Properties:

The probability of survival or reliability R (t) at time 't' has the following properties.

1. $0 \le R(t) \le 1$
2. $R(0) = 1$ and $R(\infty) = 0$

As special case, when the probability function is exponential and failure rate is constant. Then

$$Z(t) = \frac{f(t)}{R(t)} = \frac{\lambda e^{-\lambda t}}{e^{-\lambda t}}$$
$$= \lambda$$

When the failure rate is constant, hazard rate is also constant and is equal to the failure rate.

Mean Time To Failure (MTTF) :

The Mean Time To Failure will assume to be the same for all the components, which are identical in the design and operate under the conditions, which are identical.

MTTF is given by the mathematical expectation of the random variable 'T' describing the MTTF of the component.

Therefore, $$\text{MTTF} = E(T) = \int_0^\infty f(t)\,dt$$

This is the mean or expected value of the probability distribution defined by $F(t)$.

We have the probability density function (PDF)

$$F(t) = \frac{d.f(t)}{dt} = \frac{-d\,R(t)}{dt}$$

$$\text{MTTF} = \int_0^\infty \frac{-d\,R(t)}{dt} t.dt$$

Using integration by parts

$$\text{MTTF} = -t.R(t)\Big|_0^\infty + \int_0^\infty R(t)\,dt$$

$$= \int_0^\infty R(t)\,dt$$

Since, $$\lim_{x\to\infty} t.R(t) = \lim_{x\to\infty} t.\exp\left[-\int_0^t \lambda(t^1).dt^1 = 0\right]$$

and $R(0) = 0$.

Constant Hazard Model

The constant hazard model can be expressed as

$$Z(t) = \lambda$$

where λ is constant and independent of time.

An item with constant hazard rate will have the following reliability and associated functions.

$$f(t) = \lambda . e^{-\lambda t}$$

$$R(t) = e^{-\lambda t}$$

$$F(t) = 1 - e^{-\lambda t}$$

The mean time to failure of the item is

$$\text{MTTF} = \int_0^\infty e^{-\lambda t} dt = \frac{1}{\lambda}$$

RELIABILITY AND HAZARD FUNCTIONS FOR WELL KNOWN DISTRIBUTIONS

Exponential Distribution

Exponential distribution is widely used in reliability. A constant failure rate model for continuously operating system leads to an exponential distribution. Replacing a time dependent failure rate $\lambda(t)$ by a constant λ in the *pdf* equation.

We have

$$F(t) = \lambda . e^{-\lambda t}$$

Similarly, CDF becomes

$$F(t) = 1 - e^{-\lambda t}$$

and the reliability can be written as

$$R(t) = e^{-\lambda t}$$

$$\text{MTTF} = \frac{1}{\lambda}$$

The exponential failure density function is represented in below Figure

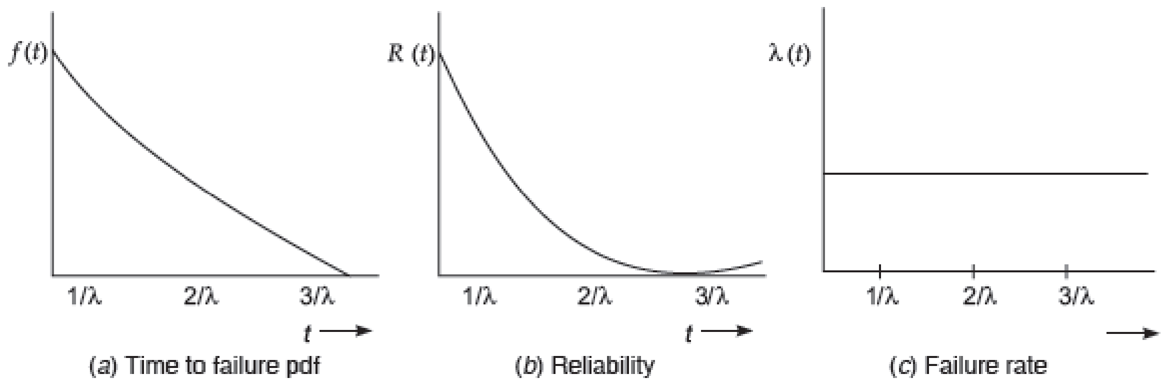

Fig: The exponential Distribution.

A device described by a constant failure rate and therefore by an exponential distribution of times to failure has the following property of "Memorylessness." The probability that it will fail during same period in the future is independent of its age.

