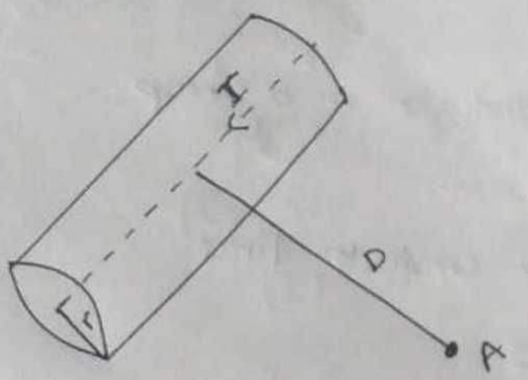
Transmission Line parameter

① inductance of single wire

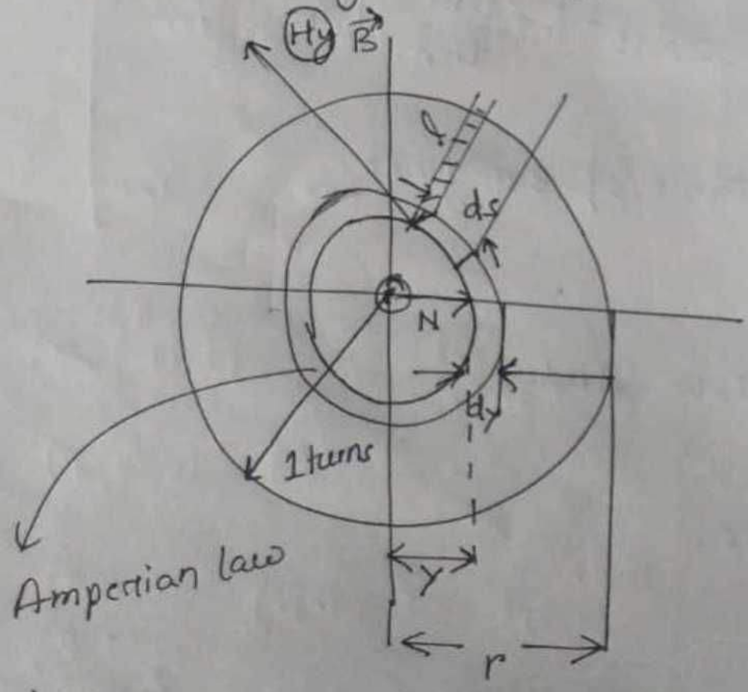


- The inductance due to flux linkages inside the conductor is called as Internal inductance.

  ⇒ Ignore skin effect

  ↳ then current is uniformly distributed

current Density $J = I/\pi r^2$



inductance ⇒ flux linkage ⇒ flux ⇒ magnetic field ⇒ ampere's law.

⇒ current enclosed in amperian loop $= J(\pi y^2)$

$= I(y/r)^2$

From ampere's law

$$\oint \vec{H}\cdot\vec{dl} = I_{enc}$$

$$\Rightarrow H\cdot 2\pi y = \frac{I y^2}{r^2}$$

$$\Rightarrow H = \frac{I y}{2\pi r^2} \longrightarrow \text{magnetic flux intensity 'H'}$$

$$\Rightarrow B = \mu H \longrightarrow \text{flux density.}$$

$$= \frac{\mu I y}{2\pi r^2}$$

$$d\phi = B\,ds$$

$$d\phi = \frac{\mu I l y\,dy}{2\pi r^2}$$

Assume that conductor represents 1 turn.

$d\psi = N d\phi$, N = no. of turns.

