Definition - Functions

1) Failure density function, $f(t)$ — / Probability Density Function

* It can be defined as derivative of cumulative failure distribution function. It can be described as:

$$f(t) = \frac{dF(t)}{dt} = -\frac{dR(t)}{dt} = \frac{dQ(t)}{dt}.$$

where, $f(t) \geq 0$; $\int_0^{\infty} f(t)\,dt = 1$

It indicates the likelihood of failure for any t, and it describes the shape of failure distribution.

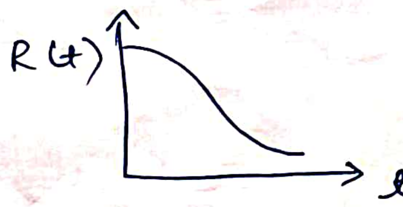
2) Reliability function $F(t)$ —

* It is the probability that an item is functioning at any time t.

Let T = "time" to - failure random variable, reliability at time t is

$$R(t) = P(T \geq t), \quad t \geq 0$$

It is monotonically decreasing.



3) Hazard Rate Function, $\lambda(t)$ —

* The instantaneous rate of failure at time t is known as hazard rate, Mathematically, can be written as,

$$\lambda(t) = \lim_{\Delta t \to 0} \frac{R(t) - R(t+\Delta t)}{R(t)\,\Delta t} = \frac{f(t)}{R(t)}$$

4.) Establish relation: $\lambda(t) = \frac{f(t)}{R(t)}$.

* The instantaneous rate of failure at time t is known as hazard rate denoted by $\lambda(t)$.

Mathematically, consider random variable T, time to failure of component, For time ~~variable~~ interval $[t, t+\Delta t]$,

$$P(t \le T \le t+\Delta t] = F[t+\Delta t) - F(t)$$
$$= [1 - R(t+\Delta t)] - [1-R(t)]$$
$$= R(t) - R(t+\Delta t)$$

Conditional probability of failure, given component has performed its intended function till time t,

$$P[t \le T \le t+\Delta t] = \frac{P[t \le T \le t+\Delta t]}{P(T > t)}$$
$$= \frac{R(t) - R(t+\Delta t)}{R(t)}$$

Short can write from here.

*

Hazard rate is instaneous rate of failure at time 't',

$$\lambda(t) = \lim_{\Delta t \to 0} \frac{R(t) - R(t+\Delta t)}{R(t)\,\Delta t}$$
$$= -\frac{1}{R(t)} \lim_{\Delta t \to 0} \frac{R(t+\Delta t) - R(t)}{\Delta t}$$
$$= -\frac{1}{R(t)} \frac{d}{dt} R(t)$$
$$= \frac{-\frac{d}{dt} R(t)}{R(t)}$$
$$\lambda(t) = \frac{f(t)}{R(t)}$$

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5) Establish the relation: $R(t) = e^{-\int_0^t \lambda(z)\,dz}$

By definition hazard rate is given by

$$\lambda(t) = \frac{f(t)}{R(t)}$$

We can write, $\lambda(t) = -\frac{1}{R(t)} \frac{d}{dt}[R(t)]$

Integrating $\lambda(t)$ w.r.t both sides within limits '$t=0$' to '$t=t$',

$$\int_0^t \lambda(t)\,dt = -\int_0^t \frac{1}{R(t)} \frac{d}{dt}[R(t)]\,dt$$

$$= -\int_0^t \frac{\frac{d}{dt}R(t)}{R(t)}\,dt$$

$$= -\left[\ln|R(t)|\right]_0^t$$

$$= -\left[\ln|R(t)| - \ln|R(0)|\right]$$

$$= -\ln|R(t)| - \ln 1$$

$$= -\ln[R(t)] - 0$$

$$= -\ln[R(t)].$$

$$\Rightarrow -\int_0^t \lambda(t)\,dt = \ln[R(t)]$$

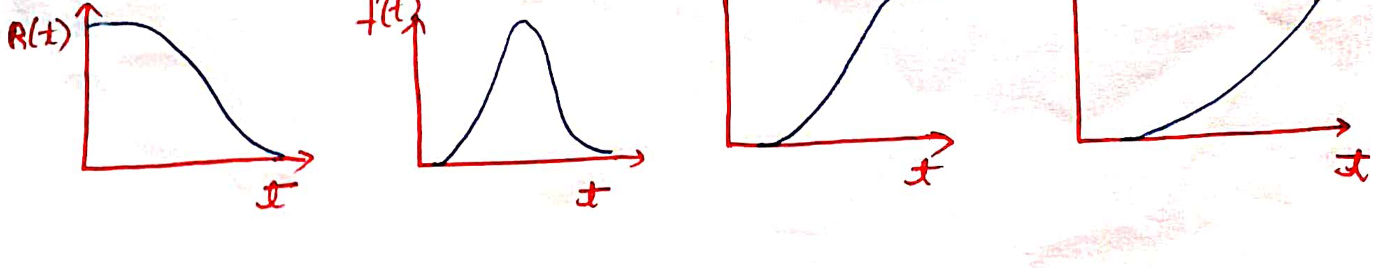
$$\Rightarrow R(t) = e^{\left(-\int_0^t \lambda(t)\,dt\right)}.$$

