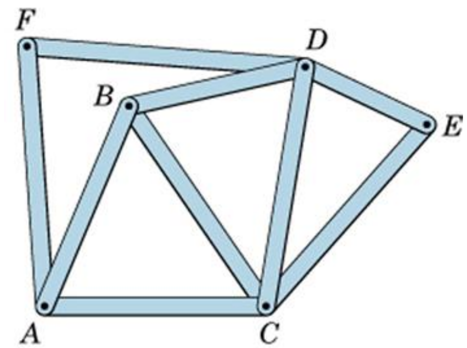


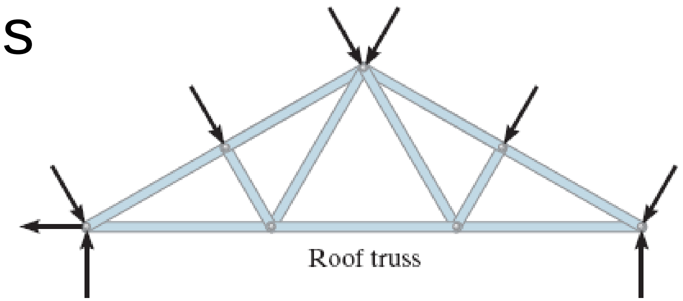
- Four or more bars polygon → Non-Rigid Frame

  How to make it rigid or stable?

  by forming more triangles!

Structures built from basic triangles →Simple Trusses

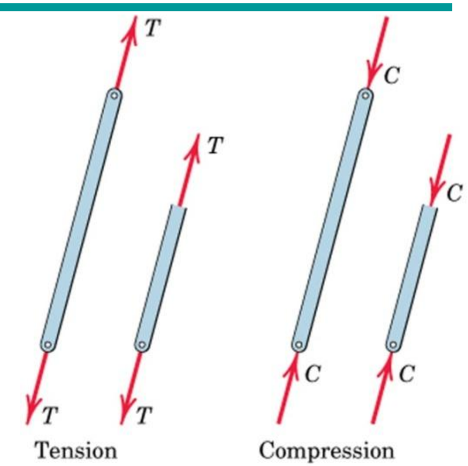




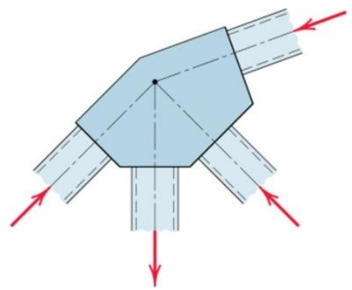
Structural Analysis: Plane Truss

Basic Assumptions in Truss Analysis

- All members are two-force members
- Weight of the members is small compared with the force it supports (weight may be considered at joints).
  - no effect of bending on members even if weight is considered
- External forces are applied at the pin connections
- Welded or riveted connections → Pin Joint if the member centerlines are concurrent at the joint

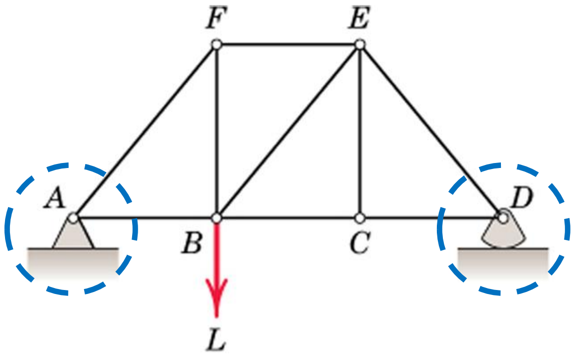


Two-Force Members



Common Practice in Large Trusses

- Roller/Rocker at one end. Why?
  - to accommodate deformations due to temperature changes and applied loads.
  - otherwise it will be a statically indeterminate truss



