Chap-5

Digital Filters (Part - I)

I) Realization of Digital Linear Systems

An LTI system can be represented in z-domain as

$$x(n) \longrightarrow \boxed{h(n)} \longrightarrow y(n) = x(n) * h(n)$$

$$X(z) \longrightarrow \boxed{H(z)} \longrightarrow Y(z)$$

System function, $H(z) = \dfrac{Y(z)}{X(z)}$

In z domain, functions $H(z)$, $Y(z)$, $X(z)$ are in form of polynomials of 'z' variable

In general system function can be expressed by factorizing the numerator & denominator polynomials -

$$H(z) = \frac{Y(z)}{X(z)} = \frac{G z^{N-M} \sum_{k=0}^{M} b_k z^{-k}}{1 + \sum_{k=1}^{N} a_k z^{-k}}$$

If $N > M$ → system has $(N-M)$ no. of trivial poles at origin. ('z' will be raised to positive integer)

If $N < M$ → System has $(M-N)$ no. of trivial zeroes at origin ('z' will be raised to negative integer)

$|G|$ → gain

For the design of FIR filters, the system function $H(z)$ or the impulse response $h(n)$ must be specified. Then the digital filter structure can be implemented or synthesized in hardware / software form by its difference effec equation obtained directly from $H(z)$ or $h(n)$.

Output of finite order linear time invariant system at time n can be expressed as –

$$y(n) = -\sum_{k=1}^{N} a_k\, y(n-k) + \sum_{k=0}^{M} b_k\, x(n-k)$$

where a_k & b_k are constants with $a_0 \neq 0$ & $M \leq N$.

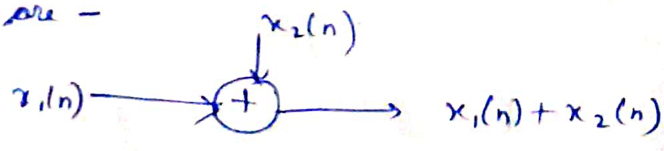
For example, the difference equation of 1st order digital system may be written as

$$y(n) = -a_1\, y(n-1) + x(n) + b_1\, x(n-1)$$

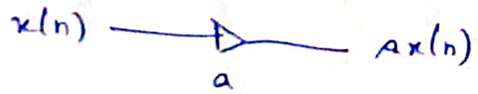
The current o/p $y(n)$ is equal to sum of past outputs, from $y(n-1)$ to $y(n-N)$, scaled by delay-dependent feedback coefficient a_k, plus the sum of future, present and past inputs, which are scaled by delay-dependent feed-forward coefficient b_k.

In order to implement the specified Difference equation of system, required basic operations are –

1) Addition



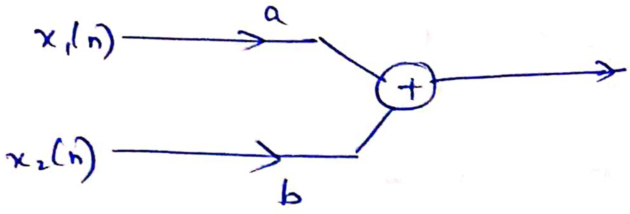
2) Multiplier



3) Unit delay

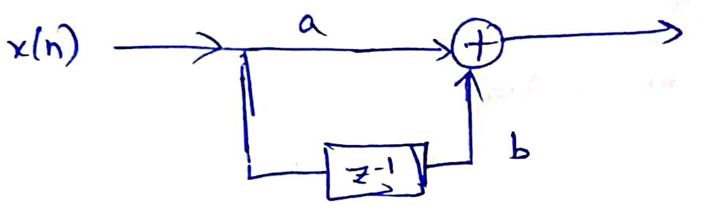
Others:

1) Adder with amplitude gain



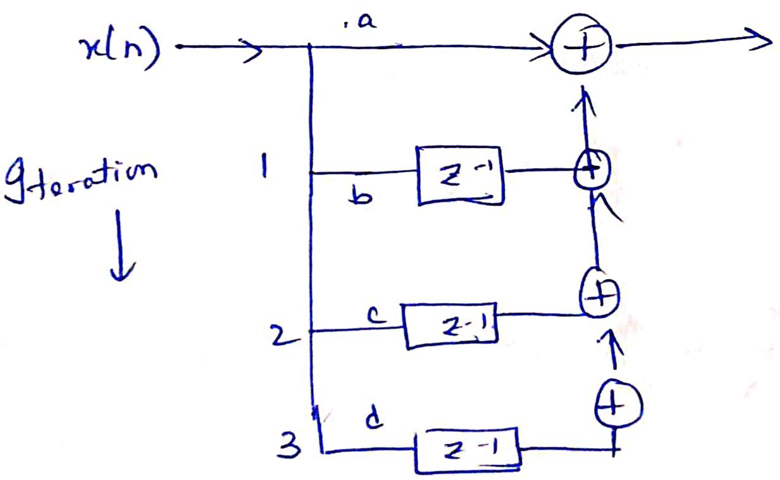
$y(n) = a\,x_1(n) + b\,x_2(n)$

2) Difference adder (Recursive system)



$y(n) = a\,x(n) + b\,x(n-1)$

3) Accumulator:



$y(n) = a\,x(n) + b\,x(n-1) + c\,x(n-2) + d\,x(n-3)$

Basic Structures for IIR Systems

① Direct form Realization of IIR systems

System function of LTI system is given as

$$H(z) = G z^{(N-M)} \frac{\sum_{k=0}^{M} (z - z_k)}{\sum_{k=0}^{N} (z - P_k)}$$

By dissolving the factors into polynomials of z,

$$H(z) = \frac{Y(z)}{X(z)} = G z^{(N-M)} \frac{\sum_{k=0}^{M} b_k z^{-k}}{\left(1 + \sum_{k=1}^{N} a_k z^{-k}\right)}$$

On rewriting the expression

$$Y(z)\left\{1 + \sum_{k=1}^{N} a_k z^{-k}\right\} = X(z)\left\{\sum_{k=0}^{M} b_k z^{-k}\right\}$$

$$Y(z) = -\sum_{k=1}^{N} a_k z^{-k} + \sum_{k=0}^{M} b_k X(z) z^{-k}$$

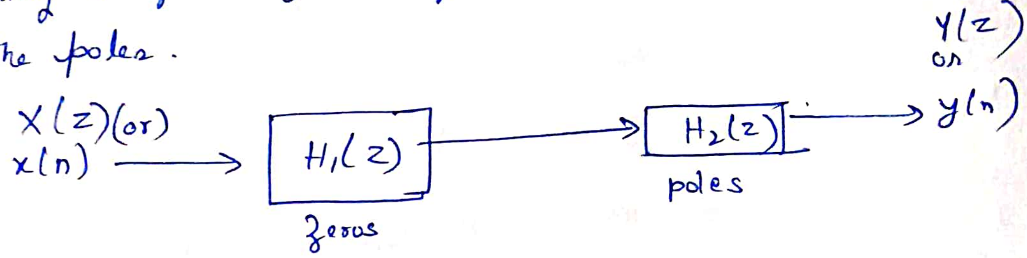
On taking inverse z-transform, we will get the result in time domain as a 'Recursive' Difference equation:

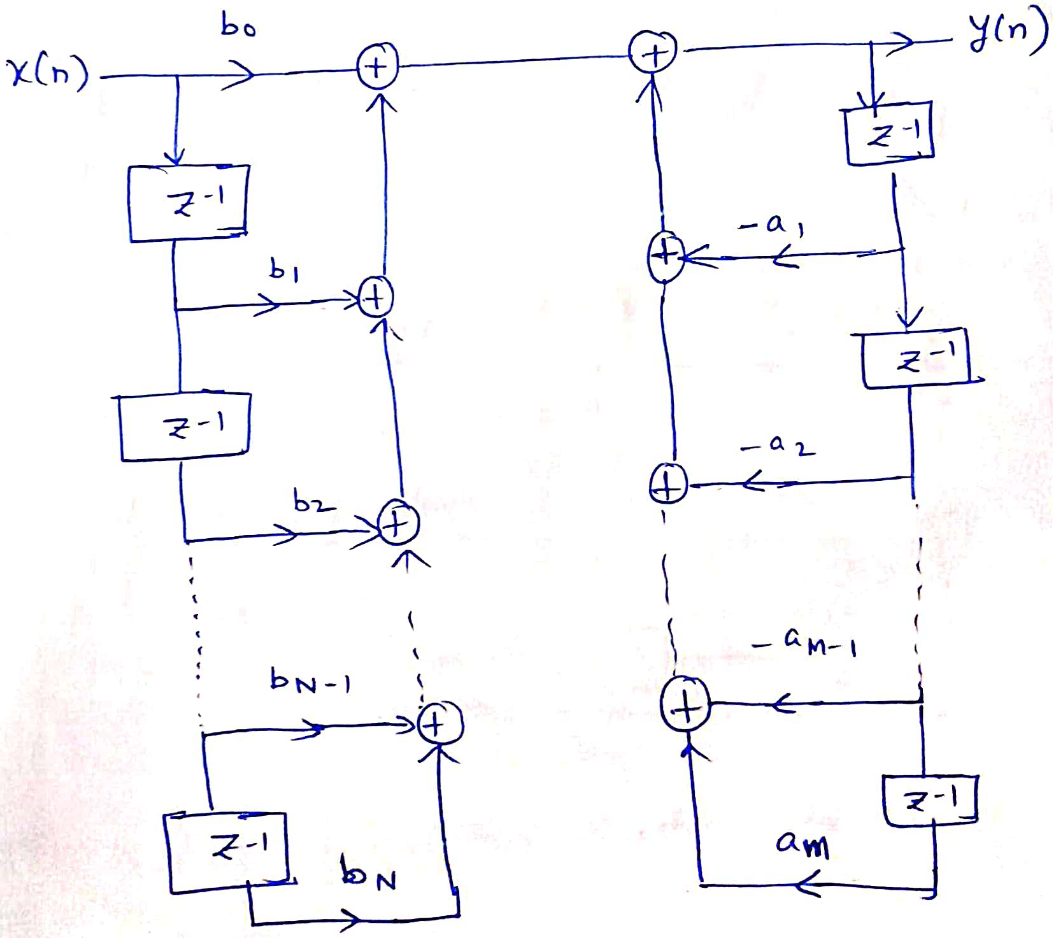
$$y(n) = -\sum_{k=1}^{N} a_k y(n-k) + \sum_{k=0}^{M} b_k x(n-k)$$

Each term z^{-k} becomes time delay $\rightarrow z^{-1} \rightarrow$ Delay blocks

Direct form I :

In this case, the system function is divided into two parts connected in cascade, the first part (~~divided into~~) containing only the zeros, followed by part containing only the poles.





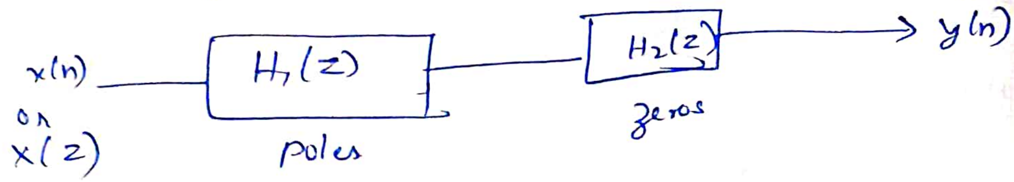
→ N-th order IIR digital transfer function is characterized by $2N+1$ → unique coefficients

$2N$ → two - input adders.