6) Relationship between $R(t)$, $f(t)$, $F(t)$, $\lambda(t)$

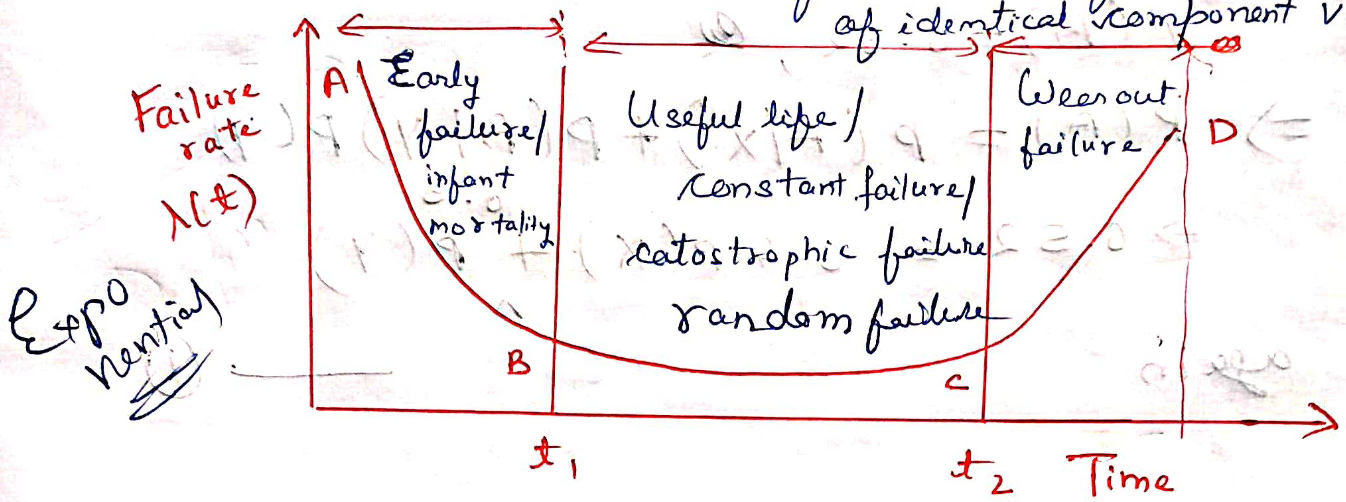
* Table:

	$R(t)$	$F(t)$	$f(t)$	$\lambda(t)$
$R(t)$	—	$1 - F(t)$	$\int_t^{\infty} f(t)\,dt$	$\exp\left[-\int_0^t \lambda(t)\,dt\right]$
$F(t)$	$1 - R(t)$	—	$\int_0^t f(t)\,dt$	$1 - \exp\left(-\int_0^t \lambda(t)\,dt\right)$
$f(t)$	$-\frac{d}{dt} R(t)$	$\frac{d}{dt} F(t)$	—	$\lambda(t) \exp\left(-\int_0^t \lambda(t)\,dt\right)$
$\lambda(t)$	$-\frac{d}{dt}\left(\ln R(t)\right)$	$\dfrac{\frac{d}{dt} F(t)}{1 - F(t)}$	$\dfrac{f(t)}{\int_0^t f(t)\,dt}$	—

Graph:



8) **Bath Tub Curve** – It is a graphical representation of failure rate of entire population of identical component vs time.



a) Phase A – B → Early failure / Infant mortality / Burn-In

'Decreasing failure rate',
Causes → manufacturing defects, welding, soldering, part defects, poor design, misuse, mishandling.

b) Phase B – C – Constant failure / Useful life

Causes → environment, random loads, human error, chance events.

c) Phase C – D → Weak out failure
Failure rate rises due to components reach end of life.
Causes → Corrosion, Fatigue, Aging, Friction, etc.

Significance: It is a model demonstrating the likely failure rate of technologies & products.

9) Differences between

a) MTTF & MTBF

MTTF → Mean-time-to failure is a measure which simply gives a number that tells on average how long the product performs its intended function successfully.

Defn

It is a mean of random variable T in absence of repair & replacement.

Mathematically, can be represented ;as -

$$MTTF = \int_0^{\infty} R(t)\,dt.$$

MTBF → It is a prediction of time that equipment is operating between breakdowns or stopages.

Difference -

MTBF is average time before repairable failure, while MTTF is the average time before total failure.

MTBF can be repaired (components) while components in MTTF cannot be repaired.

MTTR → Time to run a repair after occurrence of failure.