Structural Analysis: Plane Truss

Truss Analysis: Method of Joints

- Finding forces in members

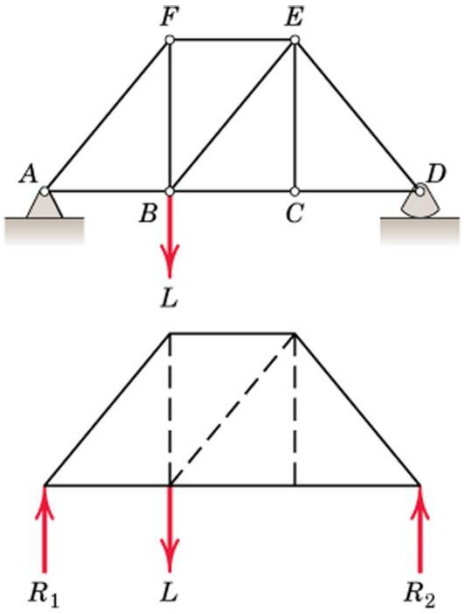
Method of Joints: Conditions of equilibrium are satisfied for the forces at each joint

– Equilibrium of concurrent forces at each joint

– only two independent equilibrium equations are involved

Steps of Analysis

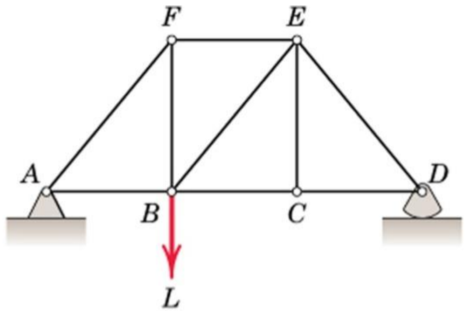
1. Draw Free Body Diagram of Truss
2. Determine external reactions by applying equilibrium equations to the whole truss
3. Perform the force analysis of the remainder of the truss by Method of Joints

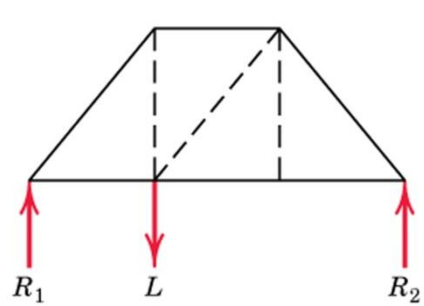


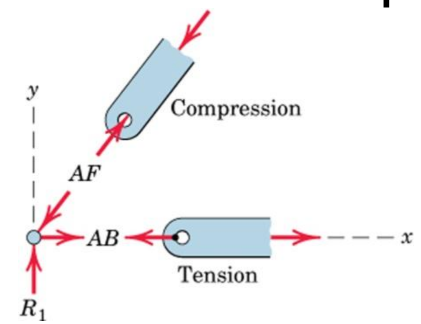
Structural Analysis: Plane Truss

Method of Joints

- Start with any joint where at least one known load exists and where not more than two unknown forces are present.







FBD of Joint A and members AB and AF: Magnitude of forces denoted as *AB* & *AF*

- Tension indicated by an arrow away from the pin
- Compression indicated by an arrow toward the pin