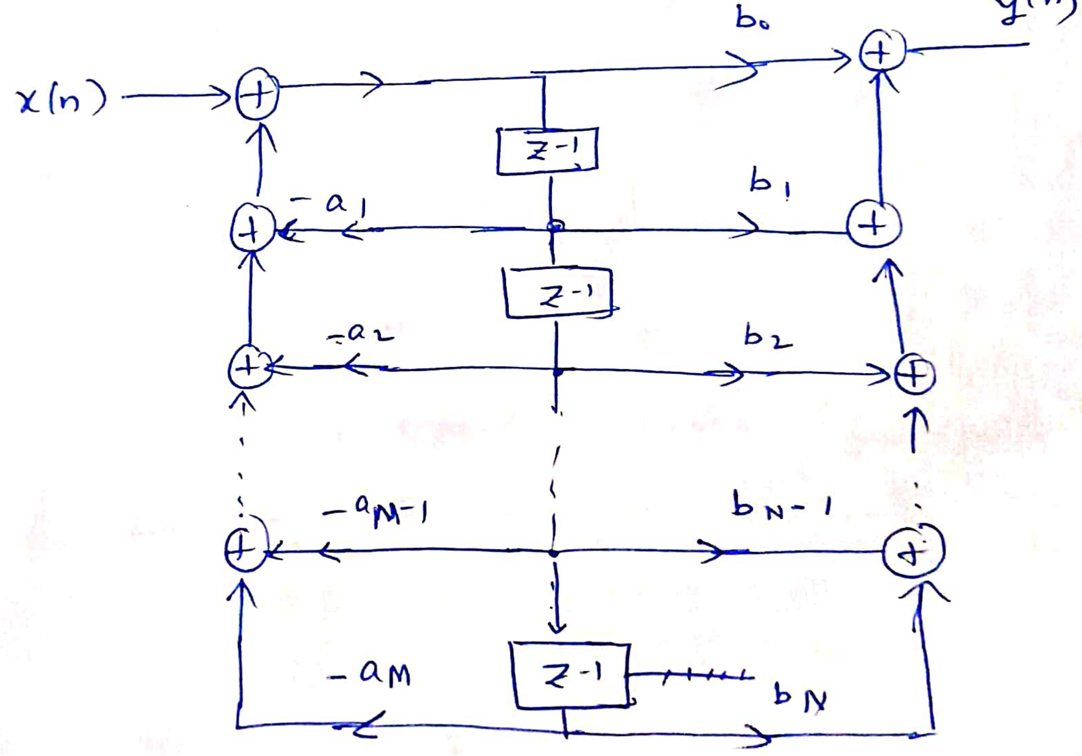
$$H(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + \dots b_N z^{-N}}{1 + a_1 z^{-1} + \dots \dots \dots a_M z^{-M}}$$

Direct form II :

In direct form II, the poles of $H(z)$ are realized first & the zeros second.



Direct form realization with combined common delays



Direct form II ~~use~~ structure has fewer numbers of blocks

Due to reduction in use of unit delay blocks $\boxed{z^{-1}}$

Delay blocks are taken common

Ex:

$$H(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2} + a_3 z^{-3}}$$

$\oplus$ $\oplus$

So write $-a_1$, $-a_2$ & $-a_3$ scalar multipliers in structure.

Hint

& if its

$$H(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{1 - a_1 z^{-1} + a_2 z^{-2}}$$

