Subject-Reliability Engineering
B.E 8^{th} Semester

BASIC PROBABILITY THEORY

Basic Concepts on Probability

Probability means whether a certain event has a good chance of occurring or not.
Its value lies between 0 and 1.

Rules for combining probabilities

1) *Independent Events*:
Two events are said to be independent if the occurrence of one event does not affect the probability of occurrence of the other event.
Example: Throwing a dice and tossing coin are independent events.

2) Mutually exclusive events:
Two events are said to be mutually exclusive or disjoint if they cannot happen at the same time.
Example: (i) When throwing a single die, the events 1, 2, 3, 4, 5 and 6 spots are all mutually exclusive because two or more cannot occur simultaneously
(ii) Similarly success and failure of a device are mutually exclusive events since they cannot occur simultaneously.

3) Complimentary Events:
Two outcomes of an event are said to be complementary, if when one outcome does not occur, the other must occur.
If the outcomes A & B have probabilities P(A) and P(B), then

$P(A) + P(B) = 1$ $\qquad$ $P(B) = P(\bar{A})$

Example: When tossing a coin, the outcomes head and tail are complementary since

$P(head) + P(tail) = 1$ or
$P(head) = P(\overline{tail})$
$P(tail) = P(\overline{head})$

Therefore we can say that two events that are complementary events are mutually exclusive also. But the converse is not necessarily true i.e, two mutually exclusive events are not necessarily complementary.

4) *Conditional Events*;
Conditional events are events which occur conditionally on the occurrence of another event or events. Consider two events A & B and also consider the probability of event A occurring under the Consider two events A & B and also consider the probability of event A occurring under the condition that event B has occurred.

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

5) Simultaneous occurrence of events:
Occurrence of both A & B - Mathematically it is represented as A∩B, A AND B, AB.

Case (i) Independent, then the probability of occurrence of each event is not influenced by the probability of occurrence of the other.
P(A/B) = P(A)
P(A∩B) = P(A) – P(B)
And P(B/A)- = P(B)

Case (ii) Events are dependent
If two events are not independent, then the probability of occurrence of one event is influenced by the probability of occurrence of the other
Therefore,
P(A∩B) = P(B/A).P(A)
= (PA/B).P(B)

Numerical Problem - 1
An engineer selects two components A & B. The probability that component A is good is 0.9 & the probability that component B is good is 0.95.What is the probability of both components being good.
P(A good ∩ B good) = P(A good) (B good)
= 0.9 x 0.95 = 0.80

Numerical Problem - 2
One card is drawn from a standard pack of 52 playing cards. Let A be the event that it is a red card and B be the event that it is a face card. What is the probability that both A & B occur.
P(A) = 26/52
Given that 'A' has occurred
Then the sample space for B is 26 states, out of which 6 are those of a face card.
Therefore, P(B/A = 6/26
P(A∩B) = 6/26 x 26/52 = 6/52
P(A∩B) = P(B/A) P(A)

6) *Occurrence of at least one of two events*:
The occurrence of atleast one of two events A and B is the occurrence of A or B or BOTH. Mathematically it is the union of the two events and is expressed as (AUB), (A or B) or (A x B)
Case (i) – Events are independent but not mutually exclusive.

P(AUB) = P(A OR B OR BOTH A AND B)
= 1 – P(NOT A AND NOT B)
$= 1 - P(\overline{A} \cap \overline{B})$
$= 1 - P(\overline{A}) . P(\overline{B})$
= 1 – (1-P(A)) (1-P(B))
= P(A) + P(B) – P(A).P(B)

Using Venn diagram
P(AUB) = P(A) + P(B) – P(A∩B)
If P(A) = 0.9 and P(B) = 0.95
P(AUB) = P(A) + P(B) – P(A).P(B)
= 0.9 + 0.95 – 0.9 X 0.95 = 0.995

Case (ii) – Events are independent and mutually exclusive In the case of events A & B being mutually exclusive, then the probability of their simultaneous occurrence P(A).P(B) must be zero by definition.
P(AUB) = P(A) + P(B)
Case (iii) – Events are not independent
If two events are not independent then
P(AUB) = P(A) + P(B) – P(A∩B)
= P(A) + P(B) – P(B/A).P(A)
= P(A) + P(B) – P(A/B).P(B)

Numerical Problem - 3
A cinema hall gets electric power from a generator run by a diesel engine. On any give day, the probability that the generator is down (event A) is 0.025 and the probability that the diesel engine is done (event B) is 0.04. What is the probability that the cinema house will have power on any given day.
Assume that the occurrence of events A & B are independent of each other.
Probability that the
Cinema hall does not have power given by the probability of the event that either the diesel engine or generator is down.
Q = Pr(AUB) = P(A) + P(B) – P(A) P(B)
= 0.025 + 0.04 – 0.025 x 0.04 = 0.064
Therefore, the probability that the cinema house have power
= R = 1- 0.064 = 0.936