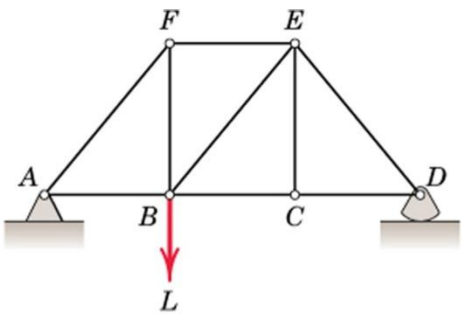
Magnitude of AF from $\Sigma F_y = 0$

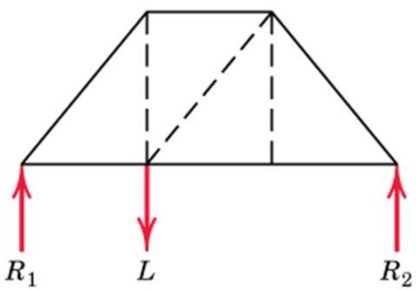
Magnitude of AB from $\Sigma F_x = 0$

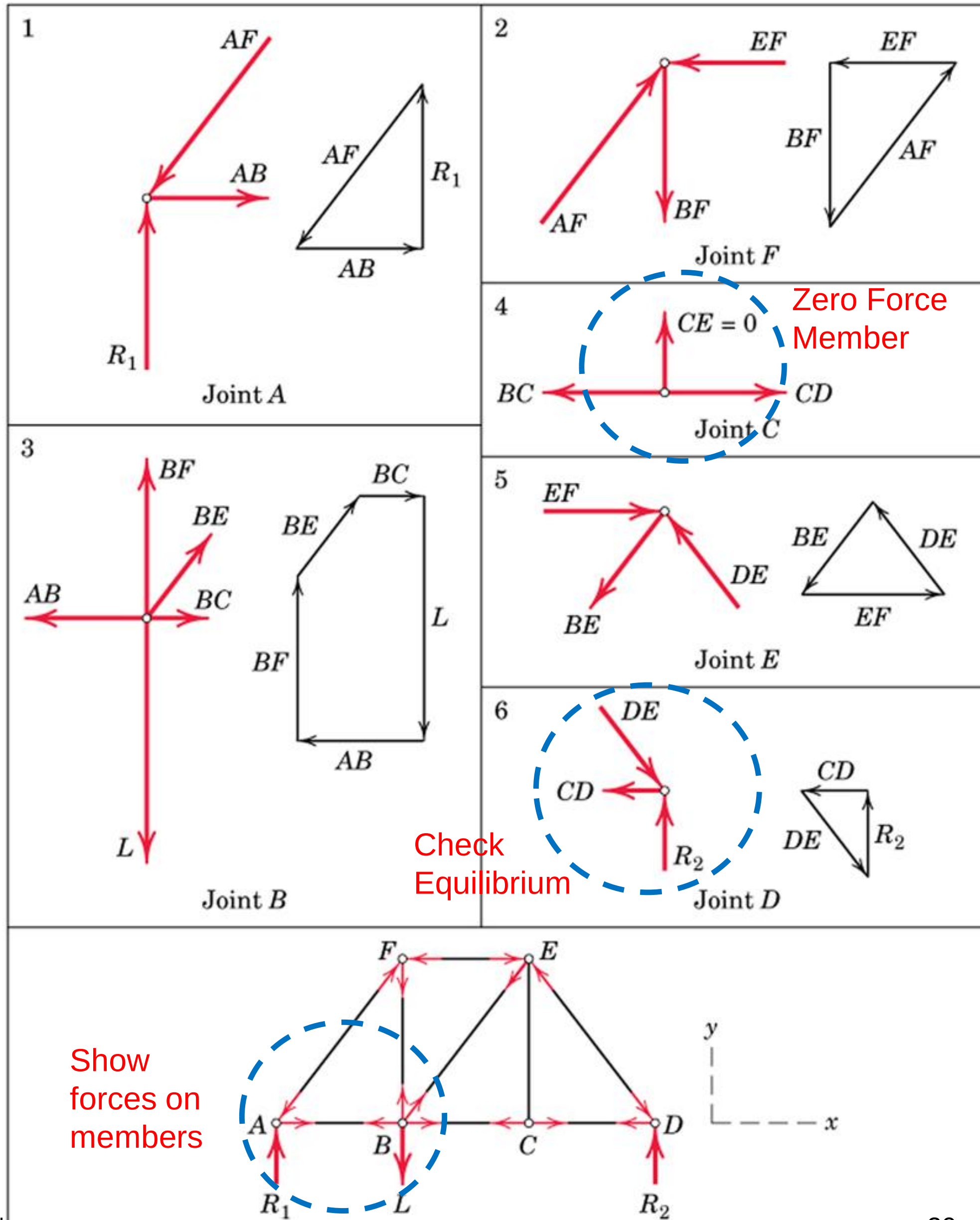
Analyze joints F, B, C, E, & D in that order to complete the analysis