Normal Distribution

The normal distribution takes the well-known bell-shape and to describe the time dependence of reliability problems. The pdf for normal distribution is given by the following equation with 't' as a random variable.

$$F(t) = \frac{1}{\sqrt{2\pi\sigma}} \exp\left[-\frac{(t-\mu)^2}{2\sigma^2}\right]$$

where μ is here is the MTTF.

$$F(t) = \int_{-\infty}^{t} \frac{1}{\sqrt{2\pi\sigma}} \exp\left[-\frac{(t-\mu)^2}{2\sigma^2}\right] dt$$

In the standardized normal form,

$$F(t) = \phi\left[\frac{t-\mu}{\sigma}\right]$$

and Reliability for normal distribution is given by

$$R(t) = 1 - \phi\left[\frac{t-\mu}{\sigma}\right]$$

and the failure rate is obtained by the equation,

$$\lambda(t) = \frac{1}{\sqrt{2\pi\sigma}} \exp\left[-\frac{1}{2}\left(\frac{t-\mu}{\sigma}\right)^2 \left(1-\phi\frac{t-\mu}{\sigma}\right)\right]^{-1}$$

The reliability and pdf are plotted for times to failure as shown in Fig

As indicated by the behaviour of failure rate, normal distributions are used to describe the reliability of the equipment to situations other than to which constant failure rates are applicable. It is useful in describing the reliability in situations in which there is a reasonably well-defined wear out time μ.

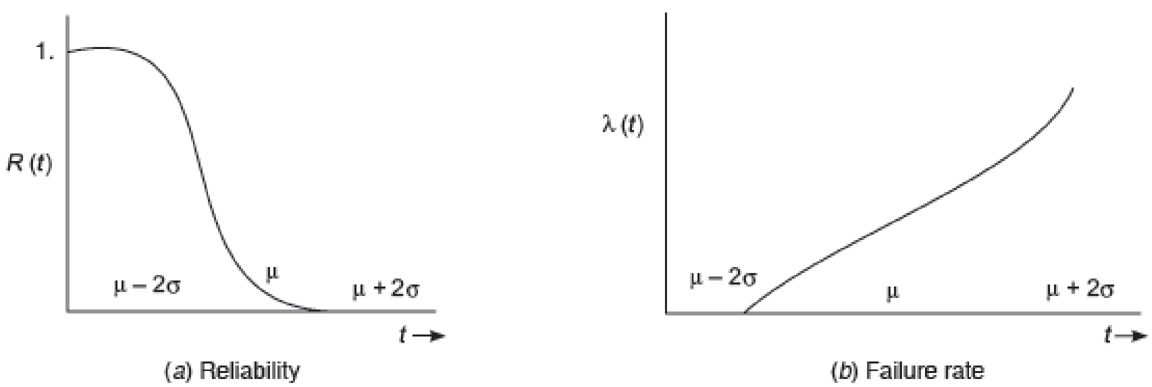

Fig. : Normal Distribution.

Log Normal Distributions:

The log normal is related distribution that has been found to be useful in describing failure distributions for a variety of situations especially when the time to failure is associated with large uncertainty. The pdf for the time to failure is given by

$$f(t) = \frac{1}{\sqrt{2\pi}St} \exp\left\{-\frac{1}{2S^2}\left[\ln\frac{t}{t_0}\right]^2\right\}$$

corresponding CDF is expressed as

$$F(t) = \phi\left[\frac{1}{S}\ln\frac{t}{t_0}\right]$$

however to is not the MTTF.

$$\text{MTTF} = \mu = \text{to} . \exp(S^2/2)$$

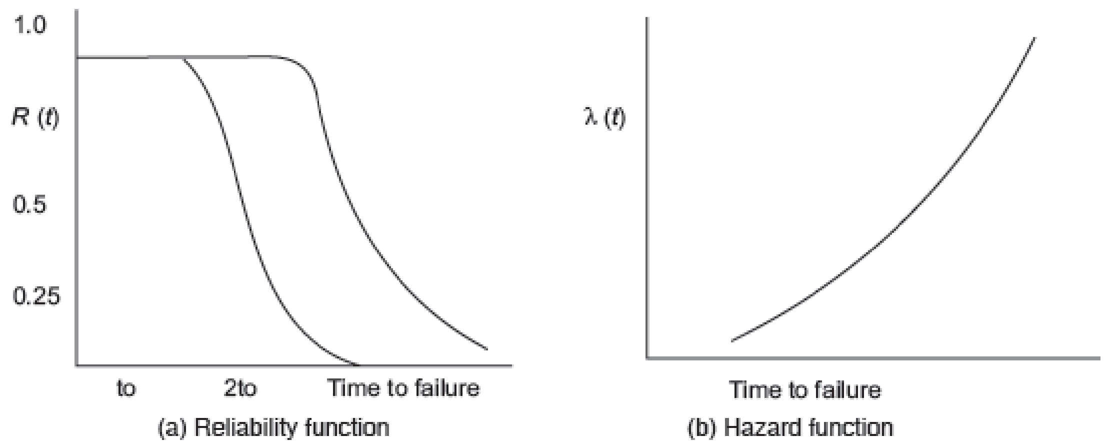

Fig: Log normal distribution

The log normal distribution is frequently used to describe fatigue and other phenomenon that are caused by ageing or wear and that result in failure rates that increase with time.