if r radius = 1 turns

y radius = $\frac{1}{\pi r^2} \times \pi y^2 = (y/r)^2$ ← hypothetical

$$d\psi = N d\phi$$

$$d\psi = \frac{\mu I l y^3 dy}{2\pi r^4}$$

$$\psi = \int_0^r \frac{\mu I l}{2\pi r^4} y^3 dy$$

$$= \frac{\mu I l}{2\pi r^4}\left[\frac{y^4}{4}\right]_0^r$$

$$= \frac{\mu I l}{8\pi}$$

$$\boxed{\frac{\psi}{I} = \frac{\mu l}{8\pi}} = \text{internal inductance}$$

$$\Rightarrow L = \frac{\mu l}{8\pi} \quad H$$

$\mu \approx \mu_0$

$$L/l = \frac{\mu_0}{8\pi} \quad H/m$$

$$L_{int} = \frac{\mu_0}{8\pi} \quad H/m$$

$$= 0.05 \times 10^{-6} \; H/m$$

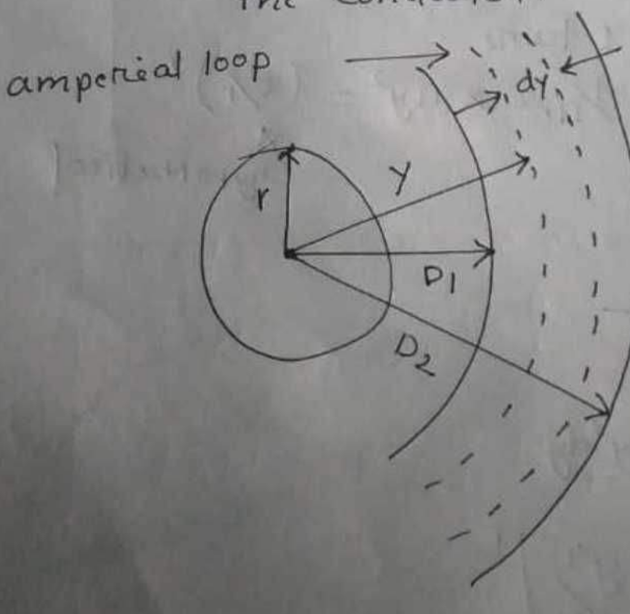
$$= \frac{0.05 \times 10^{-3} \; H}{10^3 \; m}.$$

$$= 0.05 \; mH/Km.$$

↓

Independent of r

<u>External inductance</u>: External Inductance is due to flux linkages outside the conductor because magnetic field also exists outside the conductor.



- Current enclosed = I
- By Ampere's law

$$\oint \vec{H} \cdot \vec{dl} = I_{enc}$$

$$H \times 2\pi y = I$$

$$H = \frac{I}{2\pi y}$$

$$B = \mu_0 H$$

$$= \frac{\mu_0 I}{2\pi y}$$

$$d\phi = B\,ds = \frac{\mu_0 I}{2\pi y}\, l\,dy$$

$$d\phi = \frac{\mu_0 I l\,dy}{2\pi y}$$

∴ no. of turns inside amperian loop = 1

∴ $d\psi = N\,d\phi = d\phi$

$$d\psi = \frac{\mu_0 I l\,dy}{2\pi y}$$

∴ $$\psi = \frac{\mu_0 I l}{2\pi} \int_{D_1}^{D_2} \frac{1}{y}\,dy$$

$$= \frac{\mu_0 I l}{2\pi} \ln D_2/D_1$$

if ψ is determined b/w r & D

$r = D_1$, $D = D_2$

$$\psi = \frac{\mu_0 I l}{2\pi} \ln \frac{D}{r}$$

$\frac{\psi}{I}$ = external inductance

$= \mu_0 l/2\pi \ln D/r$ H

$$\boxed{L_{ext} = \frac{\mu_0}{2\pi} \ln \frac{D}{r} \text{ H/m}}$$

(5)

$\Rightarrow$ Total inductance

$$L = L_{internal} + L_{external}$$

$$= \frac{\mu_0}{8\pi} + \frac{\mu_0}{2\pi} \ln \frac{D}{r}$$

$$= \frac{\mu_0}{2\pi}\left[\frac{1}{4} + \ln \frac{D}{r}\right]$$

$$= \frac{\mu_0}{2\pi}\left[\ln e^{1/4} + \ln \frac{D}{r}\right]$$

$$= \frac{\mu_0}{2\pi} \ln \frac{De^{1/4}}{r}$$

$$\boxed{L = \frac{\mu_0}{2\pi} \ln \frac{D}{re^{-1/4}}}$$

$$= \frac{\mu_0}{2\pi} \ln \frac{D}{r'}$$

$$r' = re^{-1/4} = 0.7788r$$

$\hookrightarrow$ geometric mean radius (GMR)

$$L_{int} = \mu_0/8\pi = 0.05\ mH/km$$

$$L_{ext} = \mu_0/2\pi \ \ln \frac{D}{r} = 0.2 \ln D/r \ \ mH/km$$

$$L_{Total} = \frac{\mu_0}{2\pi} \ln \frac{D}{r'}$$