Structural Analysis: Plane Truss

Method of Joints



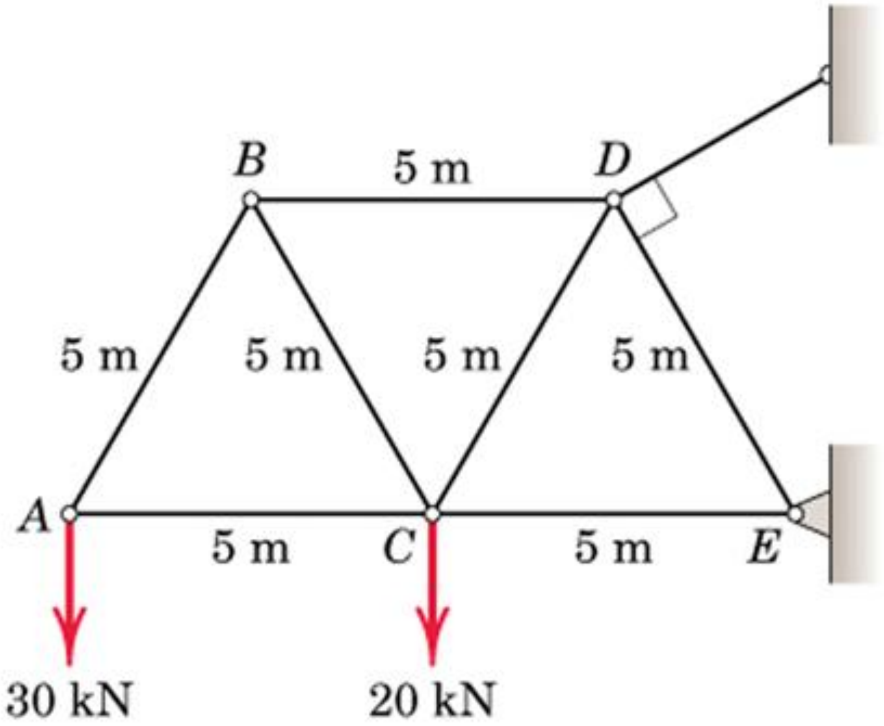




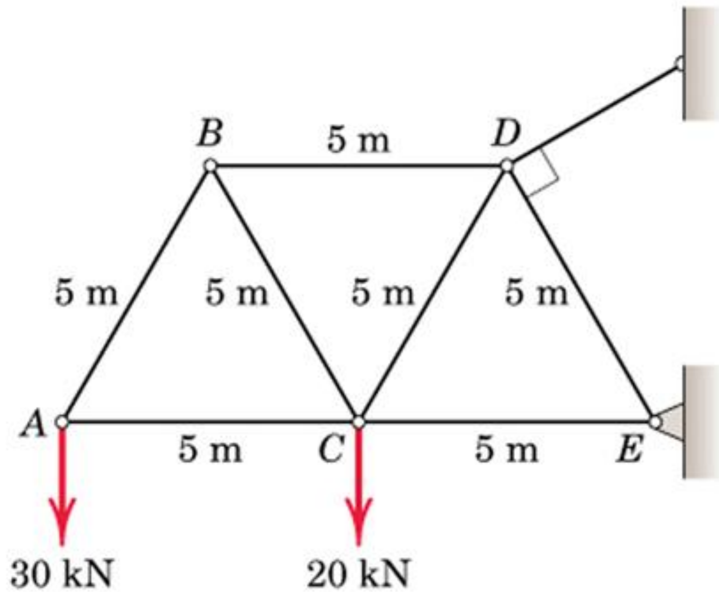
- Negative force if assumed sense is incorrect

