

When, ①, ②, ③ becomes:-

$$u^* = \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*} = \frac{\partial p^*}{\partial x^*} + \frac{1}{Re} \left[\frac{\partial^2 u^*}{\partial x^{*2}} + \frac{\partial^2 u^*}{\partial y^{*2}} \right] \rightarrow \textcircled{4}$$

$$u^* \frac{\partial v^*}{\partial x^*} + v^* \frac{\partial v^*}{\partial y^*} = \frac{\partial p^*}{\partial y^*} + \frac{1}{Re} \left[\frac{\partial^2 v^*}{\partial x^{*2}} + \frac{\partial^2 v^*}{\partial y^{*2}} \right] \rightarrow \textcircled{5}$$

$$\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} = 0 \quad \text{where } Re = \frac{\rho U_\infty L}{\mu} \rightarrow \textcircled{6}$$

By the order of magnitude analysis, $\frac{\partial^2 u^*}{\partial x^{*2}}$ can be dropped (smallest order)

$$\text{So, } \left\{ u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*} = -\frac{\partial p^*}{\partial x^*} + \frac{1}{Re} \frac{\partial^2 u^*}{\partial y^{*2}} \right\} \rightarrow \textcircled{6}$$

$$\frac{\partial p^*}{\partial y^*} = 0$$

$$\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} = 0$$

→ these are Prandtl's boundary layer equation

Boundary conditions are - Solid surface at $y^* = 0, u^* = 0 = v^*$
or at $y = 0, u = 0 = v$

= and outer edge of boundary layer -

$$\left. \begin{aligned} \text{at } y^* = (\epsilon) = \frac{\delta}{L}, u^* = 1 \\ \text{is at } y = \delta, u = U(x) \end{aligned} \right\} \rightarrow \textcircled{7}$$

At the outer edge of ~~both~~ boundary layer

$$u^* = \frac{du^*}{dx^*} = -\frac{dp^*}{dx^*}$$

or in dimensional form -

$$U \frac{du}{dx} = -\frac{1}{\rho} \frac{dp}{dx}$$

$$\text{On inner edge - } \frac{1}{2} \rho u^2 + p = \text{Constant}$$

Blasius Flow over Flat Plate

- The classical problem considered by H. Blasius was
1. Two dimensional, steady, incompressible flow over a flat plate at zero angle of incidence with respect to the uniform stream of velocity U_0 .
 2. The fluid extends to infinity in all directions from the plate.

- Blasius wanted to determine
 - (a) the velocity field solely within the boundary layer.
 - (b) the boundary layer thickness (δ).
 - (c) the shear stress distribution on the plate,
 - (d) the drag force on the plate.

• The Prandtl boundary layer equations in the case under consideration are

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} \rightarrow \textcircled{1}$$

$$\nu = \frac{\mu}{\rho}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

The boundary conditions are

$$\text{at } y=0, \quad u=v=0$$

$$\text{at } y=\infty, \quad u=U_0 \quad \} \rightarrow \textcircled{2}$$

Now, velocity at two arbitrary x locations, namely x_1 & x_2 should satisfy the equation,

$$\frac{u[x_1, \{y/g(x_1)\}]}{U(x_1)} = \frac{u[x_2, \{y/g(x_2)\}]}{U(x_2)} \rightarrow \textcircled{3}$$