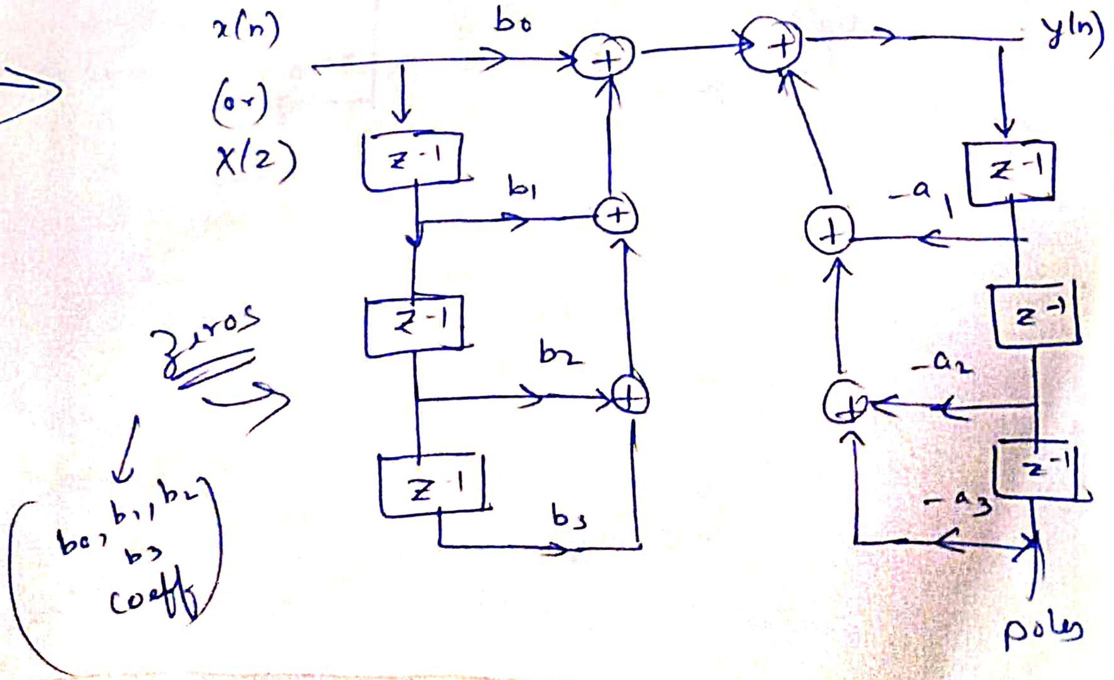
$\ominus$ $\oplus$

Write $+a_1$, $-a_2$ as scalar multipliers

Write opposite in poles (factorization)

Direct form I structure

#



Ex: Determine direct Form I & Form II realizations for a third order IIR transfer functions:

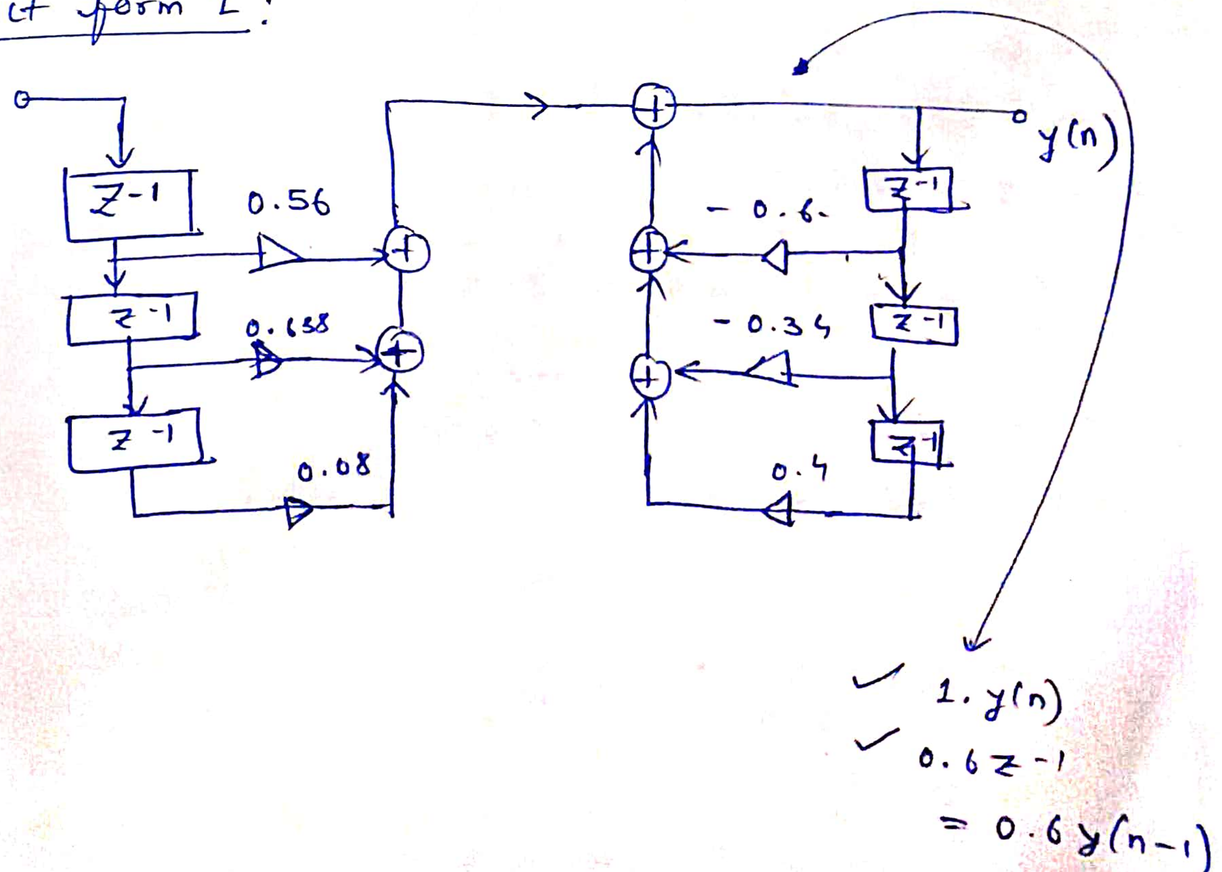
$$H(z) = \frac{0.28z^2 + 0.319z + 0.04}{0.5z^3 + 0.3z^2 + 0.17z - 0.2}$$

Sol) ① Convert into z^{-1} form.