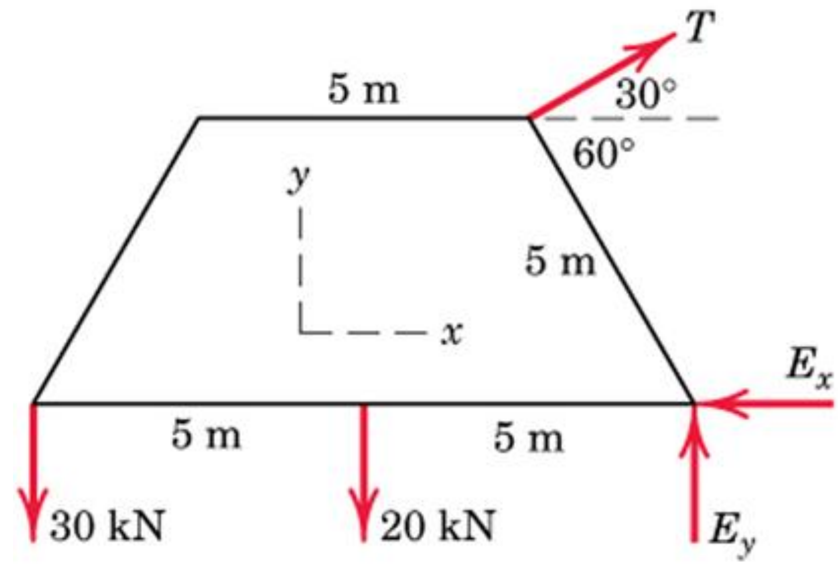
Method of Joints: Example

Determine the force in each member of the loaded truss by the **method of joints**



Method of Joints: Example

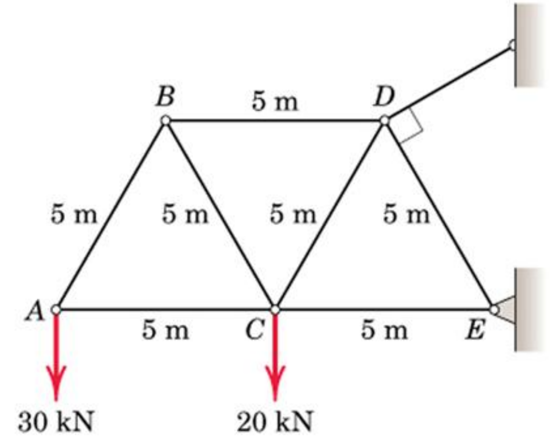




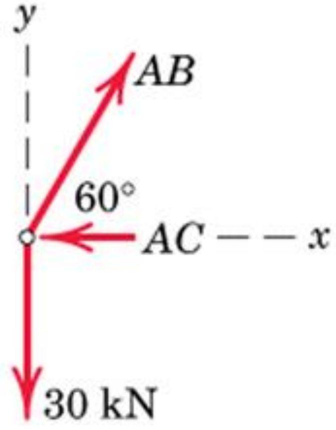
Free Body Diagram

$[\Sigma M_E = 0]$	$5T - 20(5) - 30(10) = 0$	$T = 80 \text{ kN}$
$[\Sigma F_x = 0]$	$80 \cos 30° - E_x = 0$	$E_x = 69.3 \text{ kN}$
$[\Sigma F_y = 0]$	$80 \sin 30° + E_y - 20 - 30 = 0$	$E_y = 10 \text{ kN}$

Method of Joints: Example



- Joint A



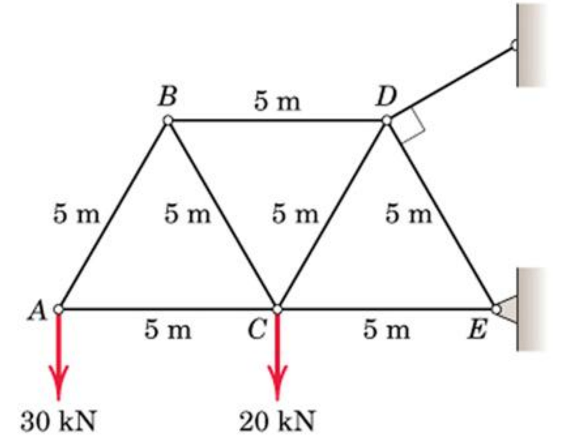
Joint A

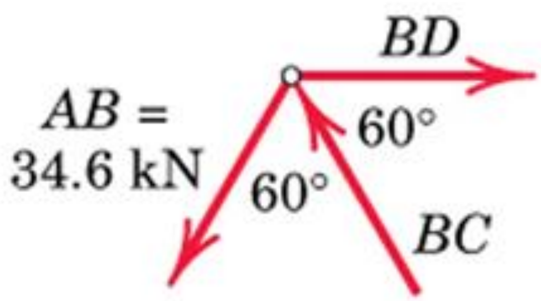
$[\Sigma F_y = 0]$ $0.866AB - 30 = 0$ $AB = 34.6$ kN T

