Random Variable

To study about a system's behavior for the application of probability theory to reliability evaluation, a series of experiments must be performed or a data collection scheme should be deduced.
To apply the probability theory to occurrence of these values or events which are random in nature, we need to study these variables called as Random Variables.

∴ Random variable is a variable quantity which denotes the result or outcome of a given random experiment.

A random variable is one that can have only a discrete number of states or countable values.
A random variable can be either "discrete" or "continuous".

A discrete random variable is one that can have only a discrete number of states or countable values.
Ex: 1.Tossing a coin - Outcomes are heads or tails.
2. Rolling a dice - Outcomes are 1,2,3,4,5 or 6.

A continuous random variable is one which takes an infinite number of values or if its range forms a continuous set of real numbers. This does not mean that the range extends from $-\infty$ to $+\infty$. It only means that there are infinite number of possibilities of the value.

Ex: 1. The life time of a light bulb.
2. If electric current have values between 5A and 10 A, then it indicates a continuous random
variable.

Probability Density Function

The probabilities associated with the random variables can be described by a formula called Probability density function or Probability mass function.

We use the notation f(x) for the probability density function.

Ex : 1. Consider the throw of a dice
Let the random variable associated with the outcome be 'X'.
The value of X are 1, 2, 3, 4, 5 and 6.
$f(1) = P(x=1) = 1/6$
$f(2) = P(x=2) = 1/6$
$\therefore f(1) = f(2) = \ldots\ldots\ldots = f(6) = 1/6$
$f(x) = 1/6$ = Constant density function.

Probability Distribution Function

If 'x' is a random variable, then for any real number x, the probability that 'x' will assume a value less than or equal to x is called Probability distribution functions.
It is indicated as F(x)
f(x) = P(x)
F(x) = P(X≤)

Ex: Consider the rolling of a single dice.

f(x) = 1/6
f(1) = f(2) = ……….f(6) = 1/6
F(1) = P (X≤1) = f(1) = 1/6
F(2) = P (X≤2) = f(1) + f(2)
= 1/6 + 1/6 = 2/6
F(3) = P (X≤3) = f(1) + f(2) + f(3)
= 1/6 + 1/6 + 1/6 = 3/6
F(4) = 4/6
F(5) = 5/6
F(6) = 6/6 = 1

Suppose a random variable X has the following density function.

X	0	1	2	3	4	5
f(x)	1/32	5/32	10/32	10/32	5/32	1/32

Then, the Probability distribution function is given by

X	0	1	2	3	4	5
F(x)	1/32	1/32 + 5/32 =6/32	6/32 + 10/32 =16/32	16/32 + 10/32 =26/32	26/32 + 5/32 =31/32	31/32 + 1/32 =32/32

Relation between Probability density function and distribution function:

F(x) = $\sum f(x)$ (Discrete Random variable)

F(x) = $\int f(x)dx$ (Continuous Random variable)

A random variable 'x' and the corresponding distribution function F(x) are said to be continuous if the following condition is satisfied for all 'x'.

$$f(x) = \frac{d}{dx} f(x)$$

Mathematical Expectation:

It is useful to describe the random behavior of a system by one or more parameters rather than as a distribution. This is particularly useful in the case of system reliability evaluation.

This parametric description can be achieved using numbers known mathematically as moments of distribution.

The most important of these moments is the expected value, which is also referred to as average mean value.

Mathematically it is the first moment of the distribution.

Consider a Probability model with outcome x1, x2, x3……xn and the probability of each is P1, P2, P3…………Pn. then the expected value of the variable is

$$E(x) = P_1x_1 + P_2X_2 + P_3X_3 + \ldots\ldots\ldots P_nX_n = \sum_{i=1}^{n} x_1 P_1$$

Expected value E(x) of a discrete random variable x having 'n' outcomes xi each with a probability of occurrence Pi is

$$E(x) = \sum_{i=1}^{n} x_i P_i \text{ where } \sum_{i}^{n} P_1 = 1$$

In case of continuous random variable, the equation can be modified from the summation to integration.

$$E(x) = \int x\, f(x) dx$$

Expected value is the weighted mean of the possible value using their Probability of occurrence as the weighing factor.

Variance and Standard Deviation

The expected value is the most important distribution parameters in reliability evaluation. But to know the amount of 'spread' or 'dispersion' of a distribution, the second moment of distribution. i.e., variane V(x) should be deduced.

The variane of a random variable 'x' is defined as the expectation of the square of deviation of 'x' from E(x).

$$m = E(x) = \int x\, f(x) dx$$

$$\text{Variance} = \sigma^2 = \int (x - m)^2\; f(x) dx$$

The quantity ' ' is called standard Deviation.

The Kth moment of a random variable 'x' about its expectation is defined as

$$M_k = E[x - E(x)]^K .$$

The second moment of distribution is known as variance V(x) (K=2)

V(x) = E[x – E(x)]2
= E[x2 – 2x E(x) + E2(x)]
= E(x2) – E(2x E(x)) + E [E2(x)]
= E(x2) – 2 E(x) E(x) + E2(x)
= E(x2) – 2 E2(x) + E2(x)
= E(x2) - E2(x)
$= \sum_{i=1}^{n} x_i^2 P_i - E^2(x)$

Properties of the binomial distribution
The binomial distribution can be represented by the general expression: +
For the expression to be applicable, four specific conditions are required. These are:
(a) There must be a fixed number of trials, i.e. n is known
(b) Each trial must result in either a success or a failure, i.e., only two outcomes are possible and p + q = 1.
(c) All trials must have identical probabilities of success and therefore of failure, i.e., the values of p and q remain constant, and
(d) All trials must be independent (this property follows from (c) since the probabilities of success in trial i must be constant and not affected by the outcome of trials 1, 2, ..., (i-1)).

In order to apply the binomial distribution and to evaluate the outcomes and their probability of occurrence of a given experiment or set of trials, the expression $(p+q)^n$ must be expanded into the form of equations and

$$(p+q)^n = p^n + np^{n-1}q + \frac{n(n-1)}{2!}p^{n-2}q^2 + \dots + \frac{n(n-1)\dots(n-r+1)}{r!}p^{n-r}q^r + \dots + q^n$$

If equation is compared with, it is seen that the coefficient of the (r+1)th term in the binomial expansion represents the number of ways, i.e., combinations, in which exactly r failures and therefore (n-r) successes can occur in n trials and is equal to nCr. Therefore each coefficient in equation can be directly evaluated from the definition of nCr as discussed and the probability of exactly r successes or (n-r) failures in n trails can be evaluated from

$$p_r = \frac{n!}{r!\,(n-r)!} p^r q^{n-r}$$

$$= {}_nC_r p^r q^{n-r}$$

$$= {}_nC_r p^r (1-p)^{n-r}$$

Substituting of equations gives

$$(p+q)^n = \sum_{r=0}^{n} c_r p^r q^{n-r} = 1$$