The log normal reliability function and hazard functions are shown in Fig. The failure can be increasing or decreasing depending on value of ‘S’

Weibull Distribution:

One of the most useful probability distribution in reality is the weibull. Weibull failure distribution may be used to model both increasing and decreasing failure rates. It is characterised by the hazard rate function of the form.

$$\lambda(t) = a \, . \, t^{b}$$

which is a power function.

The function $\lambda(t)$ is increasing for $a > 0$, $b > 0$ and is decreasing for $a > 0$, $b < 0$. For convenience, $\lambda(t)$ can be expressed as

$$\lambda(t) = \frac{\beta}{\theta}\left(\frac{t}{\theta}\right)^{\beta-1} \quad \theta > 0, \beta > 0, t \geq 0$$

$R(t)$ is expressed as

$$R(t) = \exp\left[-\int_0^t \frac{\beta}{\theta}\left(\frac{t}{\theta}\right)^{\beta-1} . dt\right]$$

and

$$f(t) = -\frac{dR(t)}{dt} = \frac{\beta}{\theta}\left[\frac{t}{\theta}\right]^{\beta-1} . e^{-(t/\theta)\beta}$$

Beta β is referred to as the shape parameter. Its effect on the distribution can be represented in the figure below for several different values β.

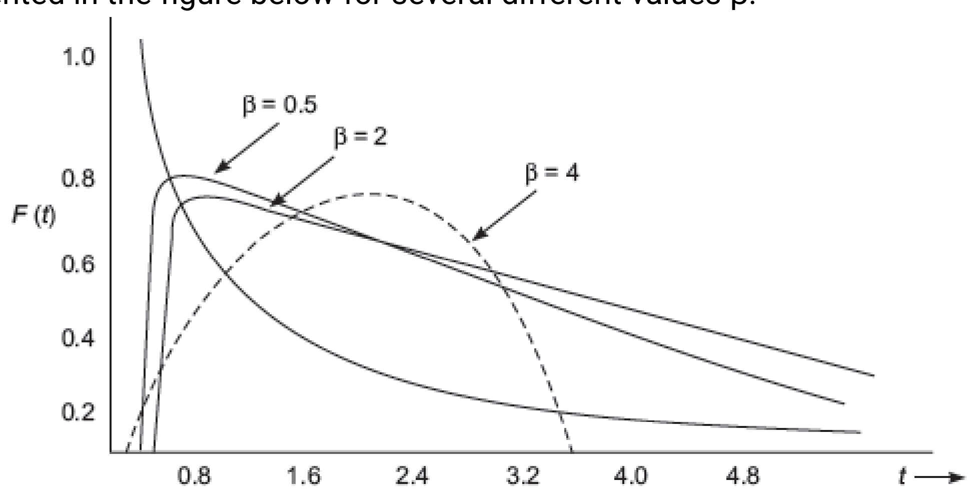

Fig: Effect of β on Weibull probability density function.

For $\beta < 1$, the pdf is similar in shape to the exponential and $\beta \geq 3$ [large values], the *pdf* is somewhat symmetrical like normal distribution.

For $1 < \beta < 3$. The density function is skewed. When $\beta = 1$, $\lambda(t)$ is constant and the distribution is identical to exponential.

Theta (θ) is a scale parameter that influences both the mean and dispersion of the distribution. As 'θ' increases the reliability increases at a given point in time. The parameter is also called the characteristic life.

MTTF and variance of the Weibull distribution is given by

$$\text{MTTF} = \theta\Gamma\left(1+\frac{1}{\beta}\right) \text{ and}$$

$$\sigma^2 = \theta^2\left\{\Gamma\left(1+\frac{2}{\beta}\right)-\left[T\left(1+\frac{1}{\beta}\right)\right]^2\right\}$$

where $\Gamma(x)$ is Gamma function and is given by

$$\Gamma(x) = \int_0^{\infty} y^{x-1}.e^{-y}dy \text{ where } y = \left(\frac{t}{\theta}\right)^{\beta}$$