$[\Sigma F_x = 0]$ $AC - 0.5(34.6) = 0$ $AC = 17.32$ kN C

Method of Joints: Example

- Joint B





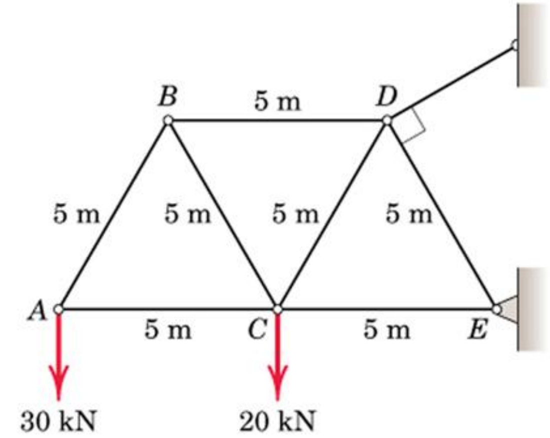
Joint B

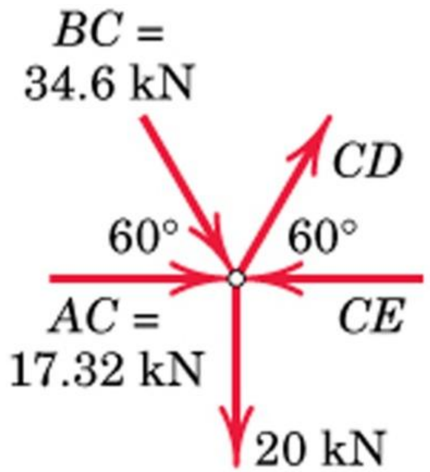
$$[\Sigma F_y = 0] \qquad 0.866BC - 0.866(34.6) = 0 \qquad BC = 34.6 \text{ kN } C$$

$$[\Sigma F_x = 0] \qquad BD - 2(0.5)(34.6) = 0 \qquad BD = 34.6 \text{ kN } T$$

Method of Joints: Example

- Joint C





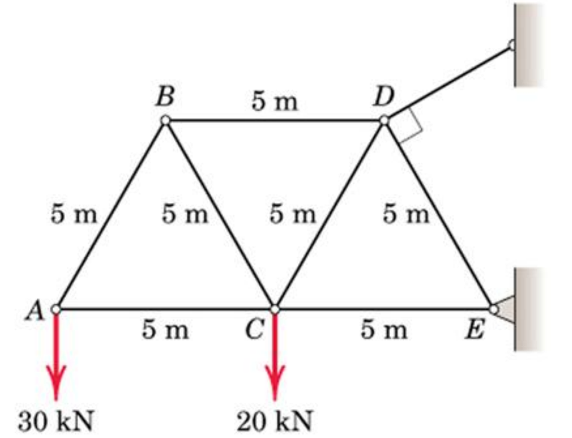
$[\Sigma F_y = 0]$ $\quad 0.866CD - 0.866(34.6) - 20 = 0$

$CD = 57.7 \text{ kN } T$

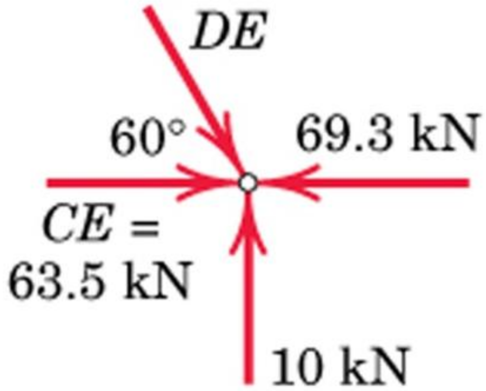
$[\Sigma F_x = 0]$ $\quad CE - 17.32 - 0.5(34.6) - 0.5(57.7) = 0$

$CE = 63.5 \text{ kN } C$

Method of Joints: Example



- Joint E



Joint E

$[\Sigma F_y = 0] \qquad 0.866DE = 10 \qquad DE = 11.55 \text{ kN } C$

and the equation $\Sigma F_x = 0$ checks.