② Multiply the transfer function numerator & denominator by $2z^{-3}$, we obtain the standard form of T.F

$$H(z) = \frac{0.56z^{-1} + 0.638z^{-2} + 0.08z^{-3}}{1 + 0.6z^{-1} + 0.34z^{-2} - 0.4z^{-3}}$$

Direct form I:



Previous Years (Solved) 2019

Ex 1) Evaluate the stability of an LTI system described by the system function H(z) given below & form II implementation of given system –

$$H(z) = \frac{1.2\,(z+1.5)(z-0.6)}{z(z-0.2)(z+0.3)(z+0.7)(z-0.5)}$$

Sol:

$$H(z) = 1.2\,\frac{z^2+0.9z-0.9}{z^5+0.3z^4-0.41z^3-0.037z^2+0.021z}$$

$$H(z) = 1.2\,\frac{z^2\{1+0.9z^{-1}-0.9z^{-2}\}}{z^5\{1+0.3z^{-1}-0.41z^{-2}-0.037z^{-3}+0.021z^{-4}}$$

$$= 1.2\,z^{(\cancel{5}+2)}\,\frac{\{1+0.9z^{-1}-0.9z^{-2}\}}{\{1+0.3z^{-1}-0.41z^{-2}-0.037z^{-3}+0.021z^{-4}}$$

Here, On comparing with T.F

$$H(z) = G\,z^{N-M}\,\frac{\sum_{k=0}^{M} b_k z^{-k}}{1+\sum_{k=1}^{N} a_k z^{-k}}$$

Here $N > M$, → system has $N-M$ no of trivial poles at origin.

∴ 3 ~~po~~ trivial poles at origin.

→ ∴ Overall gain = $G = 1.2$

5 Trivial poles & 2 Trivial zeros at origin.

In this, 2 poles cancel two of zeros & only 3 ~~p~~ trivial pole remains at origin

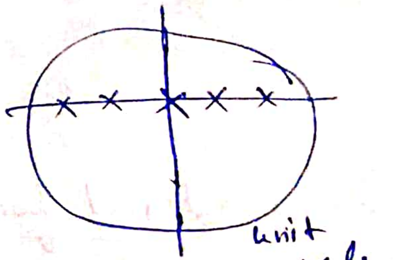
→ <u>ROC</u>: Region of space outside the circle of radius 0.021, which is the smallest coefficient in either of numerator or denominator polynomials.

∴ ROC is area outside circle of radius 0.021.

Poles : 0, 0.2, −0.3, −0.7, 0.5 (All less than unity 1)

Condition

All the poles of the system function are within the unit circle.



→ Hence, the system is stable.

$$H(z) = 1.2 \, \frac{\{1 + 0.9z^{-1} - 0.9z^{-2}\}}{1 + 0.3z^{-1} - 0.41z^{-2} - 0.037z^{-3} + 0.021z^{-4}}$$

(Neglecting Trivial poles)

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1.2 + 1.08z^{-1} - 1.08z^{-2}}{1 + 0.3z^{-1} - 0.41z^{-2} - 0.037z^{-3} + 0.021z^{-4}}$$

$$Y(z)\left[1+0.3z^{-1}-0.41z^{-2}-0.037z^{-3}+0.021z^{-4}\right] = 1.2X(z)+1.08z^{-1}X(z) - 1.08z^{-2}X(z)$$

$$Y(z) = -0.3z^{-1}+0.41z^{-2}+0.037z^{-3}-0.021z^{-4} + 1.2X(z)+1.08z^{-1}X(z)-1.08z^{-2}X(z)$$

Taking Inverse z transform, we obtain "Constant Coeff Linear Difference equation" in time domain,

$$y(n) = -0.3y(n-1)+0.41y(n-2)+0.037y(n-3) - 0.021y(n-4)+1.2x(n)+1.08x(n-1) - 1.08z^{-2}x(n-2)$$